Four-Dimensional Finsler Spaces with
T-Tensor of Some Special Forms

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Abstract: The $T$-tensor played an important role in the Finsler geometry. In this paper, we discuss a four-dimensional Finsler space whose $T$-tensor is of special forms.

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1. Introduction

Let $M^4$ be a four-dimensional smooth manifold and $F^4 = (M^4, L)$ be a four-dimensional Finsler space equipped with a metric function $L(x, y)$ on $M^4$. The normalized supporting element, the metric tensor, the angular metric tensor and Cartan tensor are defined by

$$l_i = \hat{\partial}_i L, \quad g_{ij} = \frac{1}{2} \hat{\partial}_i \hat{\partial}_j L^2, \quad h_{ij} = L \hat{\partial}_i \hat{\partial}_j L \quad \text{and} \quad C_{ijk} = \frac{1}{2} \hat{\partial}_k g_{ij}$$ respectively.

The torsion vector $C^i_j$ is defined by $C^i_j = C^i_{jk} g^{jk}$. Throughout this paper, we use the symbols $\hat{\partial}_j$ and $\hat{\partial}_i$ for $\partial / \partial y^j$ and $\partial / \partial x^i$ respectively. The Cartan connection in the Finsler space is given as $\Gamma^i_{jk} = (F^i_{jk}, G^i_j, C^i_{jk})$. The $h$- and $v$-covariant derivatives of a covariant vector $X_i(x, y)$ with respect to the Cartan connection are given by

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(1.1) \[ X_{ij} = \partial_j X_i - (\partial_k X_i)G_j^k - F_{ij}^h X_h, \]

and

(1.2) \[ X_i|_j = \partial_j X_i - C_{ij}^h X_h. \]

In 1972, H. Kawaguchi\(^1\) and M. Matsumoto\(^2\) independently found an important tensor

(1.3) \[ T_{hijk} = L C_{hij} |_k + C_{hij} l_k + C_{hjk} l_i + C_{hkl} l_j. \]

This is called the T-tensor. It is completely symmetric in its indices. The vanishing of T-tensor is called T-condition.

U.P. Singh et al.\(^3-4\) studied three-dimensional Finsler spaces with T-tensor of the following forms:

(A) \[ T_{hijk} = \rho (h_{hi} h_{jk} + h_{hj} h_{ik} + h_{hk} h_{ij}), \]

(B) \[ T_{hijk} = h_{hi} P_{jk} + h_{hj} P_{ik} + h_{hk} P_{ij} + h_{ik} P_{hj} + h_{jk} P_{hi}, \]

(C) \[ T_{hijk} = \rho C_h C_i C_j C_k + a_h C_i C_j C_k + a_i C_h C_j C_k + a_j C_h C_i C_k + a_k C_i C_j C_h, \]

where \(P_{ij}\) are the components of a tensor field, \(a_h\) are the components of a covariant vector field and \(\rho\) is a scalar. Present authors\(^6-10\) studied the theory of four-dimensional Finsler space. In this paper, we discuss four-dimensional Finsler spaces with T-tensor of such forms.

2. Four-Dimensional Finsler Space

The Miron frame for a four-dimensional Finsler space is constructed by the unit vectors \(\{e^i_1, e^i_2, e^i_3, e^i_4\}\). The first vector \(e^i_1\) is the normalized supporting element \(l^i\) and the second \(e^i_2\) is the normalized torsion vector \(m^i = C^i / c\), the third \(e^i_3 = n^i\) and the fourth \(e^i_4 = p^i\) are constructed by \(g_{ij} e^i_\alpha e^j_\beta = \delta_{\alpha\beta}\). We suppose that the length \(c\) of the vector \(C^i\) does not vanish, i.e. the space is non-Riemannian. With respect to this frame, the scalar components of an arbitrary tensor \(T^i_j\) are defined by

