

Some New Results using Soft Conflicting Bifuzzy Set Approach

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Abstract: The soft conflicting bifuzzy set theory offers a general mathematical tool for dealing with uncertainty and fuzziness. The concept of soft sets which can be seen as a new mathematical approach to vagueness is used in many applications including reliability theory, decision making and medical diagnosis problems. Later, it is generalized to soft fuzzy set. In this paper we present the definition and operations of soft conflicting bifuzzy set. Furthermore, based on the analysis of several operations on soft conflicting bifuzzy set in the study, we provide some new notions such as the restricted intersection, the restricted union and the restricted difference of two soft conflicting bifuzzy sets. We also improve the notion of complement of a soft conflicting bifuzzy set and prove certain De Morgan's laws hold in soft conflicting bifuzzy set with respect to the new definitions.

Keywords: Soft set; Soft fuzzy set; conflicting bifuzzy set; Soft conflicting bifuzzy set.

1. Introduction

Fuzzy set that allow to tackle vague information was introduced by Lotfi A. Zadeh¹. Such vagueness is reflected by the membership degree of the objects belonging to a concept². Fuzzy sets theory has been widely and successfully applied in many different areas to handle such type of vagueness. Nevertheless, it presents limitations to deal with imprecise and vague information when different sources of vagueness appear simultaneously. Due to this fact and in order to overcome such limitations, different extensions of fuzzy sets have been introduced in the literature such as conflicting bifuzzy sets, by Zamali², which allow to incorporate simultaneously the membership degree and the non-membership degree of each element. Soft set was later defined by appropriate Molodtsov³ who initiated the concept of soft theory as

a mathematical tool to deal with uncertainties. Maji *et al.*⁴ made a theoretical study on the Soft Set Theory in more detail and contributed towards the fuzzification of the notion of it and described the application of soft set theory to a decision making problem using rough sets. Recently Kong *et al.*⁵⁻⁶ applied the soft set theoretic approach in decision making problems. Soft fuzzy set was defined by Yao *et al.*⁷ followed by conflicting bifuzzyfuzzy soft set defined by Xu Yong *et al.*⁸. Alkhazaleh *et al.*⁹ introduced the concept of fuzzy parameterized interval-valued fuzzy soft set and gave its application in decision making. Alkhazaleh *et al.*¹⁰ introduced soft multisets as a generalization of Molodtsov's soft set and proposed the concept of possibility fuzzy soft set.

In the present paper we introduced some new concepts of soft conflicting bifuzzy set. There has been incredible interest in the subject due to its diverse applications, ranging from engineering and computer science to social behaviour studies. Here we define a soft conflicting bifuzzy set and its operations of equality, complement, union, intersection, OR and AND operators. We also provide some new notions such as the restricted intersection, the restricted union and the restricted difference of two soft conflicting bifuzzy sets along with examples. We also improve the notion of complement of a soft conflicting bifuzzy set and prove certain De Morgan's laws hold in soft conflicting bifuzzy set with respect to the new definitions.

2. Preliminaries

Definition 2.1 Let U be an initial set and E be a set of parameters. Let $P(U)$ denotes the power set of U and $A \subset E$ then a pair (F, A) is called a soft set over U if F is a mapping given by $F : A \rightarrow P(U)$.

Definition 2.2 Let U be an initial set and E be a set of parameters. Let $F(U)$ denotes the fuzzy power set of U and $A \subset E$ then a pair (F, A) is called a fuzzy soft set over U if F is a mapping given by $F : A \rightarrow F(U)$.

Definition 2.3 Let U be an initial set and E be a set of parameters. Let $P(U)$ denotes the power set of U and $A \subset E$ then a pair (F, A) is called a soft fuzzy set over U if F is a mapping given by $F : A \rightarrow P(U)$ and

$F(x) = \{y \in U : \tilde{R}_\alpha(x, y) \geq \alpha, x \in A, y \in U, \alpha \in [0,1]\} \subset X \times Y$
is defined as cut-set $\tilde{R} \in F(X \times Y)$.

