

Two Unit System with Two Types of Failures and Repairs under the Effect of CF and CCF

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Abstract. Present paper studies a two unit cold standby system with software and hardware failure under the effect of CF and CCF. It considers two repairman, the internal who looks after the software malfunctioning and the external repairman who is called only for hardware repairs. Supplementary variable technique has been used to obtain expression for various reliability characteristics of the system.

Introduction

The importance of the concept of circumstantial failure (CF) in reliability evaluation was introduced by Pandey and Kalra¹. The role of CF becomes more meaningful in electronic systems wherein common cause failure (CCF) is less dominant than the CF.

The paper discusses the stochastic behaviour of a two unit cold standby system that may suffer failure due to malfunctioning of the software, in addition to the usual hardware failure and / or the effect of CF and CCF. Two types of repair have also been considered, that is external and internal. Failures caused due to software malfunctioning are taken up by the internal repair facility and the external repair facility is called only for hardware failure or in the situation when internal repair is not able to settle down software problems.

The system under consideration consists of two units out of which one operates and the other works as a cold standby. In the system state transition diagram given in Fig. 1, state S_0 represents the state where both the units are in perfect conditions. If the operative unit suffers a hardware failure it transits to state S_1 , wherein the cold standby takes over as operative unit and an external repair is used to repair the failed unit. State S_2 is the outcome of the transition from S_0 upon software failure. In case external repair is needed for software malfunctioning, the system transits from S_2 to S_1 . States S_C and S_{CC} represent the failed states due to CF and CCF respectively whereas S_1 is the failed state wherein both the units become non-operative. Supplementary variable technique is used to obtain various reliability characteristics of interest.

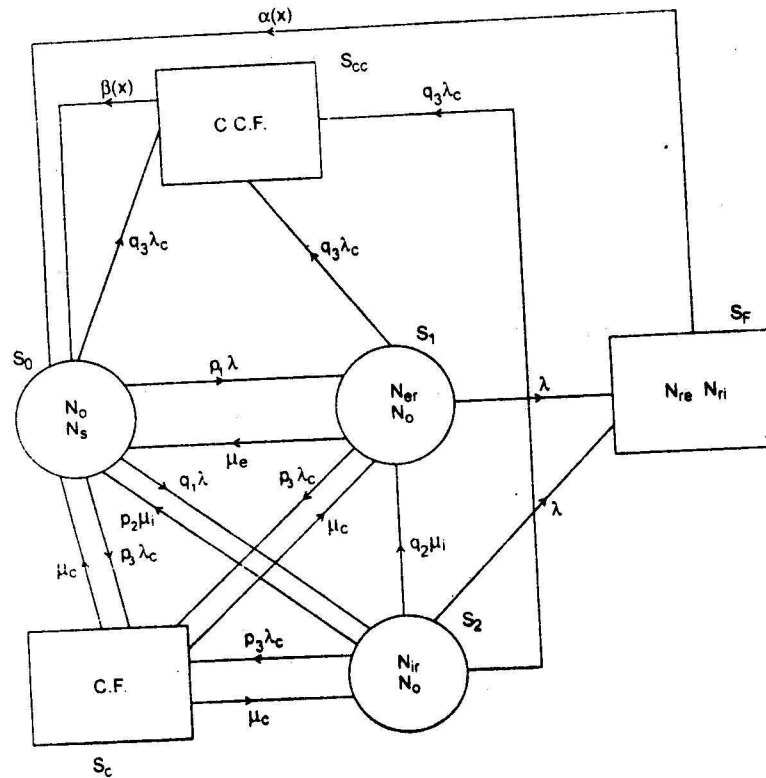


Fig.1. System State Transition Diagram

Assumptions

- (1) It is assumed that the external repairman will be called only for hardware failures and any sort of software malfunctioning shall be looked after by internal staff. External repairman will be called for software problems only when the internal staff fails to repair it.
- (2) However in case a failure of either type (hardware/software) occurs when the external/internal repairman is already busy with some repair, then the latest failed unit would be taken up by any of the two repairman who is free irrespective of type of failure.
- (3) Failure and repair rates are assumed to be constant except from the completely failed state and common cause failure state. In these cases repair distribution is taken to be general.
- (4) The repairs are assumed to be as good as new.

