

Multicriteria Decision-Making for Fund Investment Using a Modified Fermatean Cubic Aggregation Operator

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Abstract: Multi-criteria decision-making (MCDM) plays important role in investment funding, where uncertainty and imprecision are natural challenges. To improve decision-making accuracy, this paper introduces a *Modified Fermatean Cubic Fuzzy Aggregation Operator & score function*, which integrates Fermatean cubic fuzzy sets with aggregation operators. The proposed aggregation operator effectively captures higher degrees of uncertainty and expert hesitation, leading to more reliable investment evaluations. Furthermore, an investment decision-making framework is developed based on the proposed aggregation operator, enabling a systematic evaluation of funding alternatives. A numerical example is conducted to validate its effectiveness, and a comparative analysis with existing aggregation methods highlights its advantages in handling complex decision environments. The results shows that the Modified Fermatean Cubic Fuzzy Aggregation Operator is a effective tool for improving investment decision-making under uncertainty.

Keywords: Fermatean cubic fuzzy sets, aggregation operator, multi-criteria decision-making, investment funding.

1. Introduction

The choices we make have a big impact on real life. These choices have the power to shape our lives. These choices might range from minor ones like what to eat for breakfast to major ones like who we want to hang out with. These kinds of issues are referred to as decision-making issues. To address uncertainty, Bellman and Zadeh¹ introduced decision-making in a fuzzy environment in 1970. Since then, multi-criteria decision-making (MCDM)

has emerged as a framework that helps individuals choose the best option by evaluating multiple, often vague or conflicting, criteria. Hwang *et al.*² proposed MADM methods in 1981, Although intuitionistic fuzzy sets (IFS) and interval-valued IFS (IVIFS), introduced by Atanassov³, which can handle incomplete information, they are not sufficient for managing inconsistent or indeterminate data frequently encountered in practice. To overcome this limitation, Smarandache⁴ proposed neutrosophic sets (NSs) in 1999, as generalization of classical sets, FS, IVFS, IFS and IVIFS by allowing independent truth, indeterminacy, and falsity memberships. In this context, aggregation refers to the process of combining the fuzzy outputs of each rule into a single unified fuzzy set. after this, as the hybridization of the cubic set and Fermatean fuzzy set, In 2022, Riaz *et.al*⁵ introduced Fermatean cubic fuzzy sets (F-CFS). and also given the applications in MADM.

In this paper, develop a Modified Fermatean Cubic Aggregation Operator to Enhance traditional fuzzy and Fermatean fuzzy models to better handle uncertainty and vagueness in investment decision-making. After this applying Score Function for accurate ranking and selection of investment alternatives. Enhance Multicriteria Decision-Making (MCDM) Techniques for utilization of the proposed operator to improve decision-making in complex financial environments. Apply the Proposed Methodology to Fund Investment to demonstrate the effectiveness of the method in real-world investment scenarios involving multiple criteria. Compare with existing approaches for the Validation of the superiority of the proposed method through comparative analysis with traditional fuzzy, intuitionistic and Fermatean fuzzy models. Ensure Robustness and Practical Applicability and Provide a practical and computationally efficient tool for investors and financial analysts to make well-informed decisions.

This paper is divided into seven section. A few operational laws are defined in Section 2, along with some fundamental definitions of CFS, PFS, IVPFS, FFS, FCFS. In Section 3, Fermatean Cubic Fuzzy weighted aggregation procedures and a score function are suggested. In Section 4, developed a proposed methodology to solve a MCDM in a FCFS. The score function is used to rank the alternatives in order of preference and determine the alternatives which are best. The utilization of the suggested DM process is illustrated numerically in Section 5. After this, In section 6 discussed about result related to numerical computation. In the section 7, given conclusion and future scope of this paper.

Literature review

In 1996, Zadeh⁶ introduced Fuzzy logic and fuzzy system. In 2017, Jun *et. al*⁷ introduced Neutrosophic set for tackling uncertainty using the truth,