Definition 2.4 Consider U and E as a universe set and a set of parameters respectively. Let $IFS(U)$ denotes the conflicting bifuzzy power set of U and $A \subset E$ then a pair (F, A) is an conflicting bifuzzy soft set over U if the mapping F is given by $F : A \rightarrow IFS(U)$.

We recall these definitions in order to use them to introduce the concept of soft conflicting bifuzzy set and to define some operations on soft conflicting bifuzzy set namely equality, null, complement, union, intersection, AND and OR operators.

3. Soft Conflicting Bifuzzy Set

3.1 Conflicting bifuzzy set:

If a set X is fixed then a conflicting bifuzzy set Z of X is an object having the following form $Z = \{ x, u_Z(x), v_Z(x) \mid x \in X \}$ where $u_Z : X \rightarrow [0, 1]$ is the degree of positive x , ($x \in X$) with respect to Z and $u_Z : X \rightarrow [0, 1]$ is the degree of negative x , ($x \in X$) with respect to Z and $0 \leq u_Z + v_Z \leq 2$.

3.2 Soft conflicting bifuzzy set:

Let U be an initial set and E be a set of parameters. Let $P(U)$ denotes the power set of U and $A \subset E$ then a pair (F, A) is called a soft conflicting bifuzzy set over U if F is a mapping given by $F : A \rightarrow P(U)$ and

$$F(x) = \{y \in U : (x, y) \in \tilde{R}_\alpha, x \in A, y \in U, \alpha \in [0, 1]\}.$$

3.3 Relation on Soft conflicting bifuzzy set:

Let $\tilde{R}_\alpha = (\tilde{R}_{\mu_\alpha}, \tilde{R}_{\nu_\alpha})$ be a conflicting bifuzzy subset of $X \times Y$, and $\tilde{R}_\alpha$ is defined as conflicting bifuzzy relationship from X to Y written as $X \xrightarrow{\tilde{R}} Y$. Let $\tilde{R} (X \times Y)$ denotes the degree of correspondence between X and Y based on the relationship $\tilde{R}$ and $F(X \times Y)$ denotes the family of a conflicting bifuzzy relationship on X to Y .

The set $\tilde{R}_\alpha = \{(x, y) \in X \times Y : \tilde{R}_{\mu_\alpha}(x, y) \geq \alpha \text{ and } \tilde{R}_{\nu_\alpha}(x, y) \leq \alpha\} \subset X \times Y$ is defined as α - cut set if $\tilde{R} \in F(X \times Y)$ for $\alpha \in [0, 1]$.

3.4 Subset of Soft conflicting bifuzzy sets:

In this subsection we introduce the concept for subset of two soft conflicting bifuzzy sets. For two conflicting bifuzzy sets $(F, A)_{\tilde{R}}$ and $(G, B)_{\tilde{R}}$ over common universe U , we say $(F, A)_{\tilde{R}}$ is a soft conflicting bifuzzyfuzzy subset of $(G, B)_{\tilde{R}}$ if:

(i) $A \subset B$ and

(ii) $\forall \varepsilon \in A, F(\varepsilon)$ is an conflicting bifuzzy subset of $G(\varepsilon)$

i.e. $A \subset B$ iff $\forall x \in E, \mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \leq \nu_B(x)$ denoted by $(F, A) \subset (G, B)$.

3.5. Equality of Soft conflicting bifuzzy sets:

Two soft conflicting bifuzzy sets $(F, A)_{\bar{R}}$ and $(G, B)_{\bar{R}}$ over common universe U are said to be equal if $(F, A)_{\bar{R}}$ is a soft conflicting bifuzzy subset of $(G, B)_{\bar{R}}$ and $(G, B)_{\bar{R}}$ is a soft conflicting bifuzzy subset of $(F, A)_{\bar{R}}$.

3.6. Complement of Soft conflicting bifuzzy set:

The complement of soft conflicting bifuzzy set $(F, A)_{\bar{R}}$ is denoted by $(F, A)_{\bar{R}}^c$ and defined as $(F, A)_{\bar{R}}^c = (F^c, \neg A)_{\bar{R}}$, where $F^c: \neg A \rightarrow P(U)$ is a mapping, $F^c(A) =$ conflicting bifuzzy complement of $F(\neg e), \forall e \in \neg A$.

