

Solution of Unbalanced Assignment Problem using Pentagonal Fuzzy Numbers

Amit Kumar Nandan and J. P. Tripathi

University Department of Mathematics, B.R.A. Bihar University, Muzaffarpur,
Bihar, India

E-mail: jiteshmaths@gmail.com

(Received November 06, 2025, Accepted December 10, 2025)

Abstract: This paper introduces a solution mechanism for the Unbalanced Assignment Problem with fuzzy costs. Pentagonal fuzzy numbers are used to represent the fuzzy costs. In order to determine the best assignment, we first make the unbalanced assignment problem balanced, and then we use the Ranking of Pentagonal Fuzzy Number (PFN) to turn it into a crisp assignment problem.

Keywords: Fuzzy Unbalanced Assignment Problem, Pentagonal Fuzzy Number, Hungarian Method, Ranking of Pentagonal Fuzzy Number.

1. Introduction

The assignment problem is a specific type of linear programming problem. It aims to provide n work to n people at the lowest possible cost and highest possible profit. People can be assigned to occupations, courses to rooms, drivers to trucks, operators to machines, delivery routes for trucks, or problems for the research team, among other things.

One subclass of fuzzy transportation problems is the fuzzy assignment problem, which is a particular kind of fuzzy linear programming problem.

According to the problem's mathematical formulation, it is a highly degenerate 0-1 programming problem, and all of the algorithms developed to find the best solution for the transportation problem can also be applied to unbalanced assignment problems, which are commonly known as the Hungarian method, which was developed by H.W. Kuhn¹. The algorithm was examined by James Munkres², who noted that it is highly strong polynomial. The algorithm has been referred to as the Munkres assignment algorithm ever since. In order to tackle the assignment problem, Balinski, Hung, and Goldfarb³⁻⁵ adapted the simplex method for linear programming. A fuzzy assignment model, which assumes that everyone has the same talents, was

presented by Chen⁶. The fuzzy assignment problem was given a mathematical model by Long-Sheng Huang and Li-Pu Zhang⁷, who then converted the model into a particular assignment problem with qualification restrictions. In order to maximize profit, Khandelwal⁸ suggested a modified assignment strategy for solving the assignment problem. This method can also be used to solve problems with unbalanced assignments. Additionally, Khandelwal⁹ suggested a recruiting strategy that combined the Hungarian approach with evolutionary algorithms and fuzzy triangular numbers. Fuzzy ranking is used by A. K. Nandan, J. P. Tripathi *et al.*^{10,11} to determine the best answer to the assignment problem. They also propose a ranking approach that has more advantages than the fuzzy ranking technique now in use.

2. Definitions

2.1 Fuzzy Set: If a set $\tilde{A}$ from a universal set Y is mapped to the closed interval $[0,1]$ by a membership function, then the set is regarded as fuzzy. It is defined as the set of matches offered by

$$\tilde{A} = \{(y, \mu_{\tilde{A}}(y)); y \in Y\}$$

where the fuzzy set $\tilde{A}$'s membership value for $y \in Y$ is represented by $\mu_{\tilde{A}}(y)$. The value of membership is provided by

$$\mu_{\tilde{A}}(y) = \begin{cases} 1, & \text{when } y \text{ is completely in } \tilde{A} \\ 0, & \text{if } y \notin \tilde{A} \\ (0,1), & \text{if part of } y \text{ is in } \tilde{A} \end{cases}$$

2.2 Fuzzy Number: - We define a fuzzy set $\tilde{A}$ on the set of real numbers. If the following properties are present in its membership function $\mu_{\tilde{A}}: R \rightarrow [0,1]$, then R is considered a fuzzy number.

- (i) $\tilde{A}$ is normal. When $\mu_{\tilde{A}}(y) = 1$, it indicates that there is an $y \in R$.
- (ii) $\tilde{A}$ is convex. This indicates that for any $y_1, y_2 \in R$,
$$\mu_{\tilde{A}}(\lambda y_1 + (1-\lambda) y_2) \geq \min. \{\mu_{\tilde{A}}(y_1), \mu_{\tilde{A}}(y_2)\}, \lambda \in [0,1]$$
- (iii) $\mu_{\tilde{A}}$ is a semi-continuous upper limit.
- (iv) The bound of $\text{supp}(\tilde{A})$ is in R .