indeterminacy and falsity membership grades. After this in 2018, Kaur and Garg⁸⁻⁹ introduced Cubic intuitionistic fuzzy (CIF) sets combine interval-valued and intuitionistic fuzzy information, offering a richer way to express uncertainty. Since aggregation tools for CIF sets are lacking, this work introduces new Bonferroni mean-based operators for combining CIF numbers and develops a corresponding decision-making method, demonstrated through a numerical example. continuously, they developed the aggregation operator for CIF. and In 2019, Kaur and Garg¹⁰ given the fundamental properties of CIF. In 2019 Tian *et.al*¹¹ introduced Picture fuzzy sets (PFSs) model positive, neutral and negative preferences, enabling more accurate representation of decision-makers' judgments. This paper proposes new power-based Choquet geometric operators and an integrated MCDM method that manages interrelated criteria and incomplete weights using Shapley fuzzy measures. In 2019, Khan *et.al*¹² introduced Pythagorean cubic fuzzy sets, where membership and non-membership degrees are cubic fuzzy numbers and develops comparison measures, distance functions, and several aggregation operators (PCFWA, PCFWG, PCFOWA, PCFOWG). Based on these operators, a more general and accurate MCDM approach is proposed and demonstrated through detailed operational steps. In 2020, Alhazaymeh¹³ introduced cubic vague sets (CVSs) along with internal and external variants, examines their properties under P- and R-orders, and establishes conditions for union and intersection operations. A decision-making method using a CVS-based similarity measure is proposed, showing that CVSs effectively model complex two-dimensional, periodic real-world information. In 2020, Chinnadurai *et.al*¹⁴ introduced complex cubic sets (CCS), formed by integrating complex-interval valued fuzzy sets (CIVFS) and complex fuzzy sets (CFS), along with the notions of internal and external CCS and new operations such as P(R)-order, P(R)-union, and P(R)-intersection with their properties. In 2021, Qiyas¹⁵ introduced linguistic intuitionistic cubic fuzzy variables (LICFV), defining multiple weighted aggregation operators, and analyzing their properties and interrelationships. To validate the approach, a multi-criteria decision-making algorithm is developed and applied to mobile-phone selection, demonstrating improved effectiveness over existing methods. In 2022, Riaz *et.al*⁵ introduced Fermatean cubic fuzzy sets (F-CFS) a generalized fuzzy framework with broader expressive power –defines their properties, score measures, distance functions, and develops several corresponding aggregation operators. The proposed F-CFS-based operators are applied to a multi-attribute decision-making (MADM) problem, demonstrating the effectiveness of this enhanced approach. In 2022, Khan *et.al*¹⁶ introduced the cubical fuzzy set (CFS) an extension of picture and

spherical fuzzy sets with a stronger membership constraint and develops its operations, properties, distance measures, and a range of arithmetic and geometric aggregation operators. These operators are then applied to MADM problems, including an ERP system selection example, demonstrating the effectiveness of CFS in handling uncertainty. In 2024, Ozlu¹⁷ introduced Bipolar-Valued Complex Hesitant Fuzzy Sets (BVCHFSSs) to model highly inconsistent and complex information, defines their operations and score function, and develops several Dombi-based aggregation operators along with their key properties. These operators are then applied to an investment decision problem and compared with existing methods, showing the effectiveness and superiority of the proposed BVCHFSS-based approach. In 2025, Riaz¹⁸ introduced linear Diophantine fuzzy sets (LDFS) as a powerful framework for handling uncertainty and proposes two new aggregation operators—the LDF weighted average and LDF weighted geometric operators—to support optimal decision-making. In 2025 Sharma *et. al*¹⁹ introduced a new technique to solve multiobjective transportation problem under Fermatean cubic fuzzy environment.

List of Abbreviations

Notations	Full form
CS	Cubic Set
CFS	Cubic Fuzzy set
DM	Decision Making
FFS	Fermatean Fuzzy set
F-CFS	Fermatean Cubic Fuzzy set
IVFS	Interval Valued Fuzzy set
IVPFS	Interval Valued Pythagorean Fuzzy set
MD	Membership Degree
MCDM	Multicriteria Decision Making
NMD	Non Membership Degree
PFS	Pythagorean Fuzzy Sets

2. Preliminary

In this section, first give the fundamental definitions and operations of NSs and FFS.

2.1- Let U be a stable set, then **cubic fuzzy set (CFS)** is denoted and define as below-

(1)
$$\tilde{C} = \{\langle u, \delta_c(u), \gamma_c(u) \rangle | u \in U\}$$
 where $\delta_c(p)$ is IVFS and $\gamma_c(p)$ is a single fuzzy set in $\tilde{K}$.

2.2-Let U be a stable set. A **PFS (P)** on U is shown as-

$$(2) \quad P = \left\{ \langle u, P(\varphi_p(u), \omega_p(u)) \rangle / u \in U \right\}$$

here $\varphi_p(u): U \rightarrow [0,1]$ is the MD and $\omega_p(u): \rightarrow [0,1]$ is the NMD of the element $u \in U$ to P . also, $\forall u \in U$, it satisfies the following condition:

$$0 \leq \varphi_p^2(u) + \omega_p^2(u) \leq 1$$

The grade of indeterminacy of u to P is

$$\mu_p(u) = \sqrt{1 - \varphi_p^2(u) - \omega_p^2(u)}$$

2.2.1-Operations on PFS: Let $\Psi = \langle \varphi_p(u), \omega_p(u) \rangle, \psi_1 = \langle \varphi_{p_1}, \omega_{p_1} \rangle$ and $\psi_2 = \langle \varphi_{p_2}, \omega_{p_2} \rangle$ be three PFNs, next, there are the subsequent relationships:

- i. $\Psi^c = \langle \varphi_p(u), \omega_p(u) \rangle$
- ii. $\psi_1 \leq \psi_2$ if $\varphi_{p_1} \leq \varphi_{p_2}$ and $\omega_{p_1} \geq \omega_{p_2}$
- iii. $\psi_1 = \psi_2$ iff $\psi_1 \leq \psi_2$ and $\psi_1 \geq \psi_2$
- iv. $\psi_1 \cap \psi_2 = \langle \min(\varphi_{p_1}, \varphi_{p_2}), \max(\omega_{p_1}, \omega_{p_2}) \rangle$
- v. $\psi_1 \cup \psi_2 = \langle \max(\varphi_{p_1}, \varphi_{p_2}), \min(\omega_{p_1}, \omega_{p_2}) \rangle$
- vi. $\psi_1 \oplus \psi_2 = \langle \sqrt{\varphi_{p_1}^2 + \varphi_{p_2}^2 - \varphi_{p_1}^2 \varphi_{p_2}^2}, \omega_{p_1} \omega_{p_2} \rangle$
- vii. $\psi_1 \otimes \psi_2 = \langle \varphi_{p_1} \varphi_{p_2}, \sqrt{\omega_{p_1}^2 + \omega_{p_2}^2 - \omega_{p_1}^2 \omega_{p_2}^2} \rangle$
- viii. $\alpha \psi_1 = \langle \sqrt{1 - (1 - \varphi_{p_1}^2)^\alpha}, \omega_{p_1}^\alpha \rangle; \alpha > 0$
- ix. $\psi_1^\alpha = \langle \varphi_{p_1}^\alpha, \sqrt{1 - (1 - \omega_{p_1}^2)^\alpha} \rangle; \alpha > 0$
- x. $\alpha \psi_1 = \begin{cases} \langle \alpha \sqrt{1 - \varphi_{p_1}^2}, \sqrt{1 - \alpha^2 \omega_{p_1}^2} \rangle \text{ if } \alpha \in (0,1) \\ \langle (1/\alpha) \sqrt{1 - \varphi_{p_1}^2}, \sqrt{1 - (1/\alpha)^2 \omega_{p_1}^2} \rangle \text{ if } \alpha \geq 1 \end{cases}$

2.3- Let U be any Universal set. A **IVPFS (P_I)** is define as below:

$$(3) \quad P_I = \{ \langle u_i, [\eta_{1P_I}^-(u_i), \eta_{1P_I}^+(u_i)], [\eta_{2P_I}^-(u_i), \eta_{2P_I}^+(u_i)] \rangle; u_i \in U \}$$

here $0 \leq \eta_{1P_I}^-(u_i) \leq \eta_{1P_I}^+(u_i) \leq 1, 0 \leq \eta_{2P_I}^-(u_i) \leq \eta_{2P_I}^+(u_i) \leq 1$ and also it is satisfying the following condition $(\eta_{1P_I}^+(u_i))^2 + (\eta_{2P_I}^+(u_i))^2 \leq 1 \forall u \in U$. like PFSs, which correlate to membership values that are interval valued, its hesitating interval in relation to P_I is given

$$\xi_{P_I}(u_i) = [\xi_{P_I}^-(u_i), \xi_{P_I}^+(u_i)] \\ = \sqrt{1 - (\eta_{1P_I}^+(u_i))^2 - (\eta_{2P_I}^+(u_i))^2}, \sqrt{1 - (\eta_{1P_I}^-(u_i))^2 - (\eta_{2P_I}^-(u_i))^2}$$

Here $\gamma_{P_I}(u_i) = [\eta_{1P_I}^-(u_i), \eta_{1P_I}^+(u_i)]$ shown the degree of interval valued membership of u in P_I and $\delta_{P_I}(u_i) = [\eta_{2P_I}^-(u_i), \eta_{2P_I}^+(u_i)]$ shown the grade of interval valued NMD of u in F_I .

2.3.1- Operations on IVPFN: Let $P_I = [\eta_{1P_I}^-, \eta_{1P_I}^+], [\eta_{2P_I}^-, \eta_{2P_I}^+]$, $P_{I_1} = ([\eta_{1P_{I_1}}^-, \eta_{1P_{I_1}}^+][\eta_{2P_{I_1}}^-, \eta_{2P_{I_1}}^+])$, $P_{I_2} = ([\eta_{1P_{I_2}}^-, \eta_{1P_{I_2}}^+][\eta_{2P_{I_2}}^-, \eta_{2P_{I_2}}^+])$ be three IVPFN, and $\beta > 0$ then their operations are define as follows;

- i. $P_{I_1} \cup P_{I_2} = ([\max\{\eta_{1P_{I_1}}^-, \eta_{1P_{I_2}}^-\}, \max\{\eta_{1P_{I_1}}^+, \eta_{1P_{I_2}}^+\}], [\min\{\eta_{2P_{I_1}}^-, \eta_{2P_{I_2}}^-\}, \min\{\eta_{2P_{I_1}}^+, \eta_{2P_{I_2}}^+\}])$
- ii. $P_{I_1} \cap P_{I_2} = ([\min\{\eta_{1P_{I_1}}^-, \eta_{1P_{I_2}}^-\}, \min\{\eta_{1P_{I_1}}^+, \eta_{1P_{I_2}}^+\}], [\max\{\eta_{2P_{I_1}}^-, \eta_{2P_{I_2}}^-\}, \max\{\eta_{2P_{I_1}}^+, \eta_{2P_{I_2}}^+\}])$
- iii. $P_{I_1} \oplus P_{I_2} = ([\sqrt{(\eta_{1P_{I_1}}^-)^2 + (\eta_{1P_{I_2}}^-)^2 - (\eta_{1P_{I_1}}^+)^2 (\eta_{1P_{I_2}}^+)^2}, \sqrt{(\eta_{1P_{I_1}}^+)^2 + (\eta_{1P_{I_2}}^+)^2 - (\eta_{1P_{I_1}}^-)^2 (\eta_{1P_{I_2}}^-)^2}], [\eta_{2P_{I_1}}^-, \eta_{2P_{I_2}}^-, \eta_{2P_{I_1}}^+, \eta_{2P_{I_2}}^+])$
- iv. $P_{I_1} \otimes P_{I_2} = ([\eta_{1P_{I_1}}^-, \eta_{1P_{I_2}}^-, \eta_{1P_{I_1}}^+, \eta_{1P_{I_2}}^+], [\sqrt{(\eta_{2P_{I_1}}^-)^2 + (\eta_{2P_{I_2}}^-)^2 - (\eta_{2P_{I_1}}^+)^2 (\eta_{2P_{I_2}}^+)^2}, \sqrt{(\eta_{2P_{I_1}}^+)^2 + (\eta_{2P_{I_2}}^+)^2 - (\eta_{2P_{I_1}}^-)^2 (\eta_{2P_{I_2}}^-)^2}])$
- v. $u P_I = ([\sqrt{1 - (1 - (\eta_{1P_I}^-)^2)^u}, \sqrt{1 - (1 - (\eta_{1P_I}^+)^2)^u}], [(\eta_{2P_I}^-)^u, (\eta_{2P_I}^+)^u])$
- vi. $P_I^u = ([(\eta_{1P_I}^-)^u, (\eta_{1P_I}^+)^u], [\sqrt{1 - (1 - (\eta_{2P_I}^-)^2)^u}, \sqrt{1 - (1 - (\eta_{2P_I}^+)^2)^u}])$

