

Fuzzy Contra *gprw*-Continuous Mappings

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Abstract: In this manuscript new types of fuzzy mappings namely fuzzy contra *gprw*-continuous mappings have been introduced & investigated. Also we found out its relation with various other fuzzy contra mappings introduced earlier. We also introduced fuzzy contra *gprw*-open mappings and fuzzy contra *gprw*-closed mappings in this paper.

Keywords: Fuzzy contra *gprw*-continuous mappings; Fuzzy contra pre-continuous mappings; Fuzzy contra *rw*-continuous mappings; Fuzzy contra *gprw*-open mappings; Fuzzy contra *gprw*-closed mappings.

1. Introduction

The idea of fuzzy contra mappings was put forward by Ekici and Kerre in 2006 in the paper¹. Soon after that, based on various other types of fuzzy sets various fuzzy contra mappings were introduced like in 2011 fuzzy contra *rw*-continuous mappings were introduced by A.Vadivel, V. Chandrasekar and M.Saraswathi in the paper². In 2012 S.E. Abbas and I.M. Taha³ introduced the concepts of fuzzy contra-continuity, fuzzy almost contra-continuity, fuzzy contra μ continuity, fuzzy almost contra μ continuity, fuzzy contra semi-continuity and generalized fuzzy contra continuity in.

Based on fuzzy *gprw*-closed sets, we have introduced a new type of mappings namely fuzzy contra *gprw*-continuous mappings in this manuscript and have found out its relation with various other mappings introduced earlier. We found out that all fuzzy contra continuous mappings are fuzzy *gprw*-continuous mappings, All fuzzy contra pre-continuous mappings are fuzzy contra *gprw*-continuous mappings & all fuzzy contra *rw*-continuous

mappings are fuzzy contra gprw-continuous mappings. The relationship of this new mapping with other mappings have been depicted via a table figure. Also we have introduced fuzzy contra gprw-open mappings and fuzzy contra gprw-closed mappings in this paper.

2. Preliminaries

Definition 2.1⁴ A mapping f is said to be a fuzzy continuous mapping if $f^{-1}(\lambda) \in \tau X$ for each $\lambda \in \tau Y$ or, equivalently $f^{-1}(\mu)$ is a fuzzy closed set of X for each fuzzy closed set μ of Y .

Definition 2.2⁵ A function $f: X \rightarrow Y$ is said to be fuzzy contra pre continuous, if $f^{-1}(\lambda)$ is fuzzy pre-closed in X for every fuzzy open set λ of Y .

Definition 2.3⁶ Suppose X and Y are fuzzy topological spaces. A map $f: X \rightarrow Y$ is called fuzzy contra rw -continuous if the inverse image of every fuzzy open set in Y is fuzzy rw -closed in X .

Definition 2.4⁷ A function $h: H \rightarrow K$ is called fuzzy generalized pre regular weakly continuous (briefly Fgprw-continuous) if inverse image of every fuzzy closed set in fuzzy topological space K is fuzzy generalized pre regular weakly closed (Fgprw-closed) in fuzzy topological space H .

Definition 2.5⁷ A function $h: (H, \tau_1) \rightarrow (K, \tau_2)$ is said to be fuzzy generalized pre regular weakly-irresolute (briefly Fgprw-irresolute) if $h^{-1}(\{\psi\})$ is fuzzy gprw-closed for every fuzzy gprw-closed $\{\psi\}$ in (K, τ_2) .

Definition 2.6³ let $f: (X, \tau_1) \rightarrow (Y, \tau_2)$ be a mapping. Then f is fuzzy contra open mapping, if it maps every fuzzy open set in (X, τ_1) to a fuzzy closed set in (Y, τ_2) .

Definition 2.7⁵ A function f from a fuzzy topological space (X, τ) to fuzzy topological space (Y, δ) is called fuzzy contra pre-continuous (fuzzy contra α -continuous, fuzzy contra semi-continuous) if $f^{-1}(\lambda)$ is fuzzy pre-closed (fuzzy α -closed, fuzzy semi-closed resp.) in X for every fuzzy open set λ of Y .

Remark 2.8⁸ All fuzzy closed sets are fuzzy gprw-closed.

Remark 2.9⁸ All fuzzy pre-closed sets are fuzzy gprw-closed.

Remark 2.10⁸ All fuzzy rw-closed sets are fuzzy gprw-closed.