3.7. Null Soft conflicting bifuzzy set:

$(F, A)_{\bar{R}}$, a soft conflicting bifuzzy set over U is said to be a null soft conflicting bifuzzy set denoted by ϕ , if $\forall e \in A, F(e) =$ null conflicting bifuzzy of U (null-set).

3.8. Union of Soft conflicting bifuzzy set:

The union of two soft conflicting bifuzzy sets $(F, A)_{\bar{R}}$ and $(G, B)_{\bar{R}}$ over a common universe U is the soft conflicting bifuzzy set $(H, C)_{\bar{R}}$, where $C = A \cup B$ and $\forall e \in C$

$$H(e) = \begin{cases} F(e) & \text{if } e \in A - B \\ G(e) & \text{if } e \in B - A \\ F(e) \cup G(e) & \text{if } e \in A \cap B \end{cases}$$

we write $(F, A)_{\bar{R}} \dot{\cup} (G, B)_{\bar{R}} = (H, C)_{\bar{R}}$

3.9. Intersection of Soft conflicting bifuzzy set:

The intersection of two soft conflicting bifuzzy sets $(F, A)_{\bar{R}}$ and $(G, B)_{\bar{R}}$ over a common universe U is the soft conflicting bifuzzy set $(H, C)_{\bar{R}}$, where $C = A \cap B$ and $\forall e \in C$ is defined by

$$H(e) = \begin{cases} F(e) & \text{if } e \in A - B \\ G(e) & \text{if } e \in B - A \\ F(e) \cap G(e) & \text{if } e \in A \cap B \end{cases}$$

we write $(F, A)_{\bar{R}} \cap (G, B)_{\bar{R}} = (H, C)_{\bar{R}}$

3.10. AND Operation on Soft conflicting bifuzzy set:

If $(F, A)_{\bar{R}}$ and $(G, B)_{\bar{R}}$ be two soft conflicting bifuzzy sets then, " $(F, A)_{\bar{R}} \text{ AND } (G, B)_{\bar{R}}$ " is an soft conflicting bifuzzy set denoted by $(F, A)_{\bar{R}} \bar{\wedge} (G, B)_{\bar{R}}$ is defined by $(F, A)_{\bar{R}} \bar{\wedge} (G, B)_{\bar{R}} = (H, A \times B)_{\bar{R}}$, where

$$H(\alpha, \beta) = F(\alpha) \cap G(\beta), \forall (\alpha, \beta) \in A \times B$$

3.11. OR Operation on conflicting bifuzzy Soft Set:

If $(F, A)_{\bar{R}}$ and $(G, B)_{\bar{R}}$ be two soft conflicting bifuzzy sets then, " $(F, A)_{\bar{R}} \text{ OR } (G, B)_{\bar{R}}$ " is an soft conflicting bifuzzy set denoted by $(F, A)_{\bar{R}} \overset{\circ}{\vee} (G, B)_{\bar{R}}$, is defined by $(F, A)_{\bar{R}} \overset{\circ}{\vee} (G, B)_{\bar{R}} = (O, A \times B)_{\bar{R}}$, where

$$O(\alpha, \beta) = F(\alpha) \cup G(\beta), \forall (\alpha, \beta) \in A \times B.$$

4. Some New Operations in Soft Conflicting Bifuzzy Set Theory

4.1. Restricted union

Let $(F, A)_{\bar{R}}$ and $(G, B)_{\bar{R}}$ be two soft conflicting bifuzzy sets over a common universe U such that $A \cap B \neq \phi$. The restricted union of $(F, A)_{\bar{R}}$ and $(G, B)_{\bar{R}}$ is denoted by $(F, A)_{\bar{R}} \cup_{\mathfrak{R}} (G, B)_{\bar{R}}$ and is defined as $(F, A)_{\bar{R}} \cup_{\mathfrak{R}} (G, B)_{\bar{R}} = (H, C)_{\bar{R}}$ where $C = A \cap B$ and $c \in C, H(c) = F(c) \cup G(c)$.