Notations

- N_o/N_s : Operating/cold standby unit
 $P_j(t)$: Probability that the system is in state j at time t where $j = 0, 1, 2, C$
 $P_j(x, t)$: Probability density (w.r. to repair time) that the failed system is in state j and has an elapsed time x ($j = F, CC$)
 $p_1\lambda$: Constant rate of hardware failure
 $q_1\lambda$: Constant rate of software malfunctioning.
 λ_c : Constant failure rate due to common cause or circumstantial failures.
 $p_2 (= 1 - q_2)$: Probability that the internal staff is able to repair.
 $p_3 (= 1 - q_3)$: Probability that failure is circumstantial.
 $\mu_i/\mu_e/\mu_c$: Constant repair rate for internal/external/CF repairs
 $\alpha(x)/\beta(x)$: General repair distribution from state S_F/S_{CC}

$$S_k(s) : \int k(x) \exp \left[-sx - \int_0^x k(x) dx \right] dx, \quad k = \frac{\alpha}{\beta}$$

Mathematical formulation of the model : Here we use the supplementary variable technique for the mathematical formulation of the given model. The state space diagram of the model is given in Fig. 1 and the differential difference equations with boundary conditions are given below.

$$(1) \quad \frac{dP_0(t)}{dt} + (\lambda + \lambda_c)P_0(t) = \mu_c P_1(t) + p_2 \mu_i P_2(t) + \mu_c P_c(t) + \int \alpha(x) P_F(x, t) dx + \int \beta(x) P_{CC}(x, t) dx$$

$$(2) \quad \frac{dP_1(t)}{dt} + (\mu_e + \lambda + \lambda_c)P_1(t) = p_1 \lambda P_0(t) + q_2 \mu_i P_2(t) + \mu_c P_c(t)$$

$$(3) \quad \frac{dP_2(t)}{dt} + (\mu_i + \lambda + \lambda_c)P_2(t) = q_1 \lambda P_0(t) + \mu_c P_c(t)$$

$$(4) \quad \frac{dP_c(t)}{dt} + (3\mu_c)P_c(t) = p_3 \lambda_c [P_0(t) + P_1(t) + P_2(t)]$$

$$(5) \quad \left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \alpha(x) \right] P_F(x, t) = 0$$

$$(6) \quad \left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta(x) \right] P_{CC}(x, t) = 0$$

We have also assumed that initially at $t = 0$ system operates normally *i.e.* system is in the state S_0 , so we have the following initial condotions.

$$P_0(0) = 1, P_1(0) = P_2(0) = P_C(0) = P_F(0) = P_{CC}(0) = 0$$

Boundary conditions to solve the equations (5) and (6) may be taken as :

$$(7) \quad \begin{cases} P_F(0, t) = \lambda [P_1(t) + P_2(t)] \\ P_{CC}(0, t) = q_3 \lambda_c [P_0(t) + P_1(t) + P_2(t)] \end{cases}$$

Soloution of the Model. We use the Laplace transform of the above differential equations and the boundary conditions. Therefore, the above differential equations can be reduced to the following system of algebraic equations.

$$(s + \lambda + \lambda_c)P_0(s) = 1 + \mu_e P_1(s) + p_2 \mu_i P_2(s) + \mu_c P_C(s) + \int \alpha(x) P_F(x, s) dx + \int \beta(x) P_{CC}(x, s) dx$$

$$(s + \lambda + \lambda_c + \mu_e)P_1(s) = p_1 \lambda P_0(s) + q_2 \mu_i P_2(s) + \mu_c P_C(s)$$

$$(s + \lambda + \lambda_c + \mu_i)P_2(s) = q_1 \lambda P_0(s) + \mu_c P_C(s)$$

$$(s + 3\mu_c)P_C(s) = p_3 \lambda_c [P_0(s) + P_1(s) + P_2(s)]$$

$$\left(s + \frac{\partial}{\partial x} + \alpha(x) \right) P_F(x, s) = 0$$

$$\left(s + \frac{\partial}{\partial x} + \beta(x) \right) P_{CC}(x, s) = 0$$

$$P_F(0, s) = \lambda [P_1(s) + P_2(s)]$$

$$P_{CC}(s) = q_3 \lambda_c [P_0(s) + P_1(s) + P_2(s)]$$

Solving above equations for $P_j(s)$, $j = 0, 1, 2, C, F, CC$, we get the following Laplace transforms of state probabilities :