(2.1) \[ T_{\alpha\beta} = T^i_j e^i_\alpha e^j_\beta, \]

from which, we get

(2.2) \[ T^i_j = T_{\alpha\beta} e^i_\alpha e^j_\beta, \]

where summation convention is also applied to Greek indices. The scalar components of the metric tensor \(g_{ij}\) are \(\delta_{\alpha\beta}\).
Let $H_{\alpha\beta\gamma}$ and $V_{\alpha\beta\gamma} / L$ be scalar components of the $h$- and $v$-covariant derivatives $e_{\alpha}^{i})$ respectively of the vectors $e_{\alpha}$, then

\[(2.3) \quad e_{\alpha}^{i})_{,j} = H_{\alpha\beta\gamma} e_{\beta}^{j} e_{\gamma}^{i},\]

and

\[(2.4) \quad L e_{\alpha}^{i})_{,j} = V_{\alpha\beta\gamma} e_{\beta}^{j} e_{\gamma}^{i}.\]

$H_{\alpha\beta\gamma}$ and $V_{\alpha\beta\gamma}$ are called $h$- and $v$-connection scalars respectively and are positively homogeneous of degree zero in $y$. Orthogonality of the Miron frame yields $H_{\alpha\beta\gamma} = -H_{\beta\alpha\gamma}$ and $V_{\alpha\beta\gamma} = -V_{\beta\alpha\gamma}$. Also we have $H_{1\gamma\beta} = 0$ and $V_{1\gamma\beta} = \delta_{\beta\gamma} - \delta_{1\beta} \delta_{1\gamma}$.

Now we define Finsler vector fields:

\[h_i = H_{2\gamma\gamma} e_{\gamma}^{i}, \quad j_i = H_{4\gamma\gamma} e_{\gamma}^{i}, \quad k_i = H_{3\gamma\gamma} e_{\gamma}^{i},\]

and

\[u_i = V_{2\gamma\gamma} e_{\gamma}^{i}, \quad v_i = V_{4\gamma\gamma} e_{\gamma}^{i}, \quad w_i = V_{3\gamma\gamma} e_{\gamma}^{i}.\]

The vector fields $h_i, j_i, k_i$ are called $h$-connection vectors and the vector fields $u_i, v_i, w_i$ are called $v$-connection vectors. The scalars $H_{2\gamma\gamma}, H_{4\gamma\gamma}, H_{3\gamma\gamma}$ and $V_{2\gamma\gamma}, V_{4\gamma\gamma}, V_{3\gamma\gamma}$ are considered as the scalar components $h_{\gamma}, j_{\gamma}, k_{\gamma}$ and $u_{\gamma}, v_{\gamma}, w_{\gamma}$ of the $h$- and $v$-connection vectors respectively with respect to the orthonormal frame.

From (2.4), we get

\[\begin{align*}
\text{a)} \quad & L e_{\gamma}^{i})_{,j} = L l^i_{,j} = m^i m_j + n^i n_j + p^i p_j = h^i_j, \\
\text{b)} \quad & L e_{\gamma}^{2})_{,j} = L m^i_{,j} = -l^i m_j + n^i u_j - p^i v_j, \\
\text{c)} \quad & L e_{\gamma}^{3})_{,j} = L n^i_{,j} = -l^i n_j - m^i u_j + p^i w_j, \\
\text{d)} \quad & L e_{\gamma}^{4})_{,j} = L p^i_{,j} = -l^i p_j + m^i v_j - n^i w_j.
\end{align*}\]

Because of the homogeneity of $e_{\alpha}^{i})$, (2.5) gives

\[L m^i_{,j} l^j = 0 = n^i u_j l^j - p^i v_j l^j,\]

\[L n^i_{,j} l^j = 0 = -m^i u_j l^j + p^i w_j l^j.\]

These imply $u_1 = u_j l^j = 0$, $v_1 = v_j l^j = 0$, $w_1 = w_j l^j = 0$. Thus, we have:
Proposition 2.1- The first scalar components $u_i$, $v_i$ and $w_i$ of the $v$-connection vectors $u_i$, $v_i$, $w_i$ vanish identically.