Example 1.

Assume that $U = \{h_1, h_2, h_3\}$ is a set of houses under consideration and $E = \{e_1, e_2, e_3, e_4\}$ is the set of parameters where each parameter is an conflicting bifuzzy word or a sentence involving conflicting bifuzzy words $A, B \subset E$ such that $A = \{\text{beautiful}(e_1), \text{comfortable}(e_2), \text{cheap}(e_4)\}$ and $B = \{\text{beautiful}(e_1), \text{big}(e_3), \text{cheap}(e_4)\}$. The soft conflicting bifuzzy sets $(F, A)_{\bar{R}}$ and $(G, B)_{\bar{R}}$ describe the "attractiveness of the houses" which Mr. X and Mr. Y are going to buy, respectively.

Let conflicting bifuzzyfuzzy relationship for Mr. X is

$$\bar{R} = \left\{ \frac{(0.3, 0.4)}{(h_1, e_1)} + \frac{(0.5, 0.3)}{(h_1, e_2)} + \frac{(0.1, 0.4)}{(h_1, e_4)} + \frac{(0.8, 0.0)}{(h_2, e_1)} + \frac{(0.1, 0.5)}{(h_2, e_2)} + \frac{(0.3, 0.6)}{(h_2, e_4)} \right. \\ \left. + \frac{(0.0, 0.4)}{(h_3, e_1)} + \frac{(0.5, 0.5)}{(h_3, e_2)} + \frac{(0.3, 0.4)}{(h_3, e_4)} \right\}$$

Thus, we can view the soft conflicting bifuzzy set $(F, A)_{\bar{R}}$ as a collection of conflicting bifuzzyfuzzy approximations (which are under the fuzzy relationship $\bar{R}$) as below:

$$\begin{aligned}
 (F, A)_{\tilde{R}} &= \left\{ \text{beautiful house} = \left\{ \frac{(0.3, 0.4)}{h_1}, \frac{(0.8, 0.0)}{h_2}, \frac{(0.0, 0.4)}{h_3} \right\}, \right. \\
 &\quad \text{comfortable house} = \left\{ \frac{(0.5, 0.3)}{h_1}, \frac{(0.1, 0.5)}{h_2}, \frac{(0.5, 0.5)}{h_3} \right\}, \\
 &\quad \left. \text{cheap house} = \left\{ \frac{(0.1, 0.4)}{h_1}, \frac{(0.3, 0.6)}{h_2}, \frac{(0.3, 0.4)}{h_3} \right\} \right\}
 \end{aligned}$$

and suppose conflicting bifuzzyfuzzy relationship for Mr. Y is

$$\tilde{R} = \left\{ \begin{aligned} &\frac{(0.1, 0.5)}{(h_1, e_1)} + \frac{(0.4, 0.5)}{(h_1, e_3)} + \frac{(0.1, 0.6)}{(h_1, e_4)} + \frac{(0.4, 0.3)}{(h_2, e_1)} + \frac{(0.1, 0.7)}{(h_2, e_3)} + \frac{(0.3, 0.6)}{(h_2, e_4)} \\ &\quad + \frac{(0.4, 0.3)}{(h_3, e_1)} + \frac{(0.2, 0.5)}{(h_3, e_3)} + \frac{(0.3, 0.4)}{(h_3, e_4)} \end{aligned} \right\}$$

Thus, we can view the soft conflicting bifuzzy set $(G, B)_{\tilde{R}}$ as a collection of fuzzy approximations (which are under the fuzzy relationship $\tilde{R}$) as below:

$$\begin{aligned}
 (G, B)_{\tilde{R}} &= \left\{ \text{beautiful house} = \left\{ \frac{(0.3, 0.4)}{h_1}, \frac{(0.8, 0.0)}{h_2}, \frac{(0.4, 0.3)}{h_3} \right\}, \right. \\
 &\quad \text{big house} = \left\{ \frac{(0.4, 0.5)}{h_1}, \frac{(0.1, 0.7)}{h_2}, \frac{(0.2, 0.5)}{h_3} \right\}, \\
 &\quad \left. \text{cheap house} = \left\{ \frac{(0.1, 0.6)}{h_1}, \frac{(0.5, 0.3)}{h_2}, \frac{(0.3, 0.4)}{h_3} \right\} \right\}
 \end{aligned}$$