$$(8) \quad P_0(s) = \frac{1}{K(s)}$$

$$(9) \quad P_1(s) = K_1(s) P_0(s)$$

$$(10) \quad P_2(s) = K_2(s) P_0(s)$$

$$(11) \quad P_C(s) = K_3 P_0(s)$$

$$(12) \quad P_F(s) = \frac{\lambda}{s} [K_1 + K_2] P_0(s) (1 - S_\alpha(s))$$

$$(13) \quad P_{CC}(s) = \frac{q_3 \lambda_c}{s} [1 + K_1 + K_2] [1 - S_\beta(s)] P_0(s)$$

where

$$K(s) = (s + I_1) - \mu_e K_1(s) - p_2 \mu_i K_2(s) - \mu_c K_3(s) \\ - \lambda [K_1(s) + K_2(s)] S_\alpha(s) - q_3 \lambda_c [1 + K_1(s) + K_2(s)] S_\beta(s)$$

$$K_1(s) = \frac{[p_1 \lambda (s + 3\mu_c)(s + I_3) + p_3 \lambda_c \mu_c (s + I_3 + q_2 \mu_i + q_1 \lambda - p \lambda) + q_1 \lambda (s + 3\mu_c) q_2 \mu_i]}{[(s + I_2)(s + I_3)(s + 3\mu_c) - p_3 \mu_c \lambda_c (2s + I_2 + I_3 + q_2 \mu_i)]}$$

$$K_2(s) = \frac{[(s + I_2)(s + 3\mu_c) q_1 \lambda + (s + I_2 + p_1 \lambda - q_1 \lambda) q_3 \lambda_c \mu_c]}{[(s + I_2)(s + I_3)(s + 3\mu_c) - p_3 \mu_c \lambda_c (2s + I_2 + I_3 + q_2 \mu_i)]}$$

and

$$K_3 = \frac{p_3 \lambda_c}{(s + 3\mu_c)} [1 + K_1 + K_2]$$

$$I_1 = \lambda + \lambda_c$$

$$I_2 = \lambda + \lambda_c + \mu_e$$

$$I_3 = \lambda + \lambda_c + \mu_i$$

Laplace transforms of *up* and *down* state probabilities may be written as :

$$P_{up}(s) = P_0(s) + P_1(s) + P_2(s) \\ = \frac{[1 + K_1(s) + K_2(s)]}{K(s)}$$

$$P_{down}(s) = P_F(s) + P_C(s) + P_{CC}(s) \\ = \frac{\lambda}{s} [K_1 + K_2] P_0(s) \cdot (1 - S_\alpha) + K_3 P_0(s) + \frac{q_3 \lambda_c}{s} [1 + K_1 + K_2] P_0(s) (1 - S_\beta) \\ = \frac{\left[\frac{\lambda}{s} (K_1 + K_2) (1 - S_\alpha) + K_3 + \frac{q_3 \lambda_c}{s} (1 + K_1 + K_2) (1 - S_\beta) \right]}{K(s)}$$

To verify our model we can get very easily that

$$P_{\text{up}} + P_{\text{down}} = \frac{1}{s}$$

Availability Analysis. For availability analysis particularly it may be assumed that the repairs follow exponential distribution, i.e.

$$S_{\alpha}(s) = \frac{\alpha}{s + \alpha}, \quad S_{\beta}(s) = \frac{\beta}{s + \beta}$$

Taking $\alpha = \beta = \phi$ and using the above equation. The Laplace transform of state probabilities may be written as follows :

$$(15) \quad \bar{P}_0(s) = \frac{1}{\bar{K}(s)}$$

$$(16) \quad \bar{P}_1(s) = K_1(s) \bar{P}_0(s)$$

$$(17) \quad \bar{P}_2(s) = K_2(s) \bar{P}_0(s)$$

$$(18) \quad \bar{P}_c(s) = K_3(s) \bar{P}_0(s)$$

$$(19) \quad \bar{P}_f(s) = \frac{\lambda}{s} [K_1 + K_2] \bar{P}_0(s) [1 - S_{\alpha}(s)]$$

$$(20) \quad \bar{P}_{cc}(s) = \frac{q_3 \lambda_c}{s} [1 + K_1 + K_2] [1 - S_{\beta}(s)] \bar{P}_0(s)$$

where

$$(21) \quad \bar{K}(s) = (s + I_1) - \mu_e K_1(s) - p_2 \mu_i K_2(s) - \mu_c K_3(s) \\ - \lambda (K_1(s) + K_2(s)) \frac{\phi}{(s + \phi)} - q_3 \lambda_c [1 + K_1(s) + K_2(s)] \frac{\phi}{(s + \phi)}$$