Let $C_{a\beta\gamma}$ be the scalar components of $LC_{ijk}$ with respect to the Miron frame, i.e.

$$L C_{ijk} = C_{a\beta\gamma} e_{\alpha\beta} e_{\beta\gamma} e_{\gamma\delta}.$$  

The main scalars of a four-dimensional Finsler space are given by:

$$C_{222} = A, \quad C_{233} = B, \quad C_{244} = C, \quad C_{322} = D,$$
$$C_{333} = E, \quad C_{422} = F, \quad C_{433} = G, \quad C_{344} = H.$$  

We also have $C_{344} = -(D + E)$, $C_{444} = -(F + G)$ and

$$A + B + C = L.$$  

The scalar components $T_{a\beta;\gamma}$ of $LT_j^i |^k$ are written in the form:

$$T_{a\beta;\gamma} = L\left(\delta_k^j T_{a\beta}\right) e_{\gamma\delta} e^k_{\gamma\delta} + T_{\mu\beta} V_{\mu\alpha\beta} + T_{\alpha\mu} V_{\mu\beta\gamma}.$$  

The explicit form of $C_{a\beta\gamma;\delta}$ is obtained as follows

$$C_{222;\delta} = A_{,\delta} - 3D u_{,\delta} + 3F v_{,\delta};$$
$$C_{233;\delta} = B_{,\delta} + (2D - E) u_{,\delta} + G v_{,\delta} - 2H w_{,\delta},$$
$$C_{244;\delta} = C_{,\delta} + (D + E) u_{,\delta} - (3F + G) v_{,\delta} + 2H w_{,\delta},$$
$$C_{322;\delta} = D_{,\delta} + (A - 2B) u_{,\delta} + 2H v_{,\delta} - F w_{,\delta},$$
$$C_{333;\delta} = E_{,\delta} + 3B u_{,\delta} - 3G w_{,\delta},$$
$$C_{344;\delta} = F_{,\delta} - 2H u_{,\delta} - (A - 2C) v_{,\delta} + D w_{,\delta},$$
$$C_{433;\delta} = G_{,\delta} + 2H u_{,\delta} - B v_{,\delta} + (2D + 3E) w_{,\delta},$$
$$C_{234;\delta} = H_{,\delta} + (F - G) u_{,\delta} - (2D + E) v_{,\delta} + (B - C) w_{,\delta},$$
$$C_{344;\delta} = -D_{,\delta} - E_{,\delta} + C u_{,\delta} - 2H v_{,\delta} + (F + 3G) w_{,\delta},$$
$$C_{444;\delta} = -F_{,\delta} - G_{,\delta} - 3C v_{,\delta} - (3D + 3E) w_{,\delta},$$
$$C_{1\beta\gamma;\delta} = -C_{\beta\gamma\delta}.$$  

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where $A_{,\delta} = L \left( \hat{\partial}_k A \right) e^k_{\delta}$. From (2.7) and (2.9), we get

$$
\begin{align*}
C_{222,\delta} + C_{333,\delta} + C_{444,\delta} &= A_{,\delta} + B_{,\delta} + C_{,\delta} = (L \circ)_{,\delta}, \\
C_{322,\delta} + C_{333,\delta} + C_{444,\delta} &= (A + B + C) u_{,\delta} = L \circ u_{,\delta}, \\
C_{422,\delta} + C_{433,\delta} + C_{444,\delta} &= -(A + B + C) v_{,\delta} = -L \circ v_{,\delta}.
\end{align*}
$$

From (2.6), it follows that

$$L^2 C_{hi|k} + L C_{hi} l_k = C_{afty;\delta} e_{a|y} e_{f|y} e_{t|y} e_{y|k};$$

which implies

$$L^2 C_{hi|k} = (C_{afty;\delta} - C_{afty} \delta_{i|\delta}) e_{a|y} e_{f|y} e_{t|y} e_{y|k};$$

