

Analytic Behaviour of a Parallel Redundant Complex System Involving Environmental Failure under Head of Line Repair Discipline

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Abstract: Reliability requirements is now more important than ever as electronic products become more reliable and customers become more sophisticated in their needs and demand. In a complex system, if the components are connected in such a way that if one component fails, there is the complete breakdown of the system. In this paper, we discussed the analytical behaviour of a complex system with parallel redundancy under different priority repairs incorporating the concept of reduced efficiency in the class of redundantly connected components under environmental failure. The present mathematical model considers a complex system, composed of two subsystems L_1 and L_2 where L_1 consists of N non-identical components in series and L_2 consists of M identical components in parallel redundancy.

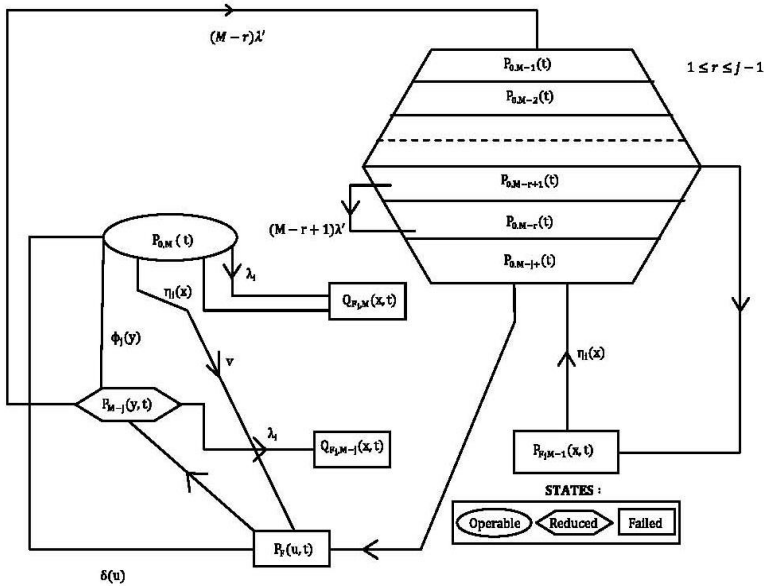
Keywords: Reliability, Redundancy, complex system, Parallel Redundancy.

1. Introduction

In this paper we study of Analytic Behavior of a parallel redundant complex system involving environmental behaviour under head of the line repair discipline. In this policy and failed component from either subsystem i.e. from A or B if taken up for repair, will be repaired first and only after this other failed components will be taken up for repair. This implies that this repair policy is 'first come first served'. Laplace transform and supplementary variable techniques have been utilised to obtain the probabilities of the system being in various states. The ergodic behaviour of the system and some more reliability parameters have also been evaluated.

Notations

N, M	- Number of L_1 - units and L_2 - units
i, j	- Subscript, denotes the serial number of L_1 units and L_2 units.
λ_i/λ'	- Constant failure rate of i^{th} unit of subsystem L_1 /any unit of subsystem L_2
λ	- Cumulative constant failure rate of subsystem L_1 , i.e, $\sum_{i=1}^M \lambda$
v	- Constant environmental failure rate
$\eta_i(x), S_i(x)$	- Transition rate and PdF, repair of i^{th} L_1 unit is completed at time x
$\phi_j(y), S_j(y)$	- Transition rate and PdF, repair of j^{th} L_2 unit is completed at time y
$\delta(u), S_\delta(u)$	- Transition rate and PdF, repair of i, j the L_1 and L_2 unit is completed at time u
$\bar{F}(s)$	- L.T. of $F(t)$
$P_{0.M}(t)$	- The probability that at time 't', the system is operating in the state of normal efficiency
$P_{0.M-r}(t)$	- The probability that at time 't', the system is in operable state where in r components of class L_2 have already failed
$P_{0.M-j}(y - t)\Delta$	- The probability that at time 't', the system is operating in reduced efficiency due to the failure of j components of class L_2 the elapsed repair time lies in the interval $(y, y + \Delta)$.
$P_{F_i.M}(x, t)\Delta$	- The probability that at time 't', the system is in the failed state due to the failure of i^{th} component of class L_1 and all the M components of class L_2 are in operable state and the elapsed repair time lies in the interval $(x, x + \Delta)$.
$P_{F_j.M-r}(x, t)\Delta$	- The probability that at time 't', the system is in the failed state due to the failure of the i^{th} component of class L_1 and $(M - r)$ components of class L_2 are in operable state and the elapsed repair time for class L_1 lies in the interval $(x, x + \Delta)$ conditioned that no repair is carried out for class L_2 .
$P_{F_j.M-j}(y, t)\Delta$	- The probability that at time 't', the system is in the failed state due to the failure of the i^{th} component of class L_1 and any j components of class L_2 , the elapsed repair time of any j components of class L_2 lies in the interval $(y, y + \Delta)$ whereas class L_1 is awaiting repair.
$P_F(u, t)\Delta$	- The probability that at time 't', the system is in failed state due to the environmental failure. The elapsed repair time lies in the interval $(u, u + \Delta)$.