Now, here we have

$$\bar{P}_{\text{up}}(s) = \bar{P}_0(s) + \bar{P}_1(s) + \bar{P}_2(s)$$

From the equation no (15), (16), (17) and (21) we may have

$$(22) \quad P_{\text{up}}(s) = \frac{c_0 s^4 + c_1 s^3 + c_2 s^2 + c_3 s + c_4}{d_0 s^5 + d_1 s^4 + d_2 s^3 + d_3 s^2 + d_4 s + d_5}$$

where

$$(23) \quad \left. \begin{aligned} c_0 &= 1, & c_1 &= L_1, & c_2 &= L_2 + L_3 \phi + L_4 \\ c_3 &= 3\mu_c L_5 + L_6 \phi + L_7, & c_4 &= 3\mu_c \phi L_8 \\ d_0 &= 1, & d_1 &= J_1, & d_2 &= 3\mu_c J_2 + J_3 \phi + J_4 \\ d_3 &= 3\mu_c J_5 - p_3 \mu_c \lambda_c J_6 - \mu_c J_7 - p_2 \mu_i q_1 \lambda J_8 + J_9 \\ d_4 &= 3\mu_c J_{10} - p_3 \mu_c \lambda_c J_{11} - \mu_c J_{12} - p_2 \mu_c q_1 \lambda J_{13} - \lambda \phi J_{14} \\ d_5 &= [J_{15} - p_3 \mu_c \lambda_c J_{16} - \mu_c J_{17} - 3\mu_c \lambda J_{18}] \phi \end{aligned} \right\}$$

The notations which have been used in above expression may be written as :

$$\begin{aligned}
 & \left. \begin{aligned}
 L_1 &= 3\mu_c + I_2 + I_3 + \phi + \lambda, \quad L_2 = I_2 I_3 + 3\mu_c (I_2 + I_3) \\
 L_3 &= I_2 + I_3 + 3\mu_c, \quad L_4 = 3\mu_c \lambda + p_1 \lambda I_3 + p_1 \lambda \phi + q_1 \lambda I_2 + q_1 q_2 \lambda \mu_i \\
 (24) \quad L_5 &= I_2 I_3 + \lambda \phi + p_1 \lambda I_3 + q_1 \lambda I_2 + q_1 q_2 \lambda \mu_i \\
 L_6 &= I_2 I_3 + 3\mu_c (I_2 + I_3), \quad L_7 = p_1 \lambda I_3 \phi + q_1 \lambda I_2 \phi + q_1 q_2 \lambda \mu_i \phi \\
 L_8 &= I_2 I_3 + p_1 \lambda I_3 + q_1 \lambda I_2 + q_1 q_2 \lambda \mu_i
 \end{aligned} \right\} \\
 \\
 & \left. \begin{aligned}
 J_1 &= I_1 + I_2 + I_3 + 3\mu_c + \phi, \quad J_2 = I_1 + I_2 + I_3, \quad J_3 = I_1 + I_2 + I_3 + 3\mu_c \\
 J_4 &= I_1 I_2 + I_2 I_3 + I_3 I_1 - 3p_3 \mu_c \lambda_c - \mu_c \lambda p_1 - p_2 \mu_i q_1 \lambda \\
 J_5 &= I_1 I_2 + I_2 I_3 + I_3 I_1 + (I_1 + I_2 + I_3) \phi \\
 J_6 &= 2(I_1 + I_2 + I_3) + 3\phi + \mu_c + \mu_i + \lambda \\
 J_7 &= 3\mu_c p_1 \lambda + p_1 \lambda I_3 + p_1 \lambda \phi + q_1 q_2 \lambda \mu_i \\
 J_8 &= I_2 + 3\mu_c + \phi, \quad J_9 = I_1 I_2 I_3 + \phi (I_1 I_2 + I_2 I_3 + I_3 I_1) \\
 (25) \quad J_{10} &= I_1 I_2 I_3 + (I_1 I_2 + I_2 I_3 + I_3 I_1) \phi \\
 J_{11} &= 2(I_1 + I_2 + I_3) \phi + I_1 I_2 + I_2 I_3 + I_3 I_1 + q_2 \mu_i I_1 + q_2 \mu_i \phi \\
 &\quad + \mu_c (I_3 + q_2 \mu_i + q_1 \lambda - p_1 \lambda + \phi) + p_2 \mu_i (I_2 + \phi + p_1 \lambda - q_1 \lambda) \\
 &\quad + 3\lambda \phi + \lambda I_2 + q_1 q_2 \lambda \mu_i \\
 J_{12} &= 3\mu_c p_1 \lambda I_3 + 3q_1 q_2 \lambda \mu_i \mu_c + 3\mu_c p_1 \lambda \phi + p_1 \lambda I_3 \phi + q_1 q_2 \lambda \mu_i \phi \\
 J_{13} &= I_2 \phi + 3\mu_c I_2 + 3\mu_c \phi, \quad J_{14} = 3\mu_c \lambda + p_1 \lambda I_3 + q_1 \lambda I_2 + q_1 q_2 \lambda \mu_i \\
 J_{15} &= 3\mu_c I_1 I_2 I_3 - 3\mu_c I_2 p_2 \mu_i q_1 \lambda \quad J_{16} = I_1 I_2 + I_2 I_3 + I_3 I_1 + \\
 &\quad q_2 \mu_i (I_1 + \mu_c + \lambda + q_1 \lambda); \quad J_{17} = 3p_1 I_3 \mu_c + 3q_1 q_2 \lambda \mu_c \mu_i \\
 J_{18} &= p_1 \lambda I_3 + q_1 \lambda I_2 + q_1 q_2 \lambda \mu_i
 \end{aligned} \right\}
 \end{aligned}$$