To find the $(F, A)_{\tilde{R}} \cup_{\mathfrak{R}} (G, B)_{\tilde{R}} = (H, C)_{\tilde{R}}$

We have $C = \{\text{beautiful}(e_1), \text{cheap}(e_4)\}$, then

$$\begin{aligned}
 H(e_1) &= F(e_1) \cup G(e_1) \\
 &= \left\{ \frac{\max(0.2, 0.1), \min(0.8, 0.6)}{h_1}, \frac{\max(0.4, 0.5), \min(0.5, 0.3)}{h_2}, \frac{\max(0.6, 0.3), \min(0.2, 0.4)}{h_3} \right\} \\
 &= \left\{ \frac{(0.3, 0.4)}{h_1}, \frac{(0.8, 0.0)}{h_2}, \frac{(0.4, 0.3)}{h_3} \right\}
 \end{aligned}$$

and

$$\begin{aligned}
 H(e_4) &= F(e_4) \cup G(e_4) \\
 &= \left\{ \frac{\max(0.2, 0.1), \min(0.8, 0.6)}{h_1}, \frac{\max(0.4, 0.5), \min(0.5, 0.3)}{h_2}, \frac{\max(0.6, 0.3), \min(0.2, 0.4)}{h_3} \right\} \\
 &= \left\{ \frac{(0.2, 0.6)}{h_1}, \frac{(0.5, 0.3)}{h_2}, \frac{(0.6, 0.2)}{h_3} \right\}
 \end{aligned}$$

Thus, we can view the soft conflicting bifuzzy set as a collection of fuzzy approximations (which are under the fuzzy relationship $\tilde{R}$) as below:

$$(H, C)_{\tilde{R}} = \left\{ \begin{array}{l} \text{beautiful house} = \left\{ \frac{(0.3, 0.4)}{h_1}, \frac{(0.8, 0.0)}{h_2}, \frac{(0.4, 0.3)}{h_3} \right\}, \\ \text{cheap house} = \left\{ \frac{(0.2, 0.6)}{h_1}, \frac{(0.5, 0.3)}{h_2}, \frac{(0.6, 0.2)}{h_3} \right\} \end{array} \right\}$$

4.2. Restricted intersection

Let $(F, A)_{\tilde{R}}$ and $(G, B)_{\tilde{R}}$ be two soft conflicting bifuzzy sets over a common universe U such that $A \cap B \neq \phi$. The restricted intersection of $(F, A)_{\tilde{R}}$ and $(G, B)_{\tilde{R}}$ is denoted by $(F, A)_{\tilde{R}} \cap_{\mathfrak{R}} (G, B)_{\tilde{R}}$ and is defined as $(F, A)_{\tilde{R}} \cap_{\mathfrak{R}} (G, B)_{\tilde{R}} = (H, C)_{\tilde{R}}$ where $C = A \cap B$ and $c \in C, H(c) = F(c) \cap G(c)$.

Example 2.

Consider $(F, A)_{\tilde{R}}$ and $(G, B)_{\tilde{R}}$ be two soft conflicting bifuzzy sets over a common universe U as defined in example 1.

To find the $(F, A)_{\tilde{R}} \cap_{\mathfrak{R}} (G, B)_{\tilde{R}} = (H, C)_{\tilde{R}}$