2. Mathematical Model of the Problem

Probabilistic considerations and limiting procedure yields for following difference differential equations for the Stochastic process which is continuous in time and discrete in space.

$$(2.1) \quad \left[\frac{\partial}{\partial t} + \lambda + M\lambda' + v \right] P_{0,M}(t) = \int_0^\infty P_{0,M-j}(y, t) \vartheta_j(y) dy \\ + \sum_{i=1}^N \int_0^\infty P_{F_i,M}(x, t) \eta_j(x) dx + \int_0^\infty \delta(u) \cdot \bar{P}_F(u, t) du$$

$$(2.2) \quad \left[\frac{\partial}{\partial t} + \lambda + (M-r)\lambda' + v \right] P_{0,M-r}(t) = (M-r+1)\lambda' P_{0,M-1+1}(t) + \sum_{i=1}^N \int_0^\infty P_{F_1 M-r}(x, t) \eta_1(x) dx$$

$$(2.3) \quad \left[\frac{\partial}{\partial y} + \frac{\partial}{\partial t} + \lambda + \phi_1 + v \right] P_{0.M-j}(y, t) = 0$$

$$(2.4) \quad \left[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \eta_i(x) \right] P_{F_i, M}(x, t) = 0$$

$$(2.5) \quad \left[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \eta_1(x) \right] P_{F_i, M-r}(x, t) = 0$$

$$(2.6) \quad \left[\frac{\partial}{\partial v} + \frac{\partial}{\partial t} + \phi_j(y) \right] Q_{F_i, M-j}(y, t) = \lambda_i \cdot P_{0, M-j}(y, t)$$

$$(2.7) \quad \left[\frac{\partial}{\partial v} + \frac{\partial}{\partial t} + \delta(u) \right] P_F(u, t) = 0$$

Initial Conditions

$$(2.8) \quad P_{0,M}(0) = 1$$

So that other probability states are zero.

Boundary Conditions

$$(2.9) \quad P_{0,M-j}(0, t) = (M - j + 1)\lambda' P_{0,M-j+1}(t)$$

$$(2.10) \quad P_{F_i,M}(0, t) = \lambda_i P_{0,M}(t) + \int_0^\infty Q_{F_1M-j}(y, t) \phi_j(y) dy$$

$$(2.11) \quad P_{F_i,M}(0, t) = \lambda_i P_{0,M-r}(t)$$

$$(2.12) \quad Q_{F_i,M-j}(0, t) = 0$$

$$(2.13) \quad \bar{P}_t(0, t) = v[P_{0,M-j}(y, t)dy + P_{0,M-r}(t) + P_{0,M}(t)]$$

Solution of the Model

Taking Laplace transform of eqn. (2.1) through (2.13) with respect to 't'

$$(2.14) \quad (s + \lambda + M\lambda' + v)\bar{P}_{0,M}(s) = 1 + \int_0^\infty \bar{P}_{0,M-j}(y, s)\phi_j(y)dy \\ + \sum_{i=1}^N \int_0^\infty \bar{P}_{F_i,M}(x, s)\eta_i(x)dx + \int_0^\infty \delta(u) \cdot \bar{P}_F(u, s)du$$