Remark 2.11⁸ All fuzzy open sets are fuzzy gprw-open.

3. Fuzzy Contra *gprw*-Continuous Mappings

Definition 3.1 A mapping $r: (R, \tau_1) \rightarrow (S, \tau_2)$ is called fuzzy contra *gprw*-continuous if $r^{-1}(s): s \in \tau_2$ is fuzzy *gprw*-closed in R .

Theorem 3.2 A fuzzy contra continuous mapping $g: (G, \tau_1) \rightarrow (H, \tau_2)$ is always fuzzy contra *gprw*-continuous.

Proof: Consider $\psi \leq \tau_2$. Now, as g is fuzzy contra continuous implies $g^{-1}(\psi)$ is fuzzy closed in G . From Remark 2.8 all fuzzy closed sets are fuzzy *gprw*-closed, so $g^{-1}(\psi)$ is fuzzy *gprw*-closed in G . Hence g is fuzzy contra *gprw*-continuous.

The other way round of the above theorem need not be true, as shown in the following example.

Example 3.3 Consider $G = H = \{l, m, n\}$ and function $\eta, \psi, \chi: G \rightarrow [0,1]$ be defined as

$$\begin{aligned} \eta(g) &= \begin{cases} 1 & \text{if } g = l \\ 0 & \text{otherwise} \end{cases} & \psi(g) &= \begin{cases} 1 & \text{if } g = m \\ 0 & \text{otherwise} \end{cases} \\ \chi(g) &= \begin{cases} 1 & \text{if } g = m, n \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

Suppose $\tau_1 = \{0, 1, \eta\}$, $\tau_2 = \{0, 1, \psi, \chi\}$. Now (G, τ_1) and (H, τ_2) are fuzzy topological spaces. Now define a function $f: (G, \tau_1) \rightarrow (H, \tau_2)$ by $f(l) = m$, $f(m) = n$ and $f(n) = l$. Then f is fuzzy contra *gprw*-continuous & not fuzzy contra continuous as $f^{-1}(\psi)$ is η in (H, τ_2) & $\eta \in \tau_1$.

Theorem 3.4 A function $\zeta: (G, \tau_1) \rightarrow (H, \tau_2)$ is fuzzy contra *gprw*-continuous iff $\zeta^{-1}(\alpha)$ is fuzzy *gprw*-open in G for every $\alpha \in 1 - \tau_2$.

Proof: Suppose $\alpha \in 1 - \tau_2$, implying $1 - \alpha \in \tau_2$. Now as ζ is fuzzy contra *gprw*-continuous, implies $\zeta^{-1}(1 - \alpha)$ is fuzzy *gprw*-closed in G . Now as $\zeta^{-1}(1 - \alpha) = 1 - \zeta^{-1}(\alpha)$ implies that $\zeta^{-1}(\alpha)$ is fuzzy *gprw*-open in G .

Contrarily, assume that $\zeta^{-1}(\alpha)$ is fuzzy *gprw*-open in G for every $\alpha \in 1 - \tau_2$. Let $\beta \in \tau_2$, then $1 - \beta$ is fuzzy closed in H . By hypothesis $\zeta^{-1}(1 - \beta) = 1 - \zeta^{-1}(\beta)$ is fuzzy *gprw* open in G , implying $\zeta^{-1}(\beta)$ is fuzzy *gprw*-closed in G . Which proves the result.

Theorem 3.5 All fuzzy contra pre-continuous functions are fuzzy contra *gprw*-continuous.

Proof: Let $g: (G, \tau_1) \rightarrow (H, \tau_2)$ be fuzzy contra pre-continuous and suppose $\lambda \in \tau_2$. So $g^{-1}(\lambda)$ is fuzzy pre-closed in G . Now by Remark 2.9 $g^{-1}(\lambda)$ is fuzzy *gprw*-closed in G . Hence g is fuzzy contra *gprw*-continuous.

The converse of the above theorem need not be true as shown in the following example.