From (1.3) and (2.11), we get

$$LT_{hijk} = (C_{afty;\delta} + C_{beta;\delta} \delta_{ta} + C_{a\gamma\delta} \delta_{i|\gamma} + C_{a\beta\gamma} \delta_{i|\gamma}) e_{a|y} e_{f|y} e_{t|y} e_{y|k};$$

Since the tensor $C_{hi|k}$ is completely symmetric in its indices, from (2.11) we get

$$C_{afty;\delta} - C_{afty} \delta_{i|\delta} = C_{afty} \delta_{i|\delta} - C_{afty} \delta_{i|\gamma}.$$

In view of (2.13), equation (2.10) gives

$$L \circ u_2 = C_{322,2} + C_{333,2} + C_{344,2} = C_{222,3} + C_{333,3} + C_{444,3} = (L \circ)_{3,3};$$

$$L \circ v_2 = C_{422,2} + C_{433,2} + C_{444,2} = C_{222,4} + C_{233,4} + C_{244,4} = (L \circ)_{4,4};$$

$$L \circ u_4 = C_{322,4} + C_{333,4} + C_{344,4} = C_{422,3} + C_{433,3} + C_{444,3} = -L \circ v_3;$$

Since $L_{,3} = L(\hat{\partial}_i L) e^i_{3} = L l_i n^i = 0$ and $L_{,4} = L(\hat{\partial}_i L) e^i_{4}) = L l_i p^i = 0$, we have:

**Proposition 2.2-** The scalar components $u_2$ and $v_2$ of the $v$-connection vectors $u_i$ and $v_i$ of a four-dimensional Finsler space are given by

$$u_2 = \xi_{,3}, \quad v_2 = -\xi_{,4};$$

and the scalar components $u_4$ and $v_3$ are related by $u_4 = -v_3$.

3. $T$-Tensor of Form (A)

A Finsler space is C-reducible if and only if the $T$-tensor is of the form (A) for $\rho \neq 0$. Let $F^4$ be a four-dimensional Finsler space with $T$-tensor of the form (A). The scalar components of the angular metric tensor $h_{ij}$ are given by
therefore in view of (2.12) and (A), we have

\[ C_{\alpha\beta\gamma\delta} \delta_{\alpha\gamma} \delta_{\alpha\delta} \delta_{\beta\delta} + C_{\alpha\beta\delta} \delta_{\alpha\delta} \delta_{\beta\gamma} + C_{\alpha\gamma\delta} \delta_{\alpha\delta} \delta_{\beta\gamma} \delta_{\gamma\delta} = \rho L \{ (\delta_{\alpha\beta} - \delta_{\alpha\delta} \delta_{\gamma\delta} \delta_{\beta\delta} ) (\delta_{\gamma\delta} - \delta_{\gamma\delta} \delta_{\delta\gamma} ) + (\delta_{\alpha\gamma} - \delta_{\alpha\delta} \delta_{\delta\gamma} ) (\delta_{\beta\delta} - \delta_{\alpha\delta} \delta_{\gamma\delta} ) \} , \]

which gives

\[
\begin{align*}
C_{222,\delta} &= 3 \rho L \delta_{2,\delta}, \\
C_{233,\delta} &= \rho L \delta_{2,\delta}, \\
C_{244,\delta} &= \rho L \delta_{2,\delta}, \\
C_{332,\delta} &= \rho L \delta_{3,\delta}, \\
C_{344,\delta} &= 3 \rho L \delta_{3,\delta}, \\
C_{433,\delta} &= \rho L \delta_{4,\delta}, \\
C_{444,\delta} &= 3 \rho L \delta_{4,\delta}.
\end{align*}
\]

(3.1)