$$(2.15) \quad (s + \lambda + (M - r)\lambda' + v)\bar{P}_{0,M-r}(s) = (m - r + 1)\lambda'\bar{P}_{0,M-r+1}(s) \\ + \sum_{i=1}^N \int_0^\infty \bar{P}_{F_i,M-r}(x, s)\eta_i(x)dx$$

$$(2.16) \quad \left[\frac{\partial}{\partial y} + s + \lambda + \phi_1(y) + v \right] \bar{P}_{0,M-j}(y, s) = 0$$

$$(2.17) \quad \left[\frac{\partial}{\partial x} + s + \eta_i(x) \right] \bar{P}_{F_i,M}(x, s) = 0$$

$$(2.18) \quad \left[\frac{\partial}{\partial x} + s + \eta_i(x) \right] \bar{P}_{F_i,M-r}(x, s) = 0$$

$$(2.19) \quad \left[\frac{\partial}{\partial x} + s + \phi_i(y) \right] \bar{Q}_{F_i,M-j}(y, s) = \lambda_i \cdot \bar{P}_{0,M-j}(y, s)$$

$$(2.20) \quad \left[\frac{\partial}{\partial u} + s + \delta(u) \right] \bar{P}_F(u, s) = 0$$

$$(2.21) \quad \bar{P}_{0,M-j}(0, s) = (M - j + 1)\lambda'\bar{P}_{0,M-j+1}(s)$$

$$(2.22) \quad \bar{P}_{F_i,M}(0, s) = P_{0,M}(s) \cdot \lambda_i + \int_0^\infty Q_{F_i,M-j}(y, s)\phi_j(y)dy$$

$$(2.23) \quad \bar{P}_{F_i,M-r}(0, s) = \lambda_i \bar{P}_{0,M-r}(s)$$

$$(2.24) \quad \bar{Q}_{F_i,M-j}(0, s) = 0$$

$$(2.25) \quad \bar{P}_F(0, s) = \left[\int_0^\infty P_{0,M-j}(y, s)dy + \bar{P}_{0,M-r}(s) + \bar{P}_{0,M}(s) \right]$$

$$(2.26) \quad \bar{P}_{0,M-j}(y, s) = e^{-(s+\lambda+v)y - \int_0^y \phi_i(y) dy} (M-j+1) \lambda' \bar{P}_{0,M-j+1}(s)$$

$$(2.27) \quad \bar{P}_{F_1,M}(x, s) = e^{-sx - \int_0^x \eta_i(x) dx} \cdot \bar{P}_{F_1,M}(0, s)$$

$$(2.28) \quad \bar{P}_{F_1,M-r}(x, s) = \lambda_i \bar{P}_{F_1,M-r}(s) e^{-sx - \int_0^x \eta_i(x) dx}$$

$$(2.29) \quad \bar{Q}_{F_1,M-j}(y, s) = \frac{\lambda_i \lambda'}{v + \lambda} (M-j+1) \bar{P}_{0,M-j+1}(s) (1 - e^{-sy - \int_0^y \phi_j(y) dy})$$

$$(2.30) \quad \bar{P}_{F_1,M}(0, s) = \bar{P}_{0,M}(s) \cdot \lambda_i + \frac{\lambda_i \lambda'}{v + \lambda} (M-j+1) \bar{P}_{0,M-j+1} [\bar{S}_j(s) - \bar{S}_j(s + \lambda + v)]$$

$$(2.31) \quad \bar{P}_{0,M-r}(s) = (\lambda')^r \prod_{p=0}^{r-1} \left(\frac{M-p}{s + \lambda + v + (M-p-1)\lambda' - \sum_{i=1}^N \lambda_i \bar{S}_i(s)} \right) \bar{P}_{0,M}(s)$$

$$(2.32) \quad \bar{P}_F(0, s) = v \left[(M-j+1) \lambda' \frac{1 - \bar{S}_j(s + \lambda + v)}{(s + \lambda + v)} \cdot (\lambda')^{j-1} \right. \\ \times \prod_{p=1}^{j-1} \frac{M-p+1}{[s + \lambda + v + (M-p)\lambda' - \sum_{i=1}^N \lambda_i \bar{S}_i(s)]} \\ \left. + (\lambda')^r \prod_{p=0}^{r-1} \left[\frac{(M-p)}{[s + \lambda + v + (M-p)\lambda' - \sum \lambda_i \bar{S}_i(s)]} + 1 \right] \bar{P}_{0,M}(s) \right]$$