Example 3.6 Consider $G = H = \{l, m, n\}$ and function $\omega, \eta, \psi, \chi: G \rightarrow [0,1]$ be defined as

$$\begin{aligned}\omega(g) &= \begin{cases} 1 & \text{if } g = l \\ 0 & \text{otherwise} \end{cases} & \psi(g) &= \begin{cases} 1 & \text{if } g = l, n \\ 0 & \text{otherwise} \end{cases} \\ \chi(g) &= \begin{cases} 1 & \text{if } g = l, m \\ 0 & \text{otherwise} \end{cases} & \eta(g) &= \begin{cases} 1 & \text{if } g = m \\ 0 & \text{otherwise} \end{cases}\end{aligned}$$

Suppose $\tau_1 = \{0,1,\omega,\psi\}$, $\tau_2 = \{0,1,\omega,\chi\}$. Now (G, τ_1) and (H, τ_2) are fuzzy topological spaces. Now, we define a function $f: (G, \tau_1) \rightarrow (H, \tau_2)$ by $f(l) = l$, $f(m) = n$ and $f(n) = m$. Then f is fuzzy contra gprw-continuous & not fuzzy contra pre-continuous as $f^{-1}(\omega)$ in (H, τ_2) is ω , which is fuzzy gprw-closed in (G, τ_1) but not fuzzy pre-closed.

Theorem 3.7 A fuzzy contra rw-continuous mapping $g: (G, \tau_1) \rightarrow (H, \tau_2)$ is fuzzy contra gprw-continuous also.

Proof: Consider $\alpha \leq \tau_2$, Now as g is fuzzy contra rw-continuous, implies $g^{-1}(\alpha)$ is fuzzy rw-closed in G . Now from Remark 2.10 all fuzzy rw-closed sets are fuzzy gprw-closed, so $g^{-1}(\alpha)$ is fuzzy gprw-closed in G , implying g is fuzzy contra gprw-continuous.

The converse of the above theorem need not be true as shown in the following example.

Example 3.8 Consider $G = H = \{l, m, n, o, p\}$ are fuzzy spaces and functions $\eta, \alpha, \beta, \gamma: G \rightarrow [0,1]$ and $\delta: H \rightarrow [0,1]$ are defined as

$$\begin{aligned}\alpha(g) &= \begin{cases} 1 & \text{if } g = l, m \\ 0 & \text{otherwise} \end{cases} & \eta(g) &= \begin{cases} 1 & \text{if } g = p \\ 0 & \text{otherwise} \end{cases} \\ \beta(g) &= \begin{cases} 1 & \text{if } g = n, o \\ 0 & \text{otherwise} \end{cases} & \gamma(g) &= \begin{cases} 1 & \text{if } g = l, m, n, o \\ 0 & \text{otherwise} \end{cases} \\ \delta(h) &= \begin{cases} 1 & \text{if } h = l \\ 0 & \text{otherwise} \end{cases}\end{aligned}$$

Suppose $\tau_1 = \{0,1,\alpha,\beta,\gamma\}$, $\tau_2 = \{0,1,\delta\}$. With these topologies (G, τ_1) and (H, τ_2) are fuzzy topological spaces. Now, we define a function $f: (G, \tau_1) \rightarrow (H, \tau_2)$ by $f(l) = m$, $f(m) = n$, $f(n) = o$, $f(o) = p$ and $f(p) = l$. Then f is fuzzy contra gprw-continuous but not fuzzy contra rw-continuous as $f^{-1}(\delta)$ in (H, τ_2) is η , & η is fuzzy gprw-closed in (G, τ_1) but not fuzzy rw-closed.

Remark 3.9 In the following examples we prove that Fuzzy contra gprw-continuous and fuzzy contra semi-continuous mappings are independent.

Example 3.10 Consider $G = H = \{p, q, r, s\}$ are fuzzy spaces and functions $\alpha, \beta, \gamma, \delta: G \rightarrow [0,1]$ be defined as

$$\begin{aligned}\alpha(g) &= \begin{cases} 1 & \text{if } g = p \\ 0 & \text{otherwise} \end{cases} & \beta(g) &= \begin{cases} 1 & \text{if } g = q \\ 0 & \text{otherwise} \end{cases} \\ \gamma(g) &= \begin{cases} 1 & \text{if } g = p, q \\ 0 & \text{otherwise} \end{cases} & \delta(g) &= \begin{cases} 1 & \text{if } g = p, q, r \\ 0 & \text{otherwise} \end{cases}\end{aligned}$$

and $\psi, \eta: H \rightarrow [0,1]$ be defined as

$$\begin{aligned}\psi(h) &= \begin{cases} 1 & \text{if } h = r \\ 0 & \text{otherwise} \end{cases} & \eta(h) &= \begin{cases} 1 & \text{if } h = r, s \\ 0 & \text{otherwise} \end{cases}\end{aligned}$$

Suppose $\tau_1 = \{0,1, \alpha, \beta, \gamma, \delta\}$, $\tau_2 = \{0,1, \eta, \psi\}$. With these topologies (G, τ_1) and (H, τ_2) are fuzzy topological spaces. Now, we define a function $f: (G, \tau_1) \rightarrow (H, \tau_2)$ by $f(p) = r$, $f(q) = s$, $f(r) = p$, $f(s) = q$. Then f is fuzzy contra semi-continuous but not fuzzy contra gprw-continuous as $f^{-1}(\psi)$ in (H, τ_2) is α , which is fuzzy semi- closed in (G, τ_1) but not fuzzy gprw-closed.