We have $C = \{\text{beautiful}(e_1), \text{cheap}(e_4)\}$, then

$$\begin{aligned} H(e_1) &= F(e_1) \cap G(e_1) \\ &= \left\{ \frac{\min(0.3, 0.1), \max(0.4, 0.5)}{h_1}, \frac{\min(0.8, 0.4), \max(0.0, 0.3)}{h_2}, \frac{\min(0.0, 0.4), \max(0.4, 0.3)}{h_3} \right\} \\ &= \left\{ \frac{(0.1, 0.5)}{h_1}, \frac{(0.4, 0.3)}{h_2}, \frac{(0.0, 0.4)}{h_3} \right\} \end{aligned}$$

and

$$\begin{aligned} H(e_4) &= F(e_4) \cap G(e_4) \\ &= \left\{ \frac{\min(0.2, 0.1), \max(0.8, 0.6)}{h_1}, \frac{\min(0.4, 0.5), \max(0.5, 0.3)}{h_2}, \frac{\min(0.6, 0.3), \max(0.2, 0.4)}{h_3} \right\} \\ &= \left\{ \frac{(0.1, 0.8)}{h_1}, \frac{(0.4, 0.5)}{h_2}, \frac{(0.3, 0.4)}{h_3} \right\} \end{aligned}$$

Thus, we can view the soft conflicting bifuzzy set as a collection of fuzzy approximations (which are under the fuzzy relationship $\tilde{R}$) as below:

$$(H, C)_{\tilde{R}} = \left\{ \begin{array}{l} \text{beautiful house} = \left\{ \frac{(0.1, 0.5)}{h_1}, \frac{(0.4, 0.3)}{h_2}, \frac{(0.0, 0.4)}{h_3} \right\}, \\ \text{cheap house} = \left\{ \frac{(0.1, 0.8)}{h_1}, \frac{(0.4, 0.5)}{h_2}, \frac{(0.3, 0.4)}{h_3} \right\} \end{array} \right\}$$

4.3. Restricted difference

Let $(F, A)_{\bar{R}}$ and $(G, B)_{\bar{R}}$ be two soft conflicting bifuzzy sets over the same universe U such that $A \cap B \neq \phi$. The restricted difference of $(F, A)_{\bar{R}}$ and $(G, B)_{\bar{R}}$ denoted by $(F, A)_{\bar{R}} \sim_{\mathfrak{R}} (G, B)$, and is defined as $(F, A)_{\bar{R}} \sim_{\mathfrak{R}} (G, B)_{\bar{R}} = (H, C)_{\bar{R}}$, where $C = A \cap B$ and for all $c \in C$, $H(c) = F(c) - G(c)$, the difference of the sets $F(c)$ and $G(c)$.

4.4. Relative null soft conflicting bifuzzy set

Let U be an initial universe set, E be the universe set of parameters, and $A \subset E$ then $(F, A)_{\bar{R}}$ is called a relative null soft conflicting bifuzzy set (with respect to the parameter set A), denoted by ϕ_A , if $F(e) = \phi$ for all $e \in A$.

4.5. Relative whole soft conflicting bifuzzy set

Let U be an initial universe set, E be the universe set of parameters, and $A \subset E$ then $(G, A)_{\bar{R}}$ is called a relative whole soft conflicting bifuzzy set (with respect to the parameter set A), denoted by U_A , if $F(e) = U$ for all $e \in A$.

The relative whole soft conflicting bifuzzysset U_E with respect to the universe set of parameters E is called the absolute soft conflicting bifuzzy set over U .

4.6. Relative complement

The relative complement of a soft conflicting bifuzzy set $(F, A)_{\bar{R}}$ is denoted by $(F, A)_{\bar{R}}^r$ and is defined by $(F, A)_{\bar{R}}^r = (F^r, A)_{\bar{R}}$ where $F^r: A \rightarrow P(U)$ is a mapping defined by $F^r(\alpha) = U - F(\alpha)$ for all $\alpha \in A$.

Clearly, $(F, A)_{\bar{R}}^r = U_{E \sim \mathfrak{R}} (F, A)_{\bar{R}}$ and $((F, A)_{\bar{R}}^r)^r = (F, A)_{\bar{R}}$. It is worth noting that in the above definition of complement, the parameter set of the complement $(F, A)_{\bar{R}}^r$ is still the original parameter set A , instead of $\neg A$ as given in Definition 3.6. To emphasize this difference, the complement given in the Definition 3.6 will be called neg-complement (or pseudo-complement).

5. De Morgan's Laws in Soft Conflicting Bifuzzy Set Theory

In this section, we first show that the following De Morgan's type of results hold in soft set theory for the newly defined relative complement, restricted union and restricted intersection.