$$(2.33) \quad \bar{P}_{F(u,s)} = e^{-su - \int_0^u \delta(u) du} \cdot v [(M-j+1) (\lambda')^j \frac{1 - \bar{S}_j(s + \lambda + v)}{(s + \lambda + v)} \\ \times \prod_{p=1}^{j-1} \left[\frac{(M-p+1)}{[s + \lambda + v + (M-p)\lambda' - \sum \lambda_i \bar{S}_i(s)]} \right. \\ \left. + (\lambda')^r \prod_{p=0}^{r-1} \frac{(M-p)}{[s + \lambda + v + (M-p-1)\lambda' - \sum_{i=1}^N \lambda_i \bar{S}_i(s)]} + 1 \right] \bar{P}_{0,M}(s)]$$

$$(2.34) \quad P_{0,M}(s) = \frac{1}{D(s)}$$

$$\text{where } D(s) = s + \lambda + M\lambda' + v - \sum_{i=1}^N \lambda_i \bar{S}_i(s) - (M-j+1)(\lambda')^j \\ \times \prod_{p=1}^{j-1} \frac{(M-p+1)}{s + \lambda + v + (M-p)\lambda' - \sum \lambda_i \bar{S}_i(s)} \\ \times \left\{ \bar{S}_j(s + \lambda + v) + \frac{\bar{S}_j(s) - \bar{S}_j(s + \lambda + v)}{v + \lambda} \sum_{i=1}^N \lambda_i \bar{S}_i(s) \right\} \\ - v \bar{S}_\delta(s) \left\{ (\lambda')^r \prod_{p=0}^{r-1} \frac{(M-p)}{s + \lambda + v + (M-p-1)\lambda' - \sum \lambda_i \bar{S}_i(s)} \right. \\ \left. + (M-j+1)(\lambda')^j \frac{1 - \bar{S}_j(s + \lambda + v)}{s + \lambda + v} \right. \\ \left. \times \prod_{p=1}^{j-1} \frac{(M-p+1)}{s + \lambda + v + (M-p)\lambda' - \sum \lambda_i \bar{S}_i(s)} + 1 \right\}$$

$$(2.35) \quad \bar{P}_{0,M-r}(s) = \frac{(\lambda')^r}{D(s)} \prod_{P=0}^{r-1} \left(\frac{(M-P)}{s + \lambda + v + (M-P-1)\lambda' - \sum_{i=1}^N \lambda_i \bar{S}_i(s)} \right)$$

$$(2.36) \quad \bar{P}_R(s) = \bar{P}_{0,M-j}(s) = \frac{M-j+1}{D(s)} \cdot \frac{1-\bar{S}_j(s+\lambda+v)}{s+\lambda+v} (\lambda')^j \\ \times \prod_{P=1}^{j-1} \left(\frac{M-P+1}{s + \lambda + v + (M-P)\lambda' - \sum \lambda_i \bar{S}_i(s)} \right)$$

$$(2.37) \quad \bar{P}_{F_1,M}(s) = \frac{\lambda_i}{D(s)} + \frac{\lambda_i(M-j+1)}{(\lambda+v)D(s)} (\lambda') \times \prod_{P=1}^{j-1} \left(\frac{M-P+1}{s + \lambda + v + (M-P)\lambda' - \sum \lambda_i \bar{S}_i(s)} \right) \\ [\bar{S}_j(s) - \bar{S}_j(s + \lambda + v)] \times \frac{1-\bar{S}_j(s)}{s}$$

$$(2.38) \quad \bar{P}_{F_1,M-r}(s) = \frac{\lambda_i}{D(s)} \cdot \frac{1-\bar{S}_i(s)}{s} + (\lambda') \prod_{P=0}^{r-1} \left(\frac{M-P+1}{s + \lambda + v + M-P-1\lambda' - \sum \lambda_i \bar{S}_i(s)} \right) \\ [\bar{S}_j(s) - \bar{S}_j(s + \lambda + v)] \times \frac{1-\bar{S}_j(s)}{s}$$