Example 3.11 Consider fuzzy topological spaces (G, τ_1) and (H, τ_2) as defined in Example 3.10. Now, if we define a mapping $f: (G, \tau_1) \rightarrow (H, \tau_2)$ by $f(p) = r$, $f(q) = s$, $f(r) = q$, $f(s) = p$. Then f is fuzzy contra gprw-continuous & not fuzzy contra semi-continuous as $f^{-1}(\eta)$ in (H, τ_2) is γ , which is fuzzy gprw- closed in (G, τ_1) but not fuzzy semi-closed.

Theorem 3.12 Suppose $g: (G, \tau_1) \rightarrow (H, \tau_2)$ is fuzzy continuous and $h: (L, \tau_3) \rightarrow (G, \tau_1)$ is fuzzy contra gprw-continuous, then their composition map $goh: (L, \tau_3) \rightarrow (H, \tau_2)$ is fuzzy contra gprw-continuous.

Proof: Suppose $\alpha \leq \tau_2$. Since g is fuzzy continuous, implies $g^{-1}(\alpha) \leq \tau_1$. Now h is fuzzy contra gprw-continuous, so $h^{-1}(g^{-1}(\alpha))$ is fuzzy gprw-closed in (L, τ_3) . Since $(goh)^{-1}(\alpha) = h^{-1}(g^{-1}(\alpha))$. So $goh: (L, \tau_3) \rightarrow (H, \tau_2)$ is fuzzy contra gprw-continuous.

Remark 3.13 In the following examples we prove that Fuzzy contra gprw-continuous and fuzzy contra generalized continuous mappings are independent.

Example 3.14 Consider fuzzy topological spaces (G, τ_1) and (H, τ_2) as defined in Example 3.10. Now, if we define a mapping $l: (G, \tau_1) \rightarrow (H, \tau_2)$ by $l(p) = r$, $l(q) = p$, $l(r) = q$ and $l(s) = s$. Then l is fuzzy contra generalized continuous mapping but not fuzzy contra gprw-continuous as $l^{-1}(\eta)$ in (H, τ_2) is $\chi: G \rightarrow [0,1]$ defined as

$$\chi(g) = \begin{cases} 1 & \text{if } g = p, s \\ 0 & \text{otherwise} \end{cases}$$

which is fuzzy generalized closed in (G, τ_1) but not fuzzy gprw-closed.

Example 3.15 Consider fuzzy topological spaces (G, τ_1) and (H, τ_2) as defined in Example 3.10. Now, if we define a mapping $h: (G, \tau_1) \rightarrow (H, \tau_2)$ by $h(p) = r$, $h(q) = s$, $h(r) = p$ and $h(s) = q$. Then h is fuzzy contra gprw-continuous mapping but not fuzzy contra generalized continuous as $h^{-1}(\eta)$ in (H, τ_2) is γ in (G, τ_1) , which is fuzzy gprw-closed in (G, τ_1) but not fuzzy generalized closed.

Remark 3.16: From the above discussion of Results we have the following diagram of implications. Here

$A \rightarrow B$ means A implies B .

$A \leftrightarrow B$ means A & B are independent of each other.