2.3 Pentagonal Fuzzy Number: -

A fuzzy pentagonal number is described as $\tilde{A}^{PL} = (a_1, a_2, a_3, a_4, a_5)$ if its membership function is provided by

$$\mu_{\tilde{A}^{PL}}(y) = \begin{cases} 0; & y \leq a_1 \\ \frac{y-a_1}{a_2-a_1}; & a_1 \leq y \leq a_2 \\ \frac{y-a_2}{a_3-a_2}; & a_2 \leq y \leq a_3 \\ 1; & y = a_3 \\ \frac{a_4-y}{a_4-a_3}; & a_3 \leq y \leq a_4 \\ \frac{a_5-y}{a_5-a_4}; & a_4 \leq y \leq a_5 \\ 0; & y \geq a_5 \end{cases}$$

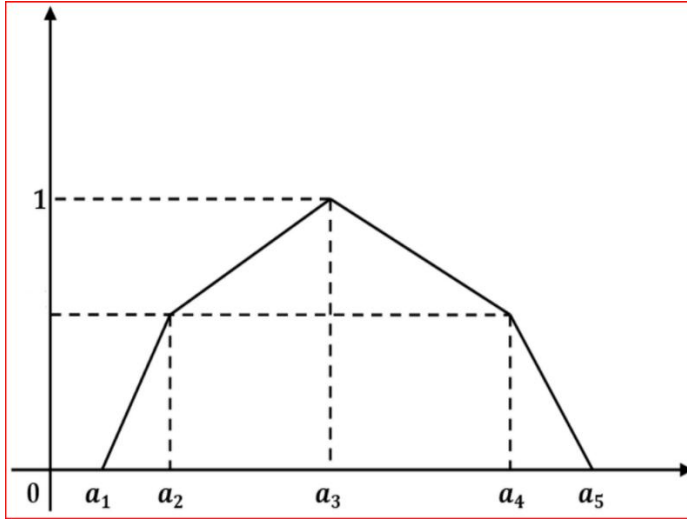


Figure 1: Pentagonal Fuzzy Number

2.4 Ranking Function: - The set of all pentagonal fuzzy numbers is represented by $f(R)$. Additionally, we design a ranking function $\mathcal{R}: f(R) \rightarrow R$ that converts any fuzzy number to a real number. If R has a linear ranking function, its ranking function is given by

$$\mathcal{R}(\tilde{A}^{PL}) = \left(\frac{a_1 + a_2 + a_3 + a_4 + a_5}{5} \right)$$

3. Fuzzy assignment problem

In terms of mathematics, the fuzzy assignment problem is

$$\text{Minimize } Z = \sum_{i=1}^n \sum_{j=1}^n C_{ij} y_{ij}.$$

Subject to the constraints

$$\sum_{i=1}^n y_{ij} = 1; j = 1, 2, 3, \dots, m$$

$$\sum_{j=1}^m y_{ij} = 1; i = 1, 2, 3, \dots, n$$

$$y_{ij} = \begin{cases} 1, & \text{if the } j\text{th job is allocated to the } i\text{th resource} \\ 0, & \text{Otherwise} \end{cases}$$

The notation y_{ij} indicates that the i^{th} individual will be allocated the j^{th} task.

4. Transformation of an Unbalanced Assignment Problem into a Balanced one

An unbalanced assignment problem occurs when the number of rows and columns is not equal. subsequently, by adding the necessary number of dummy rows or columns such that the cost matrix is square, this problem was changed into a balanced assignment problem. The entries in the dummy rows and columns are regarded as having zero values.