2.4- A FFS, F in U is shown as –

$$(4) \quad F = \{ \langle u, \gamma_f(u), \delta_f(u) \rangle : u \in U \}$$

Where the functions $\gamma_f(u): U \rightarrow [0,1]$ shown the grade of membership of u in P and $\delta_f(u): U \rightarrow [0,1]$ shown the grade of non membership of u in P and also satisfying the following condition:

$$0 \leq \gamma_f^3(u) + \delta_f^3(u) \leq 1$$

2.4.1- Let (γ_f, δ_f) , $F_1 = (\gamma_{f_1}, \delta_{f_1})$ and $F_2 = (\gamma_{f_2}, \delta_{f_2})$ be three FFSs, then their operations are defined as follows:

- i. $F_1 \cap F_2 = (\min\{\gamma_{f_1}, \gamma_{f_2}\}, \max\{\delta_{f_1}, \delta_{f_2}\})$
- ii. $F_1 \cup F_2 = (\max\{\gamma_{f_1}, \gamma_{f_2}\}, \min\{\delta_{f_1}, \delta_{f_2}\})$
- iii. $F^c = (\delta_f, \gamma_f)$
- iv. $F_1 \oplus F_2 = (\sqrt[3]{\gamma_{f_1}^3 + \gamma_{f_2}^3 - \gamma_{f_1}^3 \gamma_{f_2}^3}, \delta_{f_1} \delta_{f_2})$
- v. $F_1 \otimes F_2 = (\gamma_{f_1} \gamma_{f_2}, \sqrt[3]{\delta_{f_1}^3 + \delta_{f_2}^3 - \delta_{f_1}^3 \delta_{f_2}^3})$
- vi. $\alpha F = (\sqrt[3]{1 - (1 - (\gamma_f)^3)^\alpha}, \delta_f^\alpha) \quad , \alpha > 0$
- vii. $F^\alpha = (\gamma_f^\alpha, \sqrt[3]{1 - (1 - \delta_f^3)^\alpha}) \quad , \alpha > 0$

2.5- Let U be a fixed set. then, a **Fermatean cubic fuzzy set(F-CFS)** ($\vec{F}$) denoted and defined as below-

$$\vec{F} = (u, \Lambda \vec{F}(u), \Upsilon \vec{F}(u) | u \in U)$$

where $\Lambda \vec{F}(u) = \langle A_{\vec{F}}; \zeta_{\vec{F}}(u) \rangle = \langle [k_{\vec{F}}, l_{\vec{F}}]; \zeta_{\vec{F}}(u) \rangle$ and

$$\Upsilon \vec{F}(u) = \langle \widetilde{A}_{\vec{F}}; \xi_{\vec{F}}(u) \rangle = \langle [\widetilde{k}_{\vec{F}}, \widetilde{l}_{\vec{F}}]; \xi_{\vec{F}}(u) \rangle$$

& $\Lambda \vec{F}(u), \Upsilon \vec{F}(u)$ both are cubic fuzzy numbers and also denote the membership and non-membership degrees of F-CFNs.