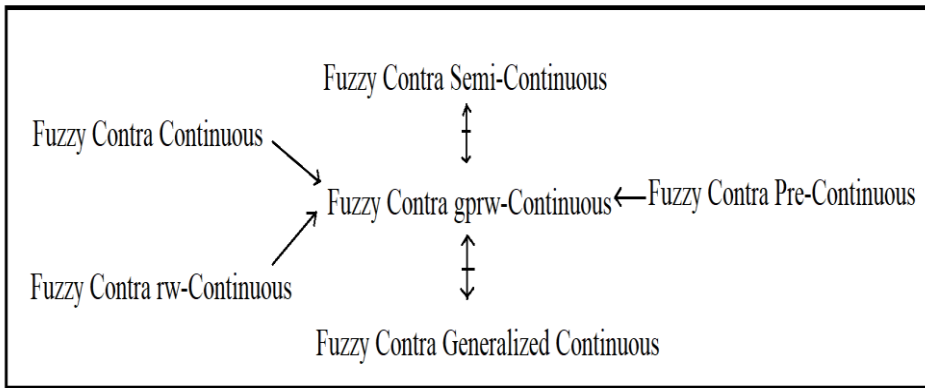


Figure 1. Relation of fuzzy contra gprw-continuous mappings with other contra mappings

Definition 3.17 Suppose (G, τ_1) and (H, τ_2) be two fuzzy topological spaces. Then a function $g: (G, \tau_1) \rightarrow (H, \tau_2)$ is called fuzzy contra gprw-contra irresolute map if $g^{-1}(h)$ is fuzzy gprw-closed in (G, τ_1) for every fuzzy gprw-open set h in (H, τ_2) .

Theorem 3.18 If $g: (G, \tau_1) \rightarrow (H, \tau_2)$ is fuzzy contra gprw-irresolute, then it is fuzzy contra gprw-continuous.

Proof: Suppose $\alpha \leq \tau_2$, Now from Remark 2.11 α is fuzzy gprw-open in (H, τ_2) . Since g is fuzzy contra gprw-irresolute, implying $g^{-1}(\alpha)$ is fuzzy gprw-closed in (G, τ_1) . Thus g is fuzzy contra gprw-continuous.

Theorem 3.19 Let (L, τ_1) , (M, τ_2) and (N, τ_3) are fuzzy topological spaces. If $l: (L, \tau_1) \rightarrow (M, \tau_2)$ is fuzzy contra gprw-irresolute and $k: (M, \tau_2) \rightarrow (N, \tau_3)$ is fuzzy gprw-continuous, then their composition $kol: (L, \tau_1) \rightarrow (N, \tau_3)$ is fuzzy contra gprw-continuous.

Proof: Suppose $\alpha \leq \tau_3$, Now as k is fuzzy gprw-continuous means $k^{-1}(\alpha)$ is fuzzy gprw open set in (M, τ_2) . Now as l is fuzzy contra gprw-irresolute, implies $l^{-1}(k^{-1}(\alpha))$ is fuzzy gprw closed set in (L, τ_1) . But $l^{-1}(k^{-1}(\alpha)) = (kol)^{-1}(\alpha)$, implies kol is fuzzy contra gprw-continuous.

Theorem 3.20 Let (L, τ_1) , (M, τ_2) and (N, τ_3) are fuzzy topological spaces. If $l: (L, \tau_1) \rightarrow (M, \tau_2)$ is fuzzy gprw-irresolute and $m: (M, \tau_2) \rightarrow (N, \tau_3)$ is fuzzy contra gprw-irresolute, then their composition $mol: (L, \tau_1) \rightarrow (N, \tau_3)$ is fuzzy contra gprw-irresolute.

Proof: Suppose α is fuzzy gprw-open in (N, τ_3) . Since m is fuzzy contra gprw-irresolute, implies $m^{-1}(\alpha)$ is fuzzy gprw-closed in (M, τ_2) . Now as l is fuzzy gprw-irresolute, implies $l^{-1}(m^{-1}(\alpha))$ is fuzzy gprw-closed in (L, τ_1) . Now $(mol)^{-1}(\alpha) = l^{-1}(m^{-1}(\alpha))$, implying $mol: (L, \tau_1) \rightarrow (N, \tau_3)$ is fuzzy contra gprw-irresolute.

4. Fuzzy Contra gprw-open Mappings and Fuzzy Contra gprw-closed Mappings

Definition 4.1 A mapping $g: (G, \tau_1) \rightarrow (H, \tau_2)$ is fuzzy contra gprw-open if the image of $\lambda \leq \tau_1$ in (G, τ_1) is fuzzy gprw-closed in (H, τ_2) .

Example 4.2 All fuzzy contra open mappings are fuzzy contra gprw-open mappings.

Proof: Suppose $l: (G, \tau_1) \rightarrow (H, \tau_2)$ be a fuzzy contra open mapping and $\alpha \leq \tau_1$, then $l(\alpha) \leq 1 - \tau_2$. Now remark 2.8 implies $l(\alpha)$ is fuzzy gprw-closed set in (H, τ_2) . Hence l is fuzzy contra gprw-open mapping.

