

A Probabilistic Consideration of Ballistic Models of Superluminal Radio Source

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Abstract. In order to examine the plausibility of ballistic model explanation of superluminal phenomenon is extragalactic sources of probability theory is used. It is shown that this approach is less probable for explanation of observed data.

Introduction

The VLBI-technique for image construction has revealed that many extragalactic sources possess two-component structure. The most important discovery pertains to quasars 3C 273, 3C 279, 3C 345 and the nucleus of Seyfert galaxy 3C 120 where the apparent relative velocities v_{exp} of their components are substantially more than the velocity of light c (cf. Table 1, Rees *et.al.*¹)

Various models have been proposed to explain this.²⁻⁸ These models can, however, be put in two groups, namely, the phase model and the ballistic model.⁹ In the latter the observed phenomenon is simply related to the fact that the apparent relative velocity in the reference frame of the observer can be more than c even if the real physical velocity v in the reference frame of the source is less than c .

If the coordinate of the source at time t_0 are (x_0, y_0) then at instant t' the source will have the coordinates

$$x_0 + v \sin \phi (t' - t_0); \quad y_0 + v \cos \phi (t' - t_0)$$

where ϕ is the angle between the velocity vector v and the direction orthogonal to the plane of the observer (Fig. 1).

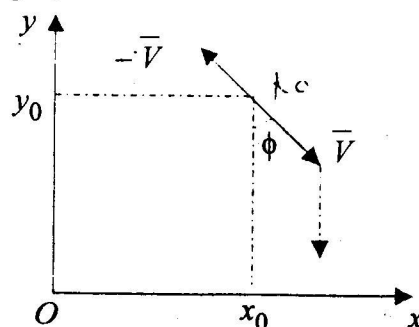


Fig.1. Plane of observer

So the coordinates of the light radiated from this point at instant t will be

$$x = x_0 + v(t' - t_0) \sin \phi$$

$$y = y_0 - v(t' - t_0) \cos \phi + c(t' - t)$$

Therefore, for the observer $y = 0$

$$t' = \frac{(y_0 + v t_0 \cos \phi - ct)}{(V \cos \phi - c)}$$

yielding

$$x = x_0 - v t_0 \sin \phi + \frac{v \sin \phi (y_0 + v t_0 \cos \phi - ct)}{(V \cos \phi - c)}$$

Therefore, the apparent velocity of the source relative to the point (x_0, y_0) is

$$\frac{V_{\text{exp}}}{c} = f_1(v) = \frac{(v/c) \sin \phi}{1 - (v/c) \cos \phi}$$

$$(1) \quad 0^\circ \leq \phi \leq 180^\circ$$

Further, we take the case when two sources move relative to each other in a straight line (where for simplicity, their velocities are assumed to be the same relative to the centre of mass). Then the apparent relative velocity is given by

$$\frac{V_{\text{exp}}}{c} = f_2(v) = \frac{2(v/c) \sin \phi}{1 - (v/c)^2 \cos \phi}$$

$$(2) \quad 0^\circ \leq \phi \leq 180^\circ$$

It should be noted that $f_1(v)$ and $f_2(v)$ can essentially be more than unity if some definite relations between (v/c) and ϕ are satisfied.

Two-Component Models

We now proceed to consider these relations for the following two-component models of the sources. They are :

- I. Stationary source, $v = 0$ and ultra-relativistic source, $v \leq c$;
- II. Two ultra-relativistic sources;
- III. Stationary source and techyonic source, $v > c$ and
- IV. Two techyonic sources.

Model I. Here $f_1(v)$ is a continuous function of ϕ_1 and its maximum value

$$f_1^{\text{max}} = \frac{(v/c)}{(1 - v^2/c^2)^{1/2}}$$

is achieved at $\phi = \cos^{-1}(v/c)$. Clearly, for $v/c = 1/\sqrt{2}$, is the apparent velocity, $v_{\text{exp}} > c$. Further, it can be shown that if the inequalities

$$\sin^{-1}\left(\frac{a}{\sqrt{1+a^2}} \cdot \frac{c}{v}\right) - \tan^{-1}a < \phi < \pi - \tan^{-1}a - \sin^{-1}\left(\frac{a}{\sqrt{1+a^2}} \cdot \frac{c}{v}\right)$$

$$(3) \quad \frac{v}{c} \geq \frac{a}{(1+a)^{1/2}}$$

are satisfied, then $f_1(v) > a$.

Model II. Here, $f_2(v)$ is continuous function of ϕ . This function attains its maximum for $v/c > 1/\sqrt{2}$.

$$f_2^{\max} = \frac{1}{\left(1 - \frac{v^2}{c^2}\right)^{1/2}}$$

at

$$\phi_1 = \sin^{-1}\left(\frac{1 - \frac{v^2}{c^2}}{\frac{v}{c}}\right) \text{ or } \phi_2 = 180^\circ - \phi_1$$

and its minimum

$$f_2^{\min} = 2v/c$$

is attained at $\phi = 90^\circ$. Here, it should be mentioned that the case $v/c < 1/\sqrt{2}$ is not of interest as $f_2^{\max} < \sqrt{2}$.

