

A Single-Server Bulk-Service Queue with Accessible and Non-Accessible Batches in Transient State

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Abstract. This paper deals with a Single-Server Poisson Queue with a new bulk service rule. This rule admits each-batch served to have not less than a and not more than b units such that the arriving in its can enter service, without affecting the service time, if the size of the batch served is less than some fixed integer d ($a \leq d \leq b$). The transforms of the transient state probabilities are obtained using a generating function.

1. Introduction

With the advances in computer and communication technology, the notions of parallel and distributed processing and gaining increased importance. Cost effective tools for performance prediction are needed to support the design of such systems. Analytical queueing network model has been regarded as one of the cost effective tools for performance analysis of parallel computer system. Thomasion¹ developed an analytical queueing network model for efficient processing of tasks in parallel processing systems. Pal² used a bulk arrival and batch service $M^x/M^k/s$ queueing system to model a centralized parallel processing system with job splitting. In such a system, message in a central queue, which is accessible by all the processors, and split into independent jobs and each job splits into tasks of fix number that can be executed on separate processors. An expression for mean response time is obtained for this centralized parallel processing system, the concept of accessibility in to batches while receiving service has been considered by Gross and Harris³, Kleinrock⁴, Chiamsiri and Leonard⁵, and Sivasamy⁶. The General Bulk Service Rule (GBSR) queues with NAB and with a number of variants have been studied by Medhi⁷, Chaudhry and Templeton⁸, Borthakur⁹, Jacob and Madhusoodanan¹⁰, and Gohan and Borthakur¹¹.

In this paper, we developed queueing model, in which units arrive in to a service facility according to a poisson process with rate λ . The units are served by a single server in batches of size n ($a \leq d \leq b$), the service times, irrespective of the batch size, are assumed to have independent identical exponential distributions with parameter. The transforms of the transient state probabilities are obtained for three cases :

$$(1) \quad a - 1 \geq n \geq 0,$$

$$(2) \quad d - 1 \geq n \geq a$$

and $(3) \quad n \geq d,$

where a is minimum numbers of units when server starts service and b is maximum capacity of units.

2. Model Description

In this model, units that are to be served, arrive at service facility as an ordinary poisson process, that they are served FCFS, that there is no waiting capacity constraint. The arrivals are served by a single server in batch of size n ($a \leq d \leq b$). The service time irrespective of the batch size, are assumed to have independent identical exponential distributors with parameter μ .

The server starts service only if at least one customer is present in the queue, and the maximum service capacity is b customers. Such a rule for bulk service may be called a GBSR. In this model, GBSR is assumed to allow the late entries to join a batch in course of on going service as busy the number of units that batch is less than d (called maximum accessible unit). At every departure epoch, before initiating the next service the server may find the queue length n which is one of the three categories :

$$(1) \quad a - 1 \geq n \geq 0,$$

$$(2) \quad d - 1 \geq n \geq a$$

and $(3) \quad n \geq d.$

For simplicity, the presented queueing model can be denoted by $M/M_{(a,d,b)}/1$. In case (1), the server can not initiate service with the number of units the queue. Hence server remains idle until units the queue length becomes a , where upon server starts service on the batch with the minimum of 'a' units. In case (2), the server takes the queue for batch while the service is on, the service of the current batch is or till the accessible batch (AB). In case (3), server takes $\min(n, b)$ units at the head of the queue for the service and does not allow further arrival in to the batch being served even if the current batch capacity is not b , i.e., when the batch size is greater than or equal to d , the batch becomes non-accessible (NAB) for late arriving units. The probability arises in case (1) and (2) and NAB in case (3).

3. Model Analysis

Assuming the transient state conditions to hold, let $P_{0,n}$ denote the probability that the system has n units, $n = 0, 1, 2, \dots, a, a+1, \dots, d-1$. Here, the server will be idle if $n = 0, 1, 2, \dots, a-1$ and busy with an AB when $n = a, a+1, \dots, d-1$. Further $P_{1,n}$ be the probability that the server is busy with a NAB and there are n units waiting in the queue (excluding those in service), $n = 0, 1, 2, \dots$.