With the following conditions:

$$0 \leq (\Lambda \vec{F}(u))^3 + (\Upsilon \vec{F}(u))^3 \leq 1$$

$$0 \leq (k_{\vec{F}}(u))^3 + (l_{\vec{F}}(u))^3 \leq 1$$

And also,

$$0 \leq (\sup \Lambda \vec{F}(u))^3 + (\sup \Upsilon \vec{F}(u))^3 \leq 1$$

The degree of indeterminacy for F-CFNs can be defined as

$$\mu_{\vec{F}(u)} = \sqrt[3]{1 - (\sup (A_{\vec{F}}(u)))^3 + (\sup (\widetilde{A}_{\vec{F}}(u)))^3; \sqrt[3]{1 - (k_{\vec{F}}(u))^3 + (l_{\vec{F}}(u))^3}}$$

2.5.1-Operations on F-CFNs

Let $\vec{F} = (\langle A_{\vec{F}}; \zeta_{\vec{F}} \rangle, \langle \widetilde{A}_{\vec{F}}; \xi_{\vec{F}} \rangle)$, $\vec{F}_1 = (\langle A_{\vec{F}_1}; \zeta_{\vec{F}_1} \rangle, \langle \widetilde{A}_{\vec{F}_1}; \xi_{\vec{F}_1} \rangle)$ and $\vec{F}_2 = (\langle A_{\vec{F}_2}; \zeta_{\vec{F}_2} \rangle, \langle \widetilde{A}_{\vec{F}_2}; \xi_{\vec{F}_2} \rangle)$ be three F-CFNs. where $A_{\vec{F}_1} = (k_{\vec{F}_1}, l_{\vec{F}_1})$, $\widetilde{A}_{\vec{F}_1} = (\widetilde{k}_{\vec{F}_1}, \widetilde{l}_{\vec{F}_1})$, $A_{\vec{F}_2} = (k_{\vec{F}_2}, l_{\vec{F}_2})$ and $\widetilde{A}_{\vec{F}_2} = (\widetilde{k}_{\vec{F}_2}, \widetilde{l}_{\vec{F}_2})$; then the following are the operational laws of F-CFNs.

$$\begin{aligned} \text{i. } \vec{F}_1 \oplus \vec{F}_2 = & \left\{ \sqrt[3]{k_{\vec{F}_1}^3 + k_{\vec{F}_2}^3 - k_{\vec{F}_1}^3 k_{\vec{F}_2}^3}, \sqrt[3]{l_{\vec{F}_1}^3 + l_{\vec{F}_2}^3 - l_{\vec{F}_1}^3 l_{\vec{F}_2}^3}, \right. \\ & \left. \sqrt[3]{\zeta_{\vec{F}_1}^3 + \xi_{\vec{F}_1}^3 - \zeta_{\vec{F}_1}^3 \xi_{\vec{F}_1}^3}, \langle [\widetilde{k}_{\vec{F}_1}, \widetilde{k}_{\vec{F}_2}, \widetilde{l}_{\vec{F}_1}, \widetilde{l}_{\vec{F}_2}]; \zeta_{\vec{F}_1}, \zeta_{\vec{F}_2} \rangle \right\} \end{aligned}$$

- ii. $\vec{F}_1 \otimes \vec{F}_2 = \{ \langle [k_{\vec{F}_1} l_{\vec{F}_2}, l_{\vec{F}_1} l_{\vec{F}_2}]; \xi_{\vec{F}_1} \xi_{\vec{F}_2} \rangle, \langle \sqrt[3]{k_{\vec{F}_1}^3 + k_{\vec{F}_2}^3 - k_{\vec{F}_1}^3 k_{\vec{F}_2}^3}, \sqrt[3]{l_{\vec{F}_1}^3 + l_{\vec{F}_2}^3 - l_{\vec{F}_1}^3 l_{\vec{F}_2}^3}; \sqrt[3]{\xi_{\vec{F}_1}^3 + \xi_{\vec{F}_2}^3 - \xi_{\vec{F}_1}^3 \xi_{\vec{F}_2}^3} \rangle \}$
- iii. $\alpha \vec{F}_1 = \{ \langle \sqrt[3]{1 - (1 - k_{\vec{F}_1}^3)^\alpha}, \sqrt[3]{1 - (1 - l_{\vec{F}_1}^3)^\alpha}; \sqrt[3]{1 - (1 - \xi_{\vec{F}_1}^3)^\alpha} \rangle, \langle (\widetilde{k_{\vec{F}_1}})^\alpha, (\widetilde{l_{\vec{F}_1}})^\alpha; \widetilde{\xi_{\vec{F}_1}^3} \rangle \}$
- iv. $\vec{F}_1^\alpha = \{ \langle (k_{\vec{F}_1})^\alpha, (l_{\vec{F}_1})^\alpha; \xi_{\vec{F}_1}^3 \rangle, \langle \sqrt[3]{1 - (1 - \widetilde{k_{\vec{F}_1}}^3)^\alpha}, \sqrt[3]{1 - (1 - \widetilde{l_{\vec{F}_1}}^3)^\alpha}; \sqrt[3]{1 - (1 - \widetilde{\xi_{\vec{F}_1}^3}^\alpha)^\alpha} \rangle \}$
- v. $\vec{F}_1^{-1} = \langle \widetilde{A_{\vec{F}}} ; \widetilde{\xi_{\vec{F}}} \rangle, \langle A_{\vec{F}} ; \xi_{\vec{F}} \rangle$

3. Proposed Aggregation Operator & score function

In this section we proposed an aggregation operator and a score function under Fermatean cubic fuzzy environment.