The other way round of the above theorem need not be true, as shown in the following example.

Example 4.3 Suppose $G = H = \{l, m, n, o\}$ are fuzzy spaces and functions $\alpha, \beta, \gamma, \delta: G \rightarrow [0, 1]$ be defined as

$$\alpha(g) = \begin{cases} 1 & \text{if } g = l \\ 0 & \text{otherwise} \end{cases} \quad \beta(g) = \begin{cases} 1 & \text{if } g = m \\ 0 & \text{otherwise} \end{cases}$$

$$\gamma(g) = \begin{cases} 1 & \text{if } g = l, m \\ 0 & \text{otherwise} \end{cases} \quad \delta(g) = \begin{cases} 1 & \text{if } g = l, m, n \\ 0 & \text{otherwise} \end{cases}$$

Consider $\tau_1 = \{0, 1, \alpha, \beta, \gamma\}$, $\tau_2 = \{0, 1, \alpha, \beta, \gamma, \delta\}$. With these topologies (G, τ_1) and (H, τ_2) are fuzzy topological spaces. Now, we define a mapping $f: (G, \tau_1) \rightarrow (H, \tau_2)$ by $f(l) = l$, $f(m) = m$, $f(n) = o$ & $f(o) = n$. Then f is fuzzy contra gprw-open mapping but not fuzzy contra open mapping, as image of $\gamma \leq \tau_1$ in (G, τ_1) is fuzzy set γ in (H, τ_2) which is fuzzy gprw-closed in (H, τ_2) but not fuzzy closed.

Example 4.4 If $l: (G, \tau_1) \rightarrow (H, \tau_2)$ is a fuzzy open map and $m: (H, \tau_2) \rightarrow (K, \tau_3)$ is fuzzy contra gprw-open, then the composition map $mol: (G, \tau_1) \rightarrow (K, \tau_3)$ is fuzzy contra gprw-open map.

Proof: Suppose $\alpha \leq \tau_1$. Now, as l is a fuzzy open map implies $l(\alpha) \leq \tau_2$. Since m is a fuzzy contra gprw-open map $m(l(\alpha))$ is fuzzy gprw-closed set in (K, τ_3) . Now $m(l(\alpha)) = (mol)(\alpha)$, implying mol is fuzzy contra gprw-open map.

Definition 4.5 Let (G, τ_1) and (H, τ_2) be two fuzzy topological spaces. A mapping $g: (G, \tau_1) \rightarrow (H, \tau_2)$ is called fuzzy contra gprw-closed if the image of $\gamma \leq 1 - \tau_1$ in (G, τ_1) is fuzzy gprw-open in (H, τ_2) .

Theorem 4.6 If $l: (G, \tau_1) \rightarrow (H, \tau_2)$ is a fuzzy contra closed mapping, then it is fuzzy contra gprw-closed mapping also.

Proof: Suppose $\lambda \leq 1 - \tau_1$, Now as l is fuzzy contra closed mapping, implies $l(\lambda) \leq \tau_2$. Now, as all fuzzy open sets are fuzzy gprw-open implying that $l(\lambda)$ is fuzzy gprw-open in (H, τ_2) . Hence l is a fuzzy contra gprw-closed mapping.

The other way round of the above theorem need not be true, as shown in the following example.

Example 4.7 Suppose $X = Y = \{l, m, n, o, p\}$ are fuzzy topological spaces with topologies $\tau_1 = \{0, 1, \chi\}$ and $\tau_2 = \{0, 1, \alpha, \beta, \gamma\}$ where $\chi: X \rightarrow [0, 1]$ and $\alpha, \beta, \gamma: Y \rightarrow [0, 1]$ are defined as

$$\begin{aligned} \chi(x) &= \begin{cases} 1 & \text{if } x = p \\ 0 & \text{otherwise} \end{cases} & \beta(y) &= \begin{cases} 1 & \text{if } y = n, o \\ 0 & \text{otherwise} \end{cases} \\ \alpha(y) &= \begin{cases} 1 & \text{if } y = l, m \\ 0 & \text{otherwise} \end{cases} & \gamma(y) &= \begin{cases} 1 & \text{if } y = l, m, n, o \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

Let the function $f: (X, \tau_1) \rightarrow (Y, \tau_2)$ be defined as $f(l) = m$, $f(m) = n$, $f(n) = o$, $f(o) = p$ and $f(p) = l$. Then f is fuzzy contra gprw-closed map but not fuzzy contra closed map as image of $\psi \leq 1 - \tau_1$ in X defined as

$$\psi(x) = \begin{cases} 1 & \text{if } x = l, m, n, o \\ 0 & \text{otherwise} \end{cases}$$

is μ in Y defined as

$$\mu(y) = \begin{cases} 1 & \text{if } y = m, n, o, p \\ 0 & \text{otherwise} \end{cases}$$

which is fuzzy gprw-open in (Y, τ_2) but not fuzzy open.