$$(2.39) \quad \bar{P}_{F_1,M-r}(s) = \frac{\lambda_i}{D(s)} \cdot \frac{1-\bar{S}_i(s)}{s} + (\lambda') \prod_{P=0}^{r-1} \left(\frac{M-P+1}{s + \lambda + v + M-P-1\lambda' - \sum \lambda_i \bar{S}_i(s)} \right)$$

$$(2.40) \quad \bar{Q}_{F_1,M-j}(s) = \frac{\lambda_i(M-j+1)(\lambda')^j}{(v+\lambda)D(s)} \prod_{P=1}^{j-1} \left(\frac{M-P+1}{s + \lambda + v + M-P\lambda' - \sum \lambda_i \bar{S}_i(s)} \right) \\ \times \left(\frac{1-\bar{S}_j(s)}{s} - \frac{1-\bar{S}_j(s+\lambda+v)}{s+\lambda+v} \right)$$

$$(2.41) \quad \bar{P}_F(s) = \frac{v}{D(s)} \cdot \frac{1-\bar{S}_\delta}{s} \left[(M-j+1)(\lambda')^j \frac{1-\bar{S}_j(s+\lambda+v)}{s+\lambda+v} \right. \\ \times \prod_{P=1}^{j-1} \frac{(M-P+1)}{[s + \lambda + v + M-P\lambda' - \sum_{i=1}^N \lambda_i \bar{S}_i(s)]} \\ \left. + (\lambda')^r \prod_{P=0}^{r-1} \left(\frac{M-P}{s + \lambda + v + (M-P-1)\lambda' - \sum_{i=1}^N \lambda_i \bar{S}_i(s)} \right) + 1 \right]$$

Evaluation of Laplace Transform of Up and Down state Probabilities

$$(2.42) \quad \bar{P}_{up}(s) = \frac{1}{D(s)} \left[1 + \sum_{r=1}^{j-1} \left\{ (\lambda')^r \prod_{P=1}^{j-1} \left(\frac{M-P}{s + \lambda + v + M-P-1\lambda' - \sum \lambda_i \bar{S}_i(s)} \right) \right\} \right] \\ + \frac{(M-j+1)}{(s+\lambda+v)} (1 - \bar{S}_j(s + \lambda + v)) (\lambda')^j \\ \times \prod_{P=1}^{j-1} \left(\frac{M-P+1}{s + \lambda + v + (M-P)\lambda' - \sum \lambda_i \bar{S}_i(s)} \right)$$

$$(2.43) \quad \bar{P}_{down}(s) = \sum_{i=1}^N \left[\frac{\lambda_i}{D(s)} \cdot \frac{1-\bar{S}_i(s)}{s} \cdot \right.$$

$$\left. \sum_{r=1}^{j-1} \left\{ (\lambda') \prod_{P=0}^{r-1} \left(\frac{M-P}{s + \lambda + v + M-P-1\lambda' - \sum_{i=1}^N \lambda_i \bar{S}_i(s)} \right) \right\} + \frac{\lambda_i}{D(s)} \right]$$

$$\begin{aligned}
& + \frac{\lambda(M-j+1)}{(\lambda+v)D(s)} (\lambda')^j \times \prod_{P=1}^{j-1} \left(\frac{M-P+1}{S+\lambda+v+M-P\lambda'-\sum_{i=1}^N \lambda_i \bar{S}_i(s)} \right) \\
& \times [\bar{S}_j(s) - \bar{S}_j(s+\lambda+v)] \times \left(\frac{1-\bar{S}_j(s)}{s} \right) + \frac{\lambda_1(M-j+1)(\lambda')^j}{(v+\lambda).D(s)} \\
& \times \prod_{P=0}^{j-1} \left(\frac{M-P+1}{s+\lambda+v+M-P\lambda'-\sum_{i=1}^N \lambda_i \bar{S}_i(s)} \right) \\
& \times \left(\frac{1-\bar{S}_j(s)}{s} - \frac{1-\bar{S}_j(s+\lambda+v)}{s+\lambda+v} \right) \Bigg] + \frac{v}{D(s)} \cdot \frac{1-\bar{S}_\delta}{s} \\
& \left[(M-j+1)(\lambda')^j \frac{1-\bar{S}_j(s+\lambda+v)}{s+\lambda+v} \times \prod_{P=1}^{j-1} \left(\frac{(M-P+1)}{s+\lambda+v+M-P\lambda'-\sum_{i=1}^N \lambda_i \bar{S}_i(s)} \right) \right. \\
& \left. + (\lambda')^r \prod_{P=0}^{r-1} \left(\frac{M-P}{s+\lambda+v+(M-P-1)\lambda'-\sum_{i=1}^N \lambda_i \bar{S}_i(s)} \right) + 1 \right]
\end{aligned}$$