Let $\vec{F} = (\langle A_{\vec{F}}; \xi_{\vec{F}} \rangle, \langle \widetilde{A_{\vec{F}}}; \widetilde{\xi_{\vec{F}}} \rangle)$ be a collection of F-CFNs for $i = 1, 2, 3, \dots, n$. Suppose $W = (w_1, w_2, \dots, w_n)^T$ be the weight vector of F_i for $i = 1, 2, 3, \dots, n$ with $w_i \geq 0$ and $w_i \in [0, 1]$ $\sum_{i=1}^n w_i = 1$. Then, the aggregation result according to our definition is also F-CFN that is given below-

$$K(\vec{F}_1, \vec{F}_2, \dots, \vec{F}_n) = [\langle (1 - \sqrt[3]{1 - \prod_{i=1}^n (1 - k_{\vec{F}_i}^3)^{w_i}}, 1 - \sqrt[3]{1 - \prod_{i=1}^n (1 - l_{\vec{F}_i}^3)^{w_i}}, 1 - \sqrt[3]{1 - \prod_{i=1}^n (1 - \xi_{\vec{F}_i}^3)^{w_i}}, \langle 1 - \prod_{i=1}^n (1 - \widetilde{k_{\vec{F}_i}}^3)^{w_i}, 1 - \prod_{i=1}^n (1 - \widetilde{l_{\vec{F}_i}}^3)^{w_i}, 1 - \prod_{i=1}^n (1 - \widetilde{\xi_{\vec{F}_i}^3})^{w_i} \rangle]$$

3.1-Properties of aggregation operator under F-CFN

- I. **Idempotency:** Let $\vec{F}_i$ ($i = 1, 2, \dots, n$) be a collection of F-CFNs. If $\vec{F}_i = \vec{F}$, then $K(\vec{F}_1, \vec{F}_2, \dots, \vec{F}_n) = \vec{F}$.
- II. **Boundedness:** $\vec{F}_{min} \leq K(\vec{F}_1, \vec{F}_2, \dots, \vec{F}_n) \leq \vec{F}_{max}$, where $\vec{F}_i = (min(\Lambda_i), max(Y_i))$ and $\vec{F}_{max} = (min(\Lambda_i), max(Y_i))$.
- III. **Continuity:** It is obvious that $K(\vec{F}_1, \vec{F}_2, \dots, \vec{F}_n)$ is continuous.
- IV. $K(0, 0, 0, \dots, 0) = (\langle 1, 1, \dots, 1 \rangle, \langle 0, 0, 0, \dots, 0 \rangle)$
- V. $K(1, 1, 1, \dots, 1) = (\langle 0, 0, 0, \dots, 0 \rangle, \langle 1, 1, 1, \dots, 1 \rangle)$

3.2-Existing score function¹⁹

Let $\vec{F}_i = (\langle A_{\vec{F}_i}; \xi_{\vec{F}_i} \rangle, \langle \widetilde{A_{\vec{F}_i}}; \widetilde{\xi_{\vec{F}_i}} \rangle)$ where $A_{\vec{F}} = [k_{\vec{F}}, l_{\vec{F}}]$, $\widetilde{A_{\vec{F}}} = [\widetilde{k_{\vec{F}}}, \widetilde{l_{\vec{F}}}]$ for $(i = 1, 2, \dots, n)$ be the family of F-CFNs;

then the existing score function for F-CFNs is define as below-

$$S(\overrightarrow{F_N}) = \left(\frac{(k_{\overrightarrow{F_l}}^3 + l_{\overrightarrow{F_l}}^3 - z_{\overrightarrow{F_l}}^3)}{3} \right)^3 - \left(\frac{(\widetilde{k}_{\overrightarrow{F_l}}^3 + \widetilde{l}_{\overrightarrow{F_l}}^3 - \widetilde{z}_{\overrightarrow{F_l}}^3)}{3} \right)^3$$

where $S(\overrightarrow{F_N}) \in [-2, 2]$.

❖ **Ranking of Score function:**

- i. If $S(F_{N_1}) > S(F_{N_2})$, then F_{N_1} is greater than F_{N_2} .
- ii. If $S(F_{N_1}) < S(F_{N_2})$, then F_{N_1} is less than F_{N_2} .
- iii. If $S(F_{N_1}) = S(F_{N_2})$, then $F_{N_1} = F_{N_2}$

4. Proposed methodology

This section introduces a Fermatean cubic fuzzy set environment based decision making strategy. The proposed method in steps are described below;