Theorem 4.8 If $l: (G, \tau_1) \rightarrow (H, \tau_2)$ and $m: (H, \tau_2) \rightarrow (K, \tau_3)$ be two maps. Then $mol: (G, \tau_1) \rightarrow (K, \tau_3)$ is fuzzy contra gprw-closed map if l is a fuzzy closed mapping and m is fuzzy contra gprw-closed mapping.

Proof: Let $\alpha \leq 1 - \tau_1$. Since l is a fuzzy closed mapping, so $l(\alpha) \leq 1 - \tau_2$. Now m is a fuzzy contra gprw-closed map, implies $m(l(\alpha))$ is fuzzy gprw-open in (K, τ_3) . But $m(l(\alpha)) = (mol)(\alpha)$, implying mol is fuzzy contra gprw-closed mapping.

Theorem 4.9 Let $l: (G, \tau_1) \rightarrow (H, \tau_2)$ and $m: (H, \tau_2) \rightarrow (K, \tau_3)$ be two mappings such that $mol: (G, \tau_1) \rightarrow (K, \tau_3)$ is fuzzy contra gprw-closed map then

- Suppose l is fuzzy continuous and onto then m is fuzzy contra gprw-closed.
- Suppose m is fuzzy gprw-irresolute and one-one then l is fuzzy contra gprw-closed.

Proof: (I) Let $\alpha \leq \tau_2$. Since l is fuzzy continuous, $l^{-1}(\alpha) \leq 1 - \tau_1$. Since mol is fuzzy contra gprw-closed map, $(mol)(l^{-1}(\alpha))$ is fuzzy gprw-open set in I . But $(mol)(l^{-1}(\alpha)) = m(\alpha)$, as l is surjective. Thus m is fuzzy contra gprw-closed.

(II) Let $\mu \leq 1 - \tau_1$. Now mol is fuzzy contra gprw-closed, implies $mol(\mu)$ is fuzzy gprw-open in I . Since m is fuzzy gprw-irresolute, so $m^{-1}(mol)(\mu)$ is fuzzy gprw-open in (H, τ_2) . But $m^{-1}(mol)(\mu) = l(\mu)$ as m is injective. Thus l is fuzzy contra gprw-closed mapping.

References

1. E. Ekici and E. Kerre, On fuzzy contra-continuities, *Advances in Fuzzy Mathematics*, 1 (2006) 35-44.
2. A.Vadivel,V. Chandrasekar and M.Saraswathi, Fuzzy contra rw-continuous functions, *Advanced in fuzzy mathematics*, 6-1 (2011) 53-60.
3. S. E. Abbas and I.M. Taha, Weaker Forms of Fuzzy Contra-continuity in Fuzzy Topological Spaces, *The Journal of Fuzzy Mathematics*, 20-4 (2012).

4. K. K. Azad On Fuzzy Semicontinuity, Fuzzy Almost Continuity and Fuzzy Weakly Continuity, *Journal Of Mathematical Analysis and Applications*, 82 (1981), 14-32.
5. S. Jafari and Nori, On contra- precontinuous functions, *Bull. Malays Math. Sci. Soc.* 25-2 (2002), 115-128.
6. A. Vadivel, V. Chandrasekar and M. Saraswathi, Fuzzy Contra rw-continuous Functions. *Advances in Fuzzy Mathematics*, 6 (2011) 53-60.
7. Firdose Habib and Khaja Moinuddin, On Fuzzy gprw-closed sets in Fuzzy Topological spaces, *Journal of the Gujarat Research Society*, 21-14, (2019).
8. Firdose Habib and Khaja Moinuddin, On fuzzy generalized pre regular weakly continuity, *South East Asian J. of Mathematics and Mathematical Sciences*, 17-2 (2021), 181-188.