It would be interesting to note that

$$(2.44) \quad \bar{P}_{up}(s) + \bar{P}_{down}(s) = \frac{1}{s}$$

Ergodic Behaviour of the system

Using Abel's Lemman,

$$\lim_{s \rightarrow 0} sF(s) = \lim_{t \rightarrow \infty} F(t) = F(\text{say}),$$

provided the limit on right exists, the system's steady state probabilities are obtained as under :

$$(2.45) \quad P_{0,M} = \frac{1}{D'(0)}$$

$$(2.46) \quad P_{0,M-r} = \frac{1}{D'(0)} \cdot \frac{M}{M-r}$$

$$(2.47) \quad P_R = \frac{M\lambda'}{D'(0).(\lambda+v)} [1 - \bar{S}'_j(\lambda+v)]$$

$$(2.48) \quad P_{F_i,M} = \frac{\lambda_i M_i}{D'(0)} \left\{ 1 + \frac{M\lambda'}{(\lambda+v)} [1 - \bar{S}'_j(\lambda+v)] \right\}$$

$$(2.49) \quad P_{F_i,M-r} = \frac{\lambda_i M_i}{D'(0)} \cdot \frac{M}{M-r}$$

$$(2.50) \quad Q_{F_i,M-j} = \frac{\lambda_i M_i \lambda'}{(v+\lambda)D'(0)} \left\{ M'_j - \frac{1-\bar{S}'_j(\lambda+v)}{\lambda+v} \right\}$$

$$(2.51) \quad P_F = \frac{vM_\delta}{D'(0)} \cdot \left[\frac{M\lambda'}{(\lambda+v)} [1 - \bar{S}'_j(\lambda+v)] + \frac{M}{M-r} \right]$$

where M_i = Mean time to repair i th, L_1 components

M'_j = Mean time to repair j th, L_2 components

and $D'(0) = \left[\frac{d}{ds} D(s) \right]_{s=0}$

Particular cases

(A) Class L_1 and consists of N components in series and class L_2 consists of M components in Paraller Redundancy, assuming that the failure of any two components out of M causes the system to work in reduce efficiency stats.

Substitution of $j = 2$ in relation (35) through (41), one gets

$$(2.52) \quad \bar{P}_{0,M}(s) = \frac{1}{E(s)} [S + \lambda + v + (M-1)\lambda' - \sum \lambda_i \bar{S}_i(s)]$$

$$(2.53) \quad \bar{P}_{0,M-1}(s) = \frac{M\lambda'}{E(s)}$$

$$(2.54) \quad \bar{P}_R(s) = \frac{M(M-1)(\lambda')^2}{E(s)} \cdot \frac{\{1 - \bar{S}_2(s + \lambda + v)\}}{(s + \lambda + v)}$$

$$(2.55) \quad \bar{P}_{F_i,M}(s) = \frac{\lambda'}{E(s)} \cdot \frac{1 - \bar{S}_i(s)}{s} \left[s + \lambda + v + \overline{M-1}\lambda' - \sum_{i=1}^N \lambda_i \bar{S}_i(s) \right. \\ \left. + \frac{M(M-1)(\lambda')^2}{(\lambda + \mu)} \cdot \{\bar{S}_2(s) - \bar{S}_2(s + \lambda + v)\} \right]$$

$$(2.56) \quad \bar{P}_{F_i,M-1}(s) = \frac{M\lambda_i\lambda'}{E(s)} \cdot \frac{1 - \bar{S}_i(s)}{s}$$