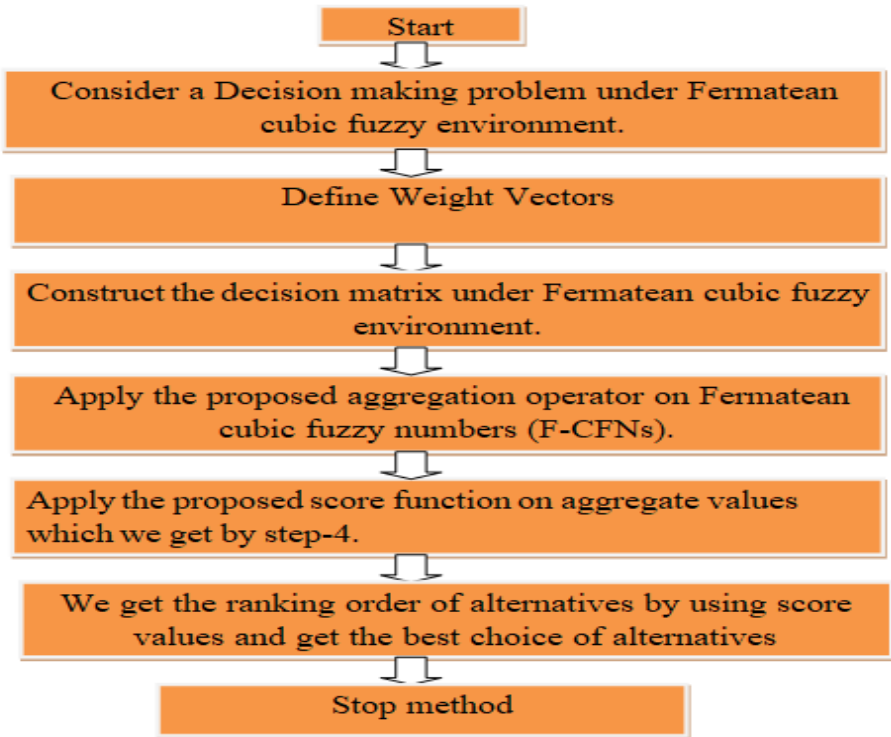


Figure 1: Representation of decision making approach.

STEP-1: First consider the DM problem under Fermatean cubic fuzzy environment.

STEP-2: After the step-1, Let us define weights vectors.

STEP-3: Now, construct the decision matrix under Fermatean cubic fuzzy environment.

STEP-4: In this step, apply the proposed aggregation operator on Fermatean cubic fuzzy numbers and collect the aggregate data by using equation (6).

STEP-5: In this step, apply the score function (eqⁿ-7) on aggregate values which we get by step-4.

STEP-6: In the last step, we find the best option among the options by ranking them according to the score values that were acquired.

STEP-7: End

5. Numerical computation of MCDM Problem

Here we present a MCDM problem and apply our modified aggregation operators to prove the applicability and smoothness of our newly introduced operators. To complete the process and find out a solution to a decision making problem.

(Example-1) To show the application of the proposed method to MCDM, we consider a practical example which is A financial institution wants to invest in one of four available funds (alternative funds): Fund A, Fund B, Fund C and Fund D. The decision is based on four criteria:

1. **Risk (C1)** (Lower is better)
2. **Return on Investment (ROI) (C2)** (Higher is better)
3. **Liquidity (C3)** (Higher is better)
4. **Market Stability (C4)** (Higher is better)

Each criterion is assigned a weight based on its importance:

$$W = (0.07, 0.28, 0.30, 0.35)$$

The decision-makers express their opinions using **Fermatean cubic numbers**, which include both membership and non-membership degrees.

Table-1: Decision making matrix under Fermatean cubic fuzzy set

Criteria	FUND A	FUND B	FUND C	FUND D
C₁ (Risk)	$\langle [0.6, 0.8]; 0.6 \rangle$ $\langle [0.5, 0.6]; 0.8 \rangle$	$\langle [0.6, 0.8]; 0.6 \rangle$ $\langle [0.5, 0.6]; 0.8 \rangle$	$\langle [0.7, 0.8]; 0.6 \rangle$ $\langle [0.5, 0.6]; 0.7 \rangle$	$\langle [0.3, 0.4]; 0.6 \rangle$ $\langle [0.8, 0.9]; 0.7 \rangle$
C₂ (ROI)	$\langle [0.5, 0.7]; 0.5 \rangle$ $\langle [0.4, 0.5]; 0.6 \rangle$	$\langle [0.3, 0.4]; 0.8 \rangle$ $\langle [0.8, 0.9]; 0.6 \rangle$	$\langle [0.5, 0.7]; 0.8 \rangle$ $\langle [0.4, 0.6]; 0.6 \rangle$	$\langle [0.6, 0.7]; 0.8 \rangle$ $\langle [0.5, 0.6]; 0.5 \rangle$
C₃ (Liquidity)	$\langle [0.5, 0.6]; 0.8 \rangle$ $\langle [0.4, 0.7]; 0.6 \rangle$	$\langle [0.6, 0.7]; 0.6 \rangle$ $\langle [0.5, 0.6]; 0.7 \rangle$	$\langle [0.6, 0.8]; 0.2 \rangle$ $\langle [0.5, 0.6]; 0.3 \rangle$	$\langle [0.6, 0.8]; 0.7 \rangle$ $\langle [0.5, 0.6]; 0.6 \rangle$
C₄ (Market Stability)	$\langle [0.6, 0.8]; 0.6 \rangle$ $\langle [0.5, 0.6]; 0.7 \rangle$	$\langle [0.6, 0.7]; 0.9 \rangle$ $\langle [0.5, 0.6]; 0.4 \rangle$	$\langle [0.5, 0.7]; 0.7 \rangle$ $\langle [0.4, 0.6]; 0.6 \rangle$	$\langle [0.4, 0.5]; 0.8 \rangle$ $\langle [0.7, 0.8]; 0.6 \rangle$

Solution-

STEP1-

Table-2: We calculate the aggregate values by using the existing proposed aggregation operator for each and every alternating fund-