Now to get the expression for reliability we substitute $\phi = 1$ in equation (21) and (22). Therefore,

$$(26) \quad R(s) = P_{up}(s) = \frac{\bar{c}_0 s^4 + \bar{c}_1 s^3 + \bar{c}_2 s^2 + \bar{c}_3 s + \bar{c}_4}{\bar{d}_0 s^5 + \bar{d}_1 s^4 + \bar{d}_2 s^3 + \bar{d}_3 s^2 + \bar{d}_4 s + \bar{d}_5}$$

where $\bar{c}_0, \bar{c}_1, \bar{c}_2, \bar{c}_3, \bar{c}_4$ and $\bar{d}_0, \bar{d}_1, \bar{d}_2, \bar{d}_3, \bar{d}_4$ and $\bar{d}_5$ can be obtained by substituting $\phi = 1$ in the equation (23), (24) and (25).

Mean Time to System Failure (MTTF) : To get the expression for MTTF first we have to obtain the expression for $R(s)$ by substituting $\phi = 1$ in equation (21) and (22), i.e.

$$R(s) = P_{up}(s) = \frac{c_0^* s^4 + c_1^* s^3 + c_2^* s^2 + c_3^* s + c_4^*}{d_0^* s^5 + d_1^* s^4 + d_2^* s^3 + d_3^* s^2 + d_4^* s + d_5^*}$$

where $c_0^*, c_1^*, c_2^*, c_3^*, c_4^*$ and $d_0^*, d_1^*, d_2^*, d_3^*, d_4^*$ and d_5^* can be obtained by substituting $\phi = 1$ in the equation (23), (24) and (25).

But on substitution $\phi = 0$ in (23), (24) and (25), we find c_4^* and d_5^* turns to be zero.

Then we have

$$(27) \quad \text{MTTF} = \lim_{s \rightarrow 0} R(s) = \frac{c_3^*}{d_4^*}$$

Numerical Computation of Results : To evaluate the availability of the system we assign the following values to different parameters used in expression (26) and take the inverse Laplace transform of $P_{up}(s)$ to get the expression for $P_{up}(t)$.

$$\lambda = 0.15, \lambda_c = 0.01, \mu_e = 0.1, \mu_i = 0.055, \mu_c = 0.085$$

$$p_1 = 0.25, p_2 = 0.75, p_3 = 0.75$$

$$q_1 = 0.75, q_2 = 0.25, q_3 = 0.25$$

Therefore,

$$P_{up}(t) = -0.3331437 \exp(-0.9878 t) + 0.06661435 \exp(0.1568 t) + 0.8132325 \exp(0.3040 t) \\ + \exp(-0.3897 t) [0.4532969 \cos(2.002 t) - 1.5369725 \sin(2.002 t)].$$

The curve for availability versus time is given in Fig. 2. As we look at the curve, we find that the availability decreases according as the increment in time t .