5. Proposed Algorithm

Steps of the algorithm are as follows:

- Based on the provided problem, find the fuzzy cost table.
- To make the cost matrix square, add as many dummy rows or columns as required.
- First, use the Pentagonal Ranking Function to transform the cost values from the FAP's pentagonal fuzzy numbers into crisp values.
- To determine the optimal fuzzy assignment, use the Hungarian technique to solve the fuzzy cost matrix.

6. Numerical Examples

Let us look at an assignment problem that is fuzzy and unbalanced, where rows represent five routes R_1, R_2, R_3, R_4 and R_5 columns that show the salesperson's S_1, S_2, S_3, S_4 , and S_5 . The cost matrix C_{ij} is provided, using pentagonal fuzzy numbers as its constituents. The problem is to choose optimal assignment to minimize overall cost of routes assignment.

Cost Matrix

$$C_{ij} = \begin{pmatrix} (-6, -2, 2, 5, 6) & (-2, 1, 2, 3, 6) & (-2, -1, 1, 3, 4) & (-3, -2, 2, 3, 5) \\ (-4, -1, 2, 3, 5) & (-2, -1, 1, 2, 5) & (-2, 1, 2, 3, 6) & (-2, -1, 1, 3, 4) \\ (-3, -2, 2, 3, 5) & (-6, -2, 2, 5, 6) & (-1, 1, 2, 3, 5) & (-4, -1, 2, 3, 5) \\ (-5, -4, 4, 5, 10) & (-4, -1, 1, 4, 10) & (-3, -2, 2, 3, 5) & (-2, -1, 1, 2, 5) \\ (-2, -1, 1, 3, 4) & (-1, 1, 2, 3, 5) & (-2, -1, 1, 3, 4) & (-4, -1, 1, 4, 10) \end{pmatrix}$$

Due to the fact that there are a different number of rows and columns in this example. Therefore, the FAP is not balanced, we add a dummy(zero) column to bring it into balance.

$C_{ij} =$

$$\begin{pmatrix} (-6, -2, 2, 5, 6) & (-2, 1, 2, 3, 6) & (-2, -1, 1, 3, 4) & (-3, -2, 2, 3, 5) & (0, 0, 0, 0, 0) \\ (-4, -1, 2, 3, 5) & (-2, -1, 1, 2, 5) & (-2, 1, 2, 3, 6) & (-2, -1, 1, 3, 4) & (0, 0, 0, 0, 0) \\ (-3, -2, 2, 3, 5) & (-6, -2, 2, 5, 6) & (-1, 1, 2, 3, 5) & (-4, -1, 2, 3, 5) & (0, 0, 0, 0, 0) \\ (-5, -4, 4, 5, 10) & (-4, -1, 1, 4, 10) & (-3, -2, 2, 3, 5) & (-2, -1, 1, 2, 5) & (0, 0, 0, 0, 0) \\ (-2, -1, 1, 3, 4) & (-1, 1, 2, 3, 5) & (-2, -1, 1, 3, 4) & (-4, -1, 1, 4, 10) & (0, 0, 0, 0, 0) \end{pmatrix}$$

Utilize a ranking function to transform the cost matrix's fuzzy numbers into crisp values.

$$\mathcal{R}(\tilde{A}^{PL}) = \left(\frac{a_1 + a_2 + a_3 + a_4 + a_5}{5} \right)$$

$$\text{Ranking of } C_{11} = R(C_{11}) = \frac{-6-2+2+5+6}{5} = \frac{5}{5} = 1$$

In the same way, the pentagonal fuzzy number ranking for the fuzzy costs is computed in the manner described below:

$$C_{ij} = \begin{pmatrix} 1 & 2 & 1 & 1 & 0 \\ 1 & 1 & 2 & 1 & 0 \\ 1 & 1 & 2 & 1 & 0 \\ 2 & 2 & 1 & 1 & 0 \\ 1 & 2 & 1 & 2 & 0 \end{pmatrix}$$