Also, if $1/\sqrt{2} < v/c < 1$, the following cases corresponding to condition $f_2(v) > a$ arise

$$(i) \quad \begin{cases} a < 2v/c < 2 \\ \phi_1 < \phi < \phi_2 \end{cases}$$

where

$$\phi_1 = \sin^{-1}\left(\frac{1 - \{1 + (v^2/c^2 - 1)a^2\}^{1/2}}{av/c}\right)$$

$$\phi_2 = \sin^{-1}\left(\frac{1 - \{1 - (v^2/c^2 - 1)a^2\}^{1/2}}{av/c}\right)$$

(ii)

$$\begin{cases} \sqrt{2} < 2v/c < a \\ \pi - \phi_2 < \phi < \pi - \phi_1 \end{cases}$$

$$(5) \quad \frac{v}{c} > \frac{(a^2 - 1)^{1/2}}{a}$$

Model III. Here $f_1(v)$ is a decreasing function of ϕ with discontinuities at $\phi = \cos^{-1}(c/v)$. Also $f_1(v)$ is negative for $0^\circ < \phi < \cos^{-1}(c/v)$ which can easily be understood. Since $v > c$ in the case under consideration, the transformation about the later states of the evolution of source is carried by the light signal first to the observer than of the earlier stages. As such it will appear to an observer that the angular distance between the stationary source and the source moving towards him decreases with time. One can easily see that

$$(6) \quad \cos^{-1}\left(\frac{v}{c}\right) < \phi < \pi - \sin^{-1}\left(\frac{a}{\sqrt{1+a^2}} \frac{c}{v}\right) - \tan^{-1} a$$

if $f_1(v) > a$

Model IV. Here $f_1(v)$ has discontinuity at two points.

$$\phi_1 = \cos^{-1}(c/v), \quad (0^\circ < \phi_1 < 90^\circ)$$

$$\phi_2 = 180^\circ - \phi_1, \quad (90^\circ < \phi_2 < 180^\circ)$$

and its local minimum is

$$f_2^{\min}(v) = 2v/c$$

at $\phi = 90^\circ$

Evidently,

$$\phi_1 = \cos^{-1}\left(\frac{c}{v}\right) < \phi < \phi_2 = \sin^{-1}\left(\frac{c/v + (a^2 + (1 - a^2)c^2/v^2)^{1/2}}{a}\right)$$

$$\pi - \phi_2 < \phi < \pi - \phi_1$$

$$(7) \quad v/c < a/2$$

should hold in order that $f_2(v) > a$.

Probability Approach

We now proceed to estimate the probability of each of the four ballistic models. It is then natural to suppose an isotropic distribution of the velocity vector, v in the system of the source. Therefore, we take for the measurement of probability the ratio of part of the unit sphere containing the orientation of v satisfying the inequalities (3), (5), (6) and (7) with the surface of the unit sphere. The elementary surface of sphere in spherical coordinates is $ds = \sin \phi d\phi d\psi$, and hence, the probability of angle ϕ lying in the interval $\phi_1 < \phi < \phi_2$ is

$$\begin{aligned}
 P &= \frac{1}{4\pi} \int_{\phi_1}^{\phi_2} \int_0^{2\pi} \sin \phi \, d\phi \, d\psi \\
 (8) \qquad &= \frac{1}{2} (\cos \phi_1 - \cos \phi_2)
 \end{aligned}$$

Then, in Model I, in view of (3)

$$(9) \qquad P(f_1 > a) = \frac{1}{1+a^2} \left\{ 1 + a^2 \left(1 - \frac{c^2}{v^2} \right) \right\}^{1/2}$$

$$(10) \qquad P(f_1 > a) \leq P_{\max} \left(= \frac{1}{1+a^2} \right)$$

Likewise for Model II in view of (4), for Model III in view of (6) and for Model IV in view of (7) we get, respectively

$$\begin{aligned}
 P(f_2 > a) &= \frac{1}{2} \left\{ \left(1 - \left[\frac{1 - (1 - a^2(1 - v^2/c^2))^{1/2}}{av/c} \right]^2 \right)^{1/2} \right. \\
 &\quad \left. - \left(1 - \left[\frac{1 + (1 - a^2(1 - v^2/c^2))^{1/2}}{av/c} \right]^2 \right)^{1/2} \right\} \\
 (11) \qquad P(f_2 > a) &\leq P_{\max} \left(= \frac{2}{a(a + (a^2 - 4)^{1/2})} \right)
 \end{aligned}$$

$$\begin{aligned}
 P(f_1 > a) &= \frac{c/v + (1 + a^2(1 - c^2/v^2))^{1/2}}{2(1 + a^2)} \\
 (12) \qquad P(f_1 > a) &\leq P_{\max} \left(= \frac{1}{2a} \right) \\
 P(f_2 > a) &= \frac{1}{2} \left\{ \frac{c}{v} - \left[1 - \left(\frac{c/v + (a^2 + (1 - a^2)c^2/v^2)^{1/2}}{a} \right)^2 \right] \right\} \\
 (13) \qquad P(f_2 > a) &\leq P_{\max} \left(= \frac{1}{a} \right)
 \end{aligned}$$

Now, in order to have an estimate of probability for the above four ballistic models, it is necessary to add to the data in Table I, the data concerning the sources which do not appear to have superluminal expansion during observation with high angular resolution. Till present time, nearly ten sources without superluminal expansion have been observed.

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