Theorem: Let $(F, A)_{\bar{R}}$ and $(G, B)_{\bar{R}}$ be two soft conflicting bifuzzy sets over the same universe U such that $A \cap B \neq \phi$. Then

- (i) $((F, A)_{\bar{R}} \cup_{\mathfrak{R}} (G, B)_{\bar{R}})^r = (F, A)_{\bar{R}}^r \cap_{\mathfrak{R}} (G, B)_{\bar{R}}^r$
- (ii) $((F, A)_{\bar{R}} \cap_{\mathfrak{R}} (G, B)_{\bar{R}})^r = (F, A)_{\bar{R}}^r \cup_{\mathfrak{R}} (G, B)_{\bar{R}}^r$

Proof.

(i) Let $(F, A)_{\bar{R}} \cup_{\mathfrak{R}} (G, B)_{\bar{R}} = (H, C)_{\bar{R}}$

where $H(c) = F(c) \cup G(c)$ for all $c \in C = A \cap B \neq \phi$

Since $((F, A)_{\bar{R}} \cup_{\mathfrak{R}} (G, B)_{\bar{R}})^r = (H, C)_{\bar{R}}^r$, by definition

$$H^r(c) = U - [F(c) \cup G(c)] = [U - F(c)] \cap [U - G(c)] \text{ for all } c \in C.$$

Now

$$(F, A)_{\bar{R}}^r \cap_{\mathfrak{R}} (G, B)_{\bar{R}}^r = (F^r, A)_{\bar{R}} \cap_{\mathfrak{R}} (G^r, B)_{\bar{R}} = (K, C)_{\bar{R}} \text{ where } C = A \cap B.$$

So by definition, we have

$$\begin{aligned} K(c) &= F^r(c) \cap G^r(c) \\ &= (U - F(c)) \cap (U - G(c)) \\ &= H^r(c) \text{ for all } c \in C. \end{aligned}$$

Hence $((F, A)_{\bar{R}} \cup_{\mathfrak{R}} (G, B)_{\bar{R}})^r = (F, A)_{\bar{R}}^r \cap_{\mathfrak{R}} (G, B)_{\bar{R}}^r$.

(ii) Let $(F, A)_{\bar{R}} \cap_{\mathfrak{R}} (G, B)_{\bar{R}} = (H, C)_{\bar{R}}$

where $H(c) = F(c) \cap G(c)$ for all $c \in C = A \cap B \neq \phi$

Since $((F, A)_{\bar{R}} \cap_{\mathfrak{R}} (G, B)_{\bar{R}})^r = (H, C)_{\bar{R}}^r$, by definition

$$H^r(c) = U - [F(c) \cap G(c)] = [U - F(c)] \cup [U - G(c)] \text{ for all } c \in C.$$

Now

$$(F, A)_{\bar{R}}^r \cup_{\mathfrak{R}} (G, B)_{\bar{R}}^r = (F^r, A)_{\bar{R}} \cup_{\mathfrak{R}} (G^r, B)_{\bar{R}} = (K, C)_{\bar{R}} \text{ where } C = A \cap B.$$

So by definition, we have

$$\begin{aligned} K(c) &= F^r(c) \cup G^r(c) \\ &= (U - F(c)) \cup (U - G(c)) \\ &= H^r(c) \text{ for all } c \in C \end{aligned}$$

Hence $((F, A)_{\bar{R}} \cap_{\mathfrak{R}} (G, B)_{\bar{R}})^r = (F, A)_{\bar{R}}^r \cup_{\mathfrak{R}} (G, B)_{\bar{R}}^r$.

Conclusion

Here we introduced new concepts of soft conflicting bifuzzy set. In this study we defined a soft conflicting bifuzzy set and its operations of equality, complement, union, intersection, OR and AND operators. We have given some new notions such as the restricted intersection, the restricted union and the restricted difference of two soft conflicting bifuzzy sets along with examples. We also improved the notion of complement of a soft conflicting

bifuzzy set and proved certain De Morgan's laws hold in soft conflicting bifuzzy set with respect to the new definitions.

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