

Generalized Auto-Correlation and its Application in Deterministic Chaos of Continuous Non-Stationary Time Series

Sanjay Kumar Palit

Mathematics Department, Calcutta Institute of Engineering and Management
24/1A Chandi Ghosh Road, Kolkata-700040
E-mail: sanjaypalit@yahoo.co.in

Sayan Mukherjee

Mathematics Department, ShivanathShastri College
23/49 Gariahat Road, Kolkata-700029

D. K. Bhattacharya

Rabindra Bharati University, Kolkata-700050

(Received December 12, 2010)

Abstract: The paper first modifies the notion of auto-correlation between two stages of a signal, so that it is applicable to any time series, stationary/ non-stationary. Next it generalizes the notion further to compare three stages of the time series simultaneously. The comparison is based on two even / uneven time-delays with two/ three dimensional correlogram diagrams. As a result three stages of the time series may be simultaneously tested to be independent. As an application, continuous non-stationary signals like ECG and music signals are investigated for the existence of deterministic chaos in two and three dimensions.

Keywords: Chaos, Attractor reconstruction, Auto-correlation, Non-stationary signals.

1. Introduction

More than four decades ago, the concept of classical time-continuous chaos was introduced¹. Since its inception, the interest in this field of research has risen rapidly and several attempts have been made to integrate this fascinating topic²⁻¹⁵. However, insights into the experimentally observed irregular behavior of systems are predominantly obtained solely with computer simulations of appropriate mathematical models. For example, we may mention the well-known mathematical model like Lorentz's system¹⁶ and its unique three dimensional attractor. This approach undermines the trustworthiness of the whole chaos paradigm since it leads to suspect that chaos is nothing more than a mathematical artifact, a phenomenon non-

existing outside the simulations of the computer. As a remedy to this problem, methods of nonlinear time series analysis were introduced to reconstruct the attractor from the time series itself. In nonlinear time series analysis, methods developed in nonlinear dynamics during the late 1970s and 1980s, are applied to time series data in order to capture as much of the underlying structure as possible. This reconstruction¹⁷⁻²⁹ of the dynamics of the system on the basis of the given data is of paramount importance, as it ensures that under certain generic conditions, such a reconstruction¹⁷⁻²⁹ is equivalent to the original phase space. This equivalence ensures that differential information is preserved and it also allows us to consider both qualitative and quantitative analysis.

The most important problem of phase space reconstruction¹⁷⁻²⁹ is the determination of the time-delay (τ) and the embedding dimension (m). Although there exist several methods to determine the proper time-delay for attractor reconstruction¹⁷⁻²⁹ still in real life problems, the only measure, which is used is either the linear auto-correlation method³⁰⁻³² or a general measure called Average Mutual Information method³³ that serves the purpose of a nonlinear measure. The linear auto-correlation³⁰⁻³² is applicable only for stationary time series. But such time series are rare in nature. Naturally, mutual information theory is most welcome in such cases. This is all about determination of the time-delay. To find out proper embedding dimension is a different problem. As we are interested in geometrical forms of the attractors, so we restrict to attractor reconstruction¹⁷⁻²⁹ in three dimensional spaces only and omit further discussion on embedding dimension. Lastly we note that a positive Lyapunov exponent³⁴ is a strong indicator of chaos for nonlinear time series, whose underlying dynamics is nonlinear. Hence the primary step for reconstruction of chaotic attractor of a signal (time series) is to check whether the underlying dynamics is nonlinear and Lyapunov exponent³⁴ is positive or not. Thus the theory of nonlinear time series analysis offers tools, which bridge the gap between experimentally observed irregular behavior and deterministic chaos theory³⁵⁻³⁸.

Again most of the time series in reality are non-stationary³⁹. We also notice that in such cases the earlier method was to first make the time series stationary³⁹ and then to apply the known methods of analysis of stationary³⁹ time series. Our purpose is to show that we can straight way deal with continuous non-stationary³⁹ signals.