Aggregate values by existing aggregation operator (Ref-15)	Aggregate values by proposed aggregation operator
$A_1 = (\langle 0.5831, 0.7287, 0.6001 \rangle, \langle 0.5892, 0.6913, 0.7334 \rangle)$	$K_1 = (\langle 0.4169, 0.2713, 0.3999 \rangle, \langle 0.2868, 0.4596, 0.4080 \rangle)$
$A_2 = (\langle 0.4337, 0.6500, 0.5702 \rangle, \langle 0.5249, 0.6634, 0.5627 \rangle)$	$K_2 = (\langle 0.5663, 0.3500, 0.4298 \rangle, \langle 0.2383, 0.4134, 0.1854 \rangle)$
$A_3 = (\langle 0.4808, 0.5944, 0.6171 \rangle, \langle 0.5024, 0.5385, 0.5086 \rangle)$	$K_3 = (\langle 0.5192, 0.4056, 0.3829 \rangle, \langle 0.1210, 0.2258, 0.2040 \rangle)$
$A_4 = (\langle 0.5166, 0.6609, 0.8087 \rangle, \langle 0.5259, 0.6634, 0.5413 \rangle)$	$K_4 = (\langle 0.4834, 0.3391, 0.1913 \rangle, \langle 0.1925, 0.3360, 0.1864 \rangle)$

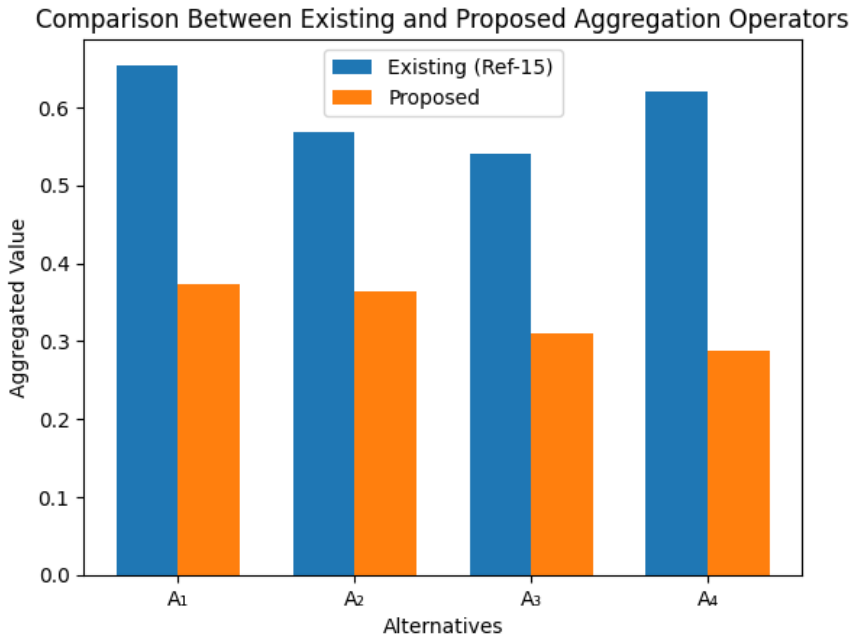


Figure 2: Graphical representation of comparison of Aggregation operators

STEP-2 : Here, we compute the score values by using the score functions-
 $S(F_{N_1}) = 0.0000031142, S(F_{N_2}) = 0.0000956722, S(F_{N_3}) = 0.0001263708, S(F_{N_4}) = 0.0001106685$

STEP-3 According to the calculating score values C is the best funding which is doing by investor. And the order of funding is given below-

$$C > D > B > A$$

6. Result and Discussion

In this study, we applied the Modified Fermatean Cubic Aggregation Operator & Score Function for multicriteria decision-making (MCDM) in the investment of funds. The criteria weights were assigned as $W = (0.07, 0.28, 0.30, 0.35)$, reflecting their relative importance in the decision-making process. In Fig.1 shows the flow chart of the improved methodology. and In Fig.2 provide a graphical representation of existing and proposed aggregate values.

After implementing the proposed aggregation operator and score function, the alternatives were evaluated and ranked accordingly. The final order of funding based on the decision model was determined as:

$$C > D > B > A$$

This ranking indicates that Alternative C is the most suitable investment choice, followed by Alternative D, then Alternative B and finally Alternative A, which is the least preferred.

7. Conclusion

The Modified Fermatean Cubic Aggregation Operator & Score Function effectively enhances multicriteria decision-making (MCDM) for fund investment by handling uncertainty and hesitation more flexibly. With the given criteria weights $W = (0.07, 0.28, 0.30, 0.35)$, the derived ranking

$$C > D > B > A$$

indicates that Alternative C is the most suitable for funding. This method provides a robust and precise approach for financial decision-making, outperforming traditional fuzzy models. It can be extended to other investment scenarios, ensuring optimal allocation of resources in uncertain environments.

Future scope of this research:

- ❖ **Hybrid MCDM Approaches** – Incorporating other MCDM methods like TOPSIS, VIKOR, or DEMATEL with Fermatean cubic operators can improve ranking stability and decision robustness.
- ❖ **Real-Time Investment Analysis** – Applying this model in real-time financial markets can help investors make dynamic and adaptive funding decisions.
- ❖ **Application in Diverse Domains** – Beyond fund investment, this approach can be extended to risk assessment, supply chain management, healthcare decision-making, and project selection.

- ❖ **Sensitivity and Comparative Analysis** – Conducting deeper sensitivity analysis and comparisons with other fuzzy-based models will validate the efficiency and reliability of the proposed method.

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