Transition equation for the system

$$(1) \quad \frac{d}{dt} P_{0,0}(t) = -\lambda P_{0,0} + \mu \sum_{n=a}^{d-1} P_{0,n}(t) + \mu P_{1,0}(t)$$

$$(2) \quad \frac{d}{dt} P_{0,n}(t) = -\lambda P_{0,n} + \lambda P_{0,n-1}(t) + \mu P_{1,n}(t) \quad (1 \leq n \leq a-1)$$

$$(3) \quad \frac{d}{dt} P_{0,n}(t) = -(\lambda + \mu) P_{0,n} + \lambda P_{0,n-1}(t) + \mu P_{1,n}(t) \quad (a \leq n \leq d-1)$$

$$(4) \quad \frac{d}{dt} P_{1,0}(t) = -\lambda P_{1,0} + \lambda P_{0,d-1}(t) + \mu \sum_{n=d}^b P_{1,n}(t)$$

$$(5) \quad \frac{d}{dt} P_{1,n}(t) = -(\lambda + \mu) P_{1,n} + \lambda P_{1,n-1}(t) + \mu P_{1,n-b}(t)$$

Taking Laplace transformation (1) to (5) and use the standard difference equation technique by setting

$$(6) \quad \bar{P}_{1,n}(s) = z^{-n} \bar{P}_{1,0}(s)$$

From (4)

$$(7) \quad S + \lambda + \mu - \lambda z - \mu z^{-b} = 0$$

Let the outside root be denoted by $z_0(s)$ and let $\rho = \lambda/\mu b$ one root outside the unit circle $|z| = 1$, when $\text{Re } s > 0$ and $\lambda < \mu b$. We find $\bar{P}_{0,n}(s)$ ($1 \leq n \leq a-1$) and $P_{0,n}(s)$ ($a \leq n \leq d-1$).

$$(8) \quad \bar{P}_{0,n}(s) = \left(\frac{\lambda}{s+\lambda} \right)^n \bar{P}_{0,0}(s) + \frac{\mu \bar{P}_{1,0}(s)}{z_0 - s - \lambda} \left[\left(\frac{\lambda}{s+\lambda} \right)^n - \left(\frac{1}{z_0} \right)^n \right] \quad (1 \leq n \leq a-1)$$

$$(9) \quad \bar{P}_{0,n}(s) = \left(\frac{\lambda}{s+\lambda+\mu} \right)^{n-a+1} \bar{P}_{0,a-1}(s) + \frac{\mu \lambda z_0}{z_0(s+\lambda+\mu)(\lambda z_0 - s - \mu - \lambda)} \left(\frac{\lambda}{s+\lambda+\mu} \right)^{n-a} \\ + \bar{P}_{1,0}(s) \left[1 - \left(\frac{s+\lambda+\mu}{\lambda z_0} \right)^{n-a+1} \right]$$

We use the normalizing condition we get

$$(10) \quad \sum_{n=1}^{a-1} \bar{P}_{0,n}(s) + \sum_{n=a}^{d-1} \bar{P}_{0,n}(s) + \sum_{n=0}^{\infty} \bar{P}_{1,n}(s) = \frac{1}{s}$$

$$\begin{aligned}
 (11) \quad \bar{P}_{l,n}(s) = & \frac{(\lambda z_0)^{n+1} \bar{P}_{0,0}(s)}{(s + \lambda + \mu - \mu z_0^{-b})^{n+2}} \left[\frac{1}{s} - \frac{\lambda}{s} \left\{ 1 - \left(\frac{\lambda}{s + \lambda} \right)^a - 1 \right\} - \left(\frac{\lambda}{s + \lambda} \right)^a \left\{ 1 - \left(\frac{\lambda}{s + \lambda + \mu} \right)^{d-a} \right\} \right] \\
 & - \frac{\frac{\lambda \mu}{s(\lambda z_0 - s - \lambda)} \left\{ 1 - \left(\frac{\lambda}{s + \lambda} \right)^{a-1} \right\} - \frac{\mu (z_0^{a-1} - 1)}{z_0^{a-1}(z_0 - 1)(\lambda z_0 - s - \lambda)} - \frac{z_0}{z_0 - 1}}{z_0^{a-1}(s + \lambda)(\lambda z_0 - s - \lambda)} \left\{ 1 - \left(\frac{\lambda}{s + \lambda + \mu} \right)^{d-1} \right\} - \frac{\mu (z_0^{d-1} - 1)}{z_0^{a-1}(z_0 - 1)(\lambda z_0 - s - \mu)} \\
 & + \frac{\lambda \mu}{z_0^{a-1}(s + \lambda)(\lambda z_0 - s - \lambda)} \left\{ 1 - \left(\frac{\lambda}{s + \lambda + \mu} \right)^{d-a} \right\} \left\{ 1 - \left(\frac{\lambda z_0}{s + \lambda} \right)^{a-1} \right\}
 \end{aligned}$$

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