Putting (3.1) into (2.10), we get

\[
\begin{align*}
& (L C)_{u,\delta} = 5 \rho L \delta_{2,\delta}, \\
& L C u_{\delta} = 5 \rho L \delta_{3,\delta}, \\
& -L C v_{\delta} = 5 \rho L \delta_{4,\delta}.
\end{align*}
\]

Also from the first equation of (3.1), we get

\[
C_{222,\delta} = A_{\delta} - 3 D u_{\delta} + 3 F v_{\delta} = 3 \rho L \delta_{2,\delta}.
\]

Thus, we have:

**Theorem 3.1** - If the T-tensor of a four-dimensional Finsler space is of the form (A) then \( \rho \) is given by

\[
\rho = \frac{A_{\delta}}{3 L} = \frac{1}{5} C_{2,2} = \frac{1}{5} C u_{3} = -\frac{1}{5} C v_{4}.
\]

**Theorem 3.2** - The scalar components of v-connection vectors \( u_{i} \) and \( v_{i} \) of a four-dimensional Finsler space with T-tensor of the form (A), are given by

\[
\begin{align*}
u_{1} &= 0, & \quad u_{2} &= 0, & \quad u_{3} &= C^{-1} c_{2,2}, & \quad u_{4} &= 0, \\
u_{1} &= 0, & \quad v_{2} &= 0, & \quad v_{3} &= 0, & \quad v_{4} &= -C^{-1} c_{2,2}.
\end{align*}
\]

4. T-Tensor of Form (B)

Ikeda\textsuperscript{13} showed that for an n-dimensional Finsler space with T-tensor of the form
(B) \[ T_{hijk} = h_{hi} P_{jk} + h_{hj} P_{ik} + h_{hk} P_{ij} + h_{ij} P_{hk}, \]

we get

\[ P_{ij} = \frac{1}{n + 3} \left\{ T_{ij} - \frac{T}{2(n + 1)} h_{ij} \right\}, \]

where \( T_{ij} = T_{hijk} g^{hk} \) and \( T = T_{ij} g^{ij} \). Therefore (B) becomes

\[ T_{hijk} = \frac{1}{n + 3} \left( h_{hi} T_{jk} + h_{hj} T_{ik} + h_{hk} T_{ij} + h_{ij} T_{hk} + h_{ik} T_{hj} + h_{jk} T_{hi} \right) \]

\[ - \frac{T}{(n + 1)(n + 3)} \left( h_{hi} h_{jk} + h_{hj} h_{ik} + h_{hk} h_{ij} \right). \]

Thus for a four-dimensional Finsler space, we have

\[ T_{hijk} = \frac{1}{7} \left( h_{hi} T_{jk} + h_{hj} T_{ik} + h_{hk} T_{ij} + h_{ij} T_{hk} + h_{ik} T_{hj} + h_{jk} T_{hi} \right) \]

\[ - \frac{T}{S} \left( h_{hi} h_{jk} + h_{hj} h_{ik} + h_{hk} h_{ij} \right) \]

Let \( T_{a\beta} \) be the scalar components of \( LT_{hi} \), i.e.

\[ LT_{hi} = T_{a\beta} e_{\alpha} e_{\beta h i} \]

In view of (2.12) and (4.1), we get

\[ C_{a\beta\gamma\delta} + C_{\beta\delta\alpha} \delta_{\alpha\gamma} + C_{\alpha\delta\gamma} \delta_{\beta\gamma} + C_{\alpha\beta\gamma} \delta_{\gamma\delta} = \frac{1}{7} \left[ \left( \delta_{\alpha\beta} - \delta_{\alpha\delta} \delta_{\beta\gamma} \right) T_{\gamma\delta} + \left( \delta_{\alpha\gamma} - \delta_{\alpha\delta} \delta_{\gamma\delta} \right) T_{\beta\delta} \right] \]

\[ + \left( \delta_{\alpha\gamma} - \delta_{\alpha\delta} \delta_{\gamma\delta} \right) T_{\beta\gamma} + \left( \delta_{\gamma\delta} - \delta_{\gamma\beta} \delta_{\delta\gamma} \right) T_{\alpha\beta} \]