$$(2.57) \quad Q_{F_1 M-2}(s) = \frac{M(M-1)(\lambda')^2}{(v + \lambda).E(s)} \left\{ \frac{1 - \bar{S}_1(s)}{s} - \frac{1 - \bar{S}_2(s + \lambda + v)}{s + \lambda + v} \right\}$$

$$(2.58) \quad \bar{P}_F(s) = \frac{v}{E(s)} \cdot \frac{1 - \bar{S}_\delta(s)}{s} \left[M(M-1)(\lambda')^2 \frac{1 - \bar{S}_2(s + \lambda + v)}{s + \lambda + v} \right. \\ \left. + (\lambda')M + (s + \lambda + v + \overline{M-1}\lambda' - \sum_{i=1}^N \lambda_i \bar{S}_i(s)) \right]$$

where,

$$E(s) = \{s + \lambda + v + M\lambda' - \sum \lambda_i \bar{S}_i(s)\} \{s + \lambda + v + (M-1)\lambda' \\ - \sum \lambda_i \bar{S}_i(s)\} - M(M-1)(\lambda')^2 \{ \bar{S}_2(s + \lambda + v) + \sum_{i=1}^N \frac{\lambda_1}{v + \lambda} \bar{S}_i(s) (S_2(s) \\ - \bar{S}_2(s + \lambda + v)) - v\bar{S}_\delta(s) \left\{ (\lambda')M + (M-1)(\lambda')^2 \cdot M \frac{1 - \bar{S}_2(s + \lambda + v)}{s + \lambda + v} \right\} \\ = s + \lambda + (M-1)\lambda' - \sum \lambda_i \bar{S}_i(s) \}$$

(B) Repair Follows Exponential Time Distribution

Setting $S_i(s) = \frac{\eta_i}{s + \eta_i}$ and $\bar{S}_j(s) = \frac{\phi_j}{s + \phi_j}$ and $\bar{S}_\delta(s) = \frac{\delta}{s + \delta}$

$$(2.59) \quad \bar{P}_{0,M}(s) = \frac{1}{F(s)}$$

$$(2.60) \quad \bar{P}_{0.M-r}(s) = \frac{(\lambda')^r}{F(s)} \prod_{P=0}^{r-1} \left(\frac{M-P}{s + \lambda + v + (M-P-1)\lambda' - \sum \frac{\lambda_i \eta_i}{S + \eta_i}} \right)$$

$$(2.61) \quad \bar{P}_R(s) = \frac{(M-j+1)(\lambda')^j}{F(s).(s + \lambda + v + \phi)} \times \prod_{P=1}^{j-1} \left\{ \frac{M-P+1}{s + \lambda + v + M-P\lambda' - \sum \frac{\lambda_i \eta_i}{S + \eta_i}} \right\}$$

$$(2.62) \quad \bar{P}_{F_i.M}(s) = \left\{ \frac{\lambda_i}{F(s)} \frac{\lambda_i(M-j+1)}{(\lambda + v).F(s)} (\lambda')^j \times \prod_{P=1}^{j-1} \left(\frac{M-P+1}{s + \lambda + v + M-P\lambda' - \sum \frac{\lambda_i \eta_i}{S + \eta_i}} \right) \right. \\ \left. \times \frac{(\lambda + v)\phi}{(s + \phi_j)(s + \lambda + v + \phi_j)} \right\} \times \frac{1}{s + \eta_i}$$

$$(2.63) \quad \bar{P}_{F_i.M-r}(s) = \frac{\lambda_i(\lambda')^r}{F(s).(S + \eta_i)} \prod_{P=0}^{r-1} \left(\frac{M-P}{s + \lambda + v + M-P-1\lambda' - \sum \frac{\lambda_i \eta_i}{S + \eta_i}} \right) \\ \times \frac{1}{(s + \phi_j)(s + \lambda + v + \phi_j)}$$

$$(2.64) \quad P_F(s) = \frac{V}{F(s)} \cdot \frac{1}{s + \delta} \left[(M - j + 1)(\lambda')^j \frac{1}{(s + \lambda + v + \phi)} \right. \\ \times \prod_{P=1}^{j-1} \left(\frac{M-P+1}{s + \lambda + v + M-P\lambda' - \sum \frac{\lambda_i \eta_i}{S + \eta_i}} \right) \\ \left. + (\lambda')^r \prod_{P=0}^{r-1} \left(\frac{M-P}{s + \lambda + v + M-P\lambda' - \sum \frac{\lambda_i \eta_i}{S + \eta_i}} \right) \right]$$