Since each row's row minimum is zero, Subtract the row's minimal value from each of the values in the row that corresponds to it, we get

$$C_{ij} = \begin{pmatrix} 1 & 2 & 1 & 1 & 0 \\ 1 & 1 & 2 & 1 & 0 \\ 1 & 1 & 2 & 1 & 0 \\ 2 & 2 & 1 & 1 & 0 \\ 1 & 2 & 1 & 2 & 0 \end{pmatrix}$$

Subtract the column's minimal value from each of the values in the column that corresponds to it, we get

$$C_{ij} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

Using the Hungarian Assignment Method, the fuzzy cost matrix

$$C_{ij} = \begin{pmatrix} (0) & 1 & 0 & 0 & 0 \\ 0 & (0) & 1 & 0 & 0 \\ 0 & 0 & 1 & (0) & 0 \\ 1 & 1 & (0) & 0 & 0 \\ 0 & 1 & 0 & 1 & (0) \end{pmatrix}$$

Thus, the assignment is

Routes	Salesman
R ₁	S ₁
R ₂	S ₂
R ₃	S ₄
R ₄	S ₃
R ₅	S ₅

Therefore, the optimal cost of an assignment is

$R(C_{11}) + R(C_{22}) + R(C_{34}) + R(C_{43}) + R(C_{55})$
 $= R(-6, -2, 2, 5, 6) + R(-2, -1, 1, 2, 5) + R(-4, -1, 2, 3, 5) + R(-3, -2, 2, 3, 5)$
 $+ R(0, 0, 0, 0, 0)$
 $= 1 + 1 + 1 + 1 + 0$
 $= 4$

7. Conclusion

In order to transform the fuzzy unbalanced assignment problem into a crisp assignment problem, this article used the ranking of Pentagonal fuzzy numbers. This approach allows us to obtain both the fuzzy optimal total cost and the optimal assignment, as demonstrated by a numerical example. Through the use of the Ranking of PFN approach, we have demonstrated that the overall cost derived is optimum. One may get the conclusion that the Pentagonal Fuzzy Number approach is an efficient way to solve fuzzy problems.

References

1. H.W. Kuhn; The Hungarian method for the assignment problem, *Neval Research Logistics*, 2 (1955), 83-97.
2. J. Munkres; Algorithms for the Assignment and Transportation Problems, *Journal of the Society for Industrial and Applied Mathematics*, 5(1), (1957), 32-38.

3. M. L. Balinski; A competitive (dual) simplex method for the assignment problem, *Mathematical Programming*, 34 (1986), 125-141.
4. M.S. Hung; A polynomial simplex method for the assignment problem, *Operations Research*, 31 (1983), 595-600.
5. D. Goldfarb; Efficient dual simplex methods for the assignment problem, *Mathematical Programming*, 33 (1985), 187-203.
6. M. S. Chen; On a Fuzzy Assignment Problem, *Jamkang Journal* 22 (1985), 407-411.
7. Long-Sheng Huang, Li-Pu Zhang; Solution Method for Fuzzy Assignment Problem with Restriction of Qualification, *Proceedings of the Sixth International Conference on Intelligent Systems Design and Applications (ISDA'06)*, 2006.
8. A. Khandelwal; A Modified Approach for Assignment Method to Maximize the throughput, *International Journal of Pure and Engg. Mathematics (IJPEM)*, 2(II) (2014), 13-20, ISSN 2348-3881.
9. A. Khandelwal and A. Agarwal; An Amalgamated Approach of Fuzzy Logic and Genetic Algorithm for Better Recruitment Process, *TNC Transactions of Network and Communications*, 3-3 (2015), 23-31, ISSN: 2054-7420.
10. Amit Kr. Nandan, J. P. Tripathi, V. K. Tiwari, R. K. Shukla; Optimal solution of fuzzy assignment problem using Ranking Technique, *International Journal of Scientific Research in Science and Technology (IJSRST)*, 11-2 (2024), 975-983.
11. Amit Kr. Nandan, J. P. Tripathi, V. K. Tiwari; An optimized approach to solve assignment problems using the fuzzy hungarian method, *Engineering and Technology Journal (ETJ)*, 9-7 (2024), 4458-4461.