\[ + \left( \delta_{\gamma\delta} - \delta_{\gamma\delta} \delta_{\beta\delta} \right) T_{\alpha\gamma} + \left( \delta_{\alpha\beta} - \delta_{\alpha\delta} \delta_{\beta\gamma} \right) \left( \delta_{\gamma\delta} - \delta_{\gamma\beta} \delta_{\delta\gamma} \right) \]

\[ + \left( \delta_{\alpha\gamma} - \delta_{\alpha\delta} \delta_{\gamma\delta} \right) \left( \delta_{\beta\gamma} - \delta_{\beta\delta} \delta_{\gamma\delta} \right) + \left( \delta_{\alpha\beta} - \delta_{\alpha\delta} \delta_{\beta\gamma} \right) \left( \delta_{\gamma\delta} - \delta_{\gamma\beta} \delta_{\delta\gamma} \right) \]

which gives
Putting (4.2) into (2.10), we get

\[
(LC)_{,\delta} = \frac{1}{7} \left\{ 3T_{2,\delta} + 3T_{22} + T_{33} + T_{44} - LT \right\} \delta_{2,\delta} + 2T_{23} \delta_{3,\delta} + 2T_{24} \delta_{4,\delta},
\]

\[
LC_{,u} = \frac{1}{7} \left\{ 8T_{5,\delta} + 2T_{23} \delta_{2,\delta} + (T_{22} + 3T_{33} + T_{44} - LT) \delta_{3,\delta} + 2T_{34} \delta_{4,\delta} \right\},
\]

\[
-LC_{,v} = \frac{1}{7} \left\{ 5T_{4,\delta} + 2T_{24} \delta_{2,\delta} + 2T_{34} \delta_{3,\delta} + (T_{22} + T_{33} + 3T_{44} - LT) \delta_{4,\delta} \right\}.
\]

Therefore

\[
(LC)_{,2} = \frac{1}{7} \left\{ 8T_{22} + T_{33} + T_{44} - LT \right\}, \quad (LC)_{,3} = T_{23}, \quad (LC)_{,4} = T_{24},
\]

\[
LC_{,u_2} = T_{23}, \quad LC_{,u_3} = \frac{1}{7} \left\{ T_{22} + 8T_{33} + T_{44} - LT \right\}, \quad LC_{,u_4} = T_{34},
\]

\[-LC_{,v_2} = T_{24}, \quad -LC_{,v_3} = T_{34}, \quad -LC_{,v_4} = \frac{1}{7} \left\{ T_{22} + T_{33} + 8T_{44} - LT \right\}.
\]

From \( T = \delta_{ij} g^{ij} \), we find

\[
LT = T_{\alpha \beta} \delta_{\alpha \beta} = T_{\alpha \alpha} = T_{22} + T_{33} + T_{44}.
\]

Thus, in view of (4.3), we have:

**Theorem 4.1.-** If the T-tensor of a four-dimensional Finsler space is of the form
(B) the scalar components of the tensor $T_{ij}$ are given by

$$T_{1a} = 0, \quad T_{22} = (L \c)_{;2}, \quad T_{33} = L \c u_3, \quad T_{44} = -L \c v_4,$$

$$T_{23} = L \c u_3 = (L \c)_{;3}, \quad T_{24} = -L \c v_2 = (L \c)_{;4}, \quad T_{34} = L \c u_4 = -L \c v_3,$$

and $T = \c u_3 + \c u_3 - \c v_4$.