where, $F(s) = s + \lambda + v + M\lambda' - \sum \frac{\lambda_i \eta_i}{S + \eta_i} - (M - j + 1)(\lambda')^j$

$$\times \prod_{P=1}^{j-1} \left(\frac{M-P+1}{s + \lambda + v + M-P\lambda' - \sum \frac{\lambda_i \eta_i}{S + \eta_i}} \right) \times \frac{\phi_j}{s + \lambda + v + \phi_j} \left\{ 1 + \frac{1}{(s + \phi)} \cdot \sum \frac{\lambda_i \eta_i}{S + \eta_i} \right\}$$

$$- \frac{v\delta}{s + \delta} \left\{ (\lambda')^r \prod_{P=0}^{r-1} \left(\frac{M-P+1}{s + \lambda + v + M-P\lambda' - \sum \frac{\lambda_i \eta_i}{S + \eta_i}} \right) \right. \\ \left. + (M - j + 1)(\lambda')^j \frac{1}{s + \lambda + v + \phi_j} \right. \\ \left. \times \prod_{P=1}^{j-1} \left(\frac{M-P+1}{s + \lambda + v + (M-P)\lambda' - \sum \frac{\lambda_i \eta_i}{S + \eta_i}} \right) + 1 \right\}$$

Numerical Computation

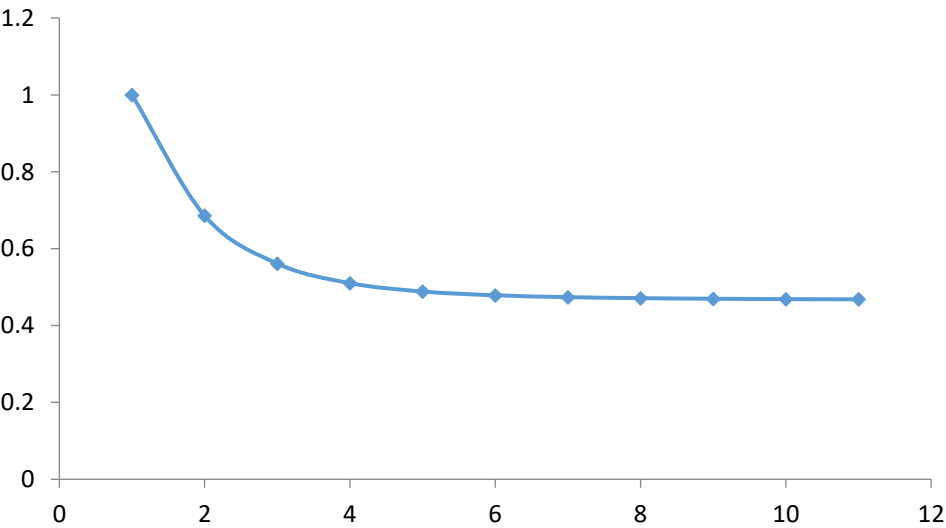
To depict the result numerically and graphically, $P_{up}^{head}(t)$ obtained from (2.41) has been plotted against t in the steady state for $N = 1$, $\lambda = 0.03$, $\lambda' =$

$0.02, v=0.01, M=10, \delta = \eta = \phi = 1, i = 1, j = 2$ for any complex system, we have

(2.65)
$$P_{up}^{head}(t = 0.466847774) \times e^{-0.375241t} + 0.053204636 \times e^{-1.1011621} \\ + 0.427207537 \times e^{-1.0117985t} \times \cos(0.024859271) t + 1.082840314 \\ \times e^{-1.01179851} \times \sin(0.024859271) t$$

Table 2.1

Time	R(t)
0	0.999990
1	0.685832
2	0.561141
3	0.510263
4	0.488560
5	0.478688
6	0.473818
7	0.471196
8	0.469670
9	0.468724
10	0.468113



Graph 2.1

3. Conclusion

A critical observation of the table and respected graph reveals that $P_{up}^{head}(t)$ decreases as the time t increases.

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