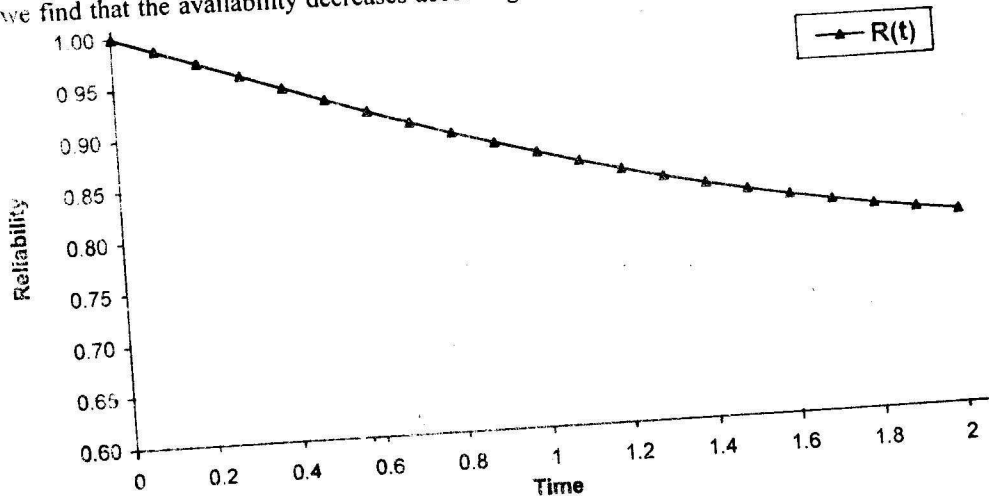


Fig. 2 Reliability Vs Time

Also to study the nature of MTTF with respect to constant failure rate λ , we give the following values to the parameters used in the problem (except λ).

$$\lambda_c = 0.01, \mu_e = 0.1, \mu_i = 0.055, \mu_c = 0.085,$$

$$p_1 = 0.25, p_2 = 0.75, p_3 = 0.75$$

$$q_1 = 0.75, q_2 = 0.25, q_3 = 0.25.$$

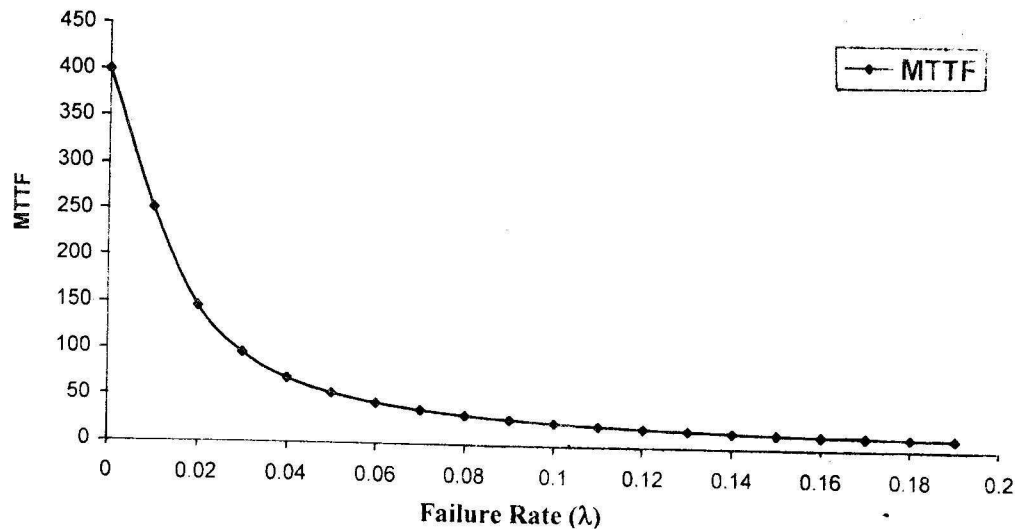


Fig. 3 MTTF Vs Failure Rate

Now for different values of λ we get, the different values of MTTF. The graph has been shown in the Fig. 3 for MTTF against λ . From this graph we find that as the failure rate λ increases a very sharp change in the values of MTTF occurs.

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