5. $T$-Tensor of Form (C)

U. P. Singh et al.\textsuperscript{4} showed that the $T$-tensor of a C-2 like Finsler space is of the form

(C) $$T_{hjk} = \rho \delta_2 c_i C_j C_k + a_{\alpha} C_i C_j C_k + a_{\alpha} C_i C_j C_k + a_{\alpha} C_i C_j C_k + a_{\alpha} C_i C_j C_k.$$ \hfill (5.1)

Let $a_{\alpha}$ be the scalar components of $L a_i$, i.e.

$$L a_i = a_{\alpha} e_{a;\alpha}.$$\hfill (2.10)

Since $e_{2;\alpha} = C_i / \c$, we get $C_i = \c \delta_2 a_{\alpha} e_{a;\alpha}$. Therefore in view of (2.12) and (C), we have

$$C_{\alpha \beta \gamma \delta} + C_{\beta \alpha \gamma \delta} \delta_{\alpha} + C_{\gamma \alpha \beta \delta} \delta_{\beta} + C_{\alpha \beta \gamma \delta} \delta_{\gamma} = \rho L \c (\delta_2 a_{\beta} \delta_2 \alpha \beta \gamma \delta)$$

$$+ C_{\alpha \beta \gamma \delta} \delta_{2 \alpha} (a_{\beta} \delta_2 \alpha \beta \gamma \delta) + a_{\beta} \delta_2 \alpha \beta \gamma \delta + a_{\gamma} \delta_{2 \alpha} \beta \gamma \delta + a_{\delta} \delta_{2 \alpha} \beta \gamma \delta),$$

which gives

$$C_{222,\delta} = \c^3 (\rho L \c + 3 a_2) \delta_{2 \delta} + \c^3 a_{\delta}, \quad C_{233,\delta} = 0, \quad C_{244,\delta} = 0,$$

$$C_{322,\delta} = \c^3 a_3 \delta_{2 \delta}, \quad C_{333,\delta} = 0, \quad C_{344,\delta} = 0,$$

$$C_{422,\delta} = \c^3 a_4 \delta_{2 \delta}, \quad C_{433,\delta} = 0, \quad C_{444,\delta} = 0.$$ \hfill (5.1)

Putting (5.1) into (2.10), we get

$$(L \c)_{;\delta} = \c^3 (\rho L \c + 3 a_2) \delta_{2 \delta} + \c^3 a_{\delta},$$

$$L \c u_{\delta} = \c^3 a_3 \delta_{2 \delta},$$

$$-L \c v_{\delta} = \c^3 a_4 \delta_{2 \delta}.$$\hfill (2.10)

Since $T_{hjk}$ is an indicatory tensor, from (C) it follows that $a_{1} = a_{1} y' = 0$. Thus we have:

**Theorem 5.1** - If the $T$-tensor of a four-dimensional Finsler space is of the form (C), the scalar components $a_{\alpha}$ of the $L a_i$ are given by

$$a_1 = 0, \quad a_2 = \frac{L}{4} (\c^3 \c_{;2} - \rho \c).$$
Theorem 5.2 - In a four-dimensional Finsler space with T-tensor of the form (C), the scalar components of v-connection vectors $u_i$ and $v_i$ are given by 

$$L \mathcal{C} u_\delta = \mathbb{C} a_3 \delta_{\delta23}, \quad -L \mathcal{C} v_\delta = \mathbb{C} a_4 \delta_{\delta24}. $$

Corollary 5.1 - In a four-dimensional Finsler space with T-tensor of the form (C), the v-connection vectors $u_i$ and $v_i$ vanish if the scalar components $a_3$ and $a_4$ of $La_i$ vanish.

6. T-2 like Finsler Space

A non-Riemannian Finsler space $F^n (n > 2)$ is called T-2 like Finsler space if the T-tensor $T_{ijk}$ is written in the form

$$(6.1) \quad T_{ijk} = \rho C_i C_j C_k.$$ 

Equation (6.1) is a particular case of (C) when $a_i = 0$. Thus we have:

Theorem 6.1 - In a T-2 like four-dimensional Finsler space, the v-connection vectors $u_i$ and $v_i$ vanish.

Theorem 6.2 - In a T-2 like four-dimensional Finsler space, $\rho$ is given by

$$\rho = \mathcal{C}_2.$$ 

References

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