Let $\{x(t)\}_{t=1}^N$ be such a non-stationary³⁹ time series. It is known⁴⁰⁻⁴¹ that the time-delay τ for which $x(t)$ and $x(t+\tau)$ are linearly independent, is the first value of τ corresponding to which the auto-correlation³⁰⁻³² coefficient

is positive minimum in the correlogram diagram¹⁸. Obviously in this case $x(t), x(t+\tau), x(t+2\tau), \dots$ are also linearly independent. We note the following limitations:

- (i) The very definition of auto-correlation³⁰⁻³² demands that the time series must be stationary³⁹, as the means and the variances are taken to be the same in all stages of the signal. Naturally the investigations are not applicable to non-stationary³⁹ signals;
- (ii) It is not possible to find out the independent co-ordinates with uneven delay of the form $(x(t), x(t+\tau_1), x(t+\tau_1+\tau_2), \dots)$, although there is no specific rule in choosing same τ in determining the m linearly independent co-ordinates $(x(t), x(t+\tau), x(t+2\tau), \dots, x(t+(m-1)\tau))$.

With a view to overcoming such shortfalls in the earlier discussions, we consider generalizations of auto-correlation³⁰⁻³² functions. Obviously to compare two and three stages of a non-stationary signal⁴¹ our modified definition of auto-correlation works. Moreover, our generalized definition of auto-correlation works well for any finite dimensional attractor reconstruction¹⁷⁻²⁹. In this paper, we apply the notions of generalized auto-correlation to study the three dimensional deterministic chaos of ECG and music signals, which are continuous and non-stationary³⁹.

2. A New Approach to Auto-Correlation of a Signal

2.1. Generalized auto-correlation between two stages

Let $\{x_1, x_2, x_3, \dots, x_N\}$ be the given time series. The generalized auto-correlation of the given time series with respect to same time-delay τ is defined by

$$(1) \quad R_x(\tau) = \frac{\sum_{t=1}^N (x_t - \overline{x_t}) \cdot (x_{t+\tau} - \overline{x_{t+\tau}})}{\sqrt{\sum_{t=1}^N (x_t - \overline{x_t})^2} \cdot \sqrt{\sum_{t=1}^N (x_{t+\tau} - \overline{x_{t+\tau}})^2}},$$

($\tau=1,2,3,4,\dots,N-1$), where $\overline{x_t}$ and $\overline{x_{t+\tau}}$ are the means of the time series $\{x_1, x_2, x_3, \dots, x_{N-\tau}\}$ and $\{x_{1+\tau}, x_{2+\tau}, x_{3+\tau}, \dots, x_N\}$ respectively.

2.2. Generalized auto-correlation amongst three stages

2.2.1. Different lags

Let $\{x_1, x_2, x_3, \dots, x_N\}$ be the given time series. The generalized auto-correlation of the given time series with respect to different time-delays τ_1, τ_2 is defined by

(2)

$$R_x(\tau_1, \tau_2) = \frac{\sum_{t=1}^{N-(\tau_1+\tau_2)} \varsigma_t \cdot \varsigma_{t+\tau_1} \cdot \varsigma_{t+\tau_1+\tau_2}}{\sqrt{\sum_{t=1}^{N-(\tau_1+\tau_2)} \varsigma_t^2} \cdot \sqrt{\sum_{t=1}^{N-(\tau_1+\tau_2)} \varsigma_{t+\tau_1}^2} \cdot \sqrt{\sum_{t=1}^{N-(\tau_1+\tau_2)} \varsigma_{t+\tau_1+\tau_2}^2}},$$

$$\text{where } \varsigma_t = (x_t - \overline{x_t}), \varsigma_{t+\tau_1} = (x_{t+\tau_1} - \overline{x_{t+\tau_1}}), \varsigma_{t+\tau_1+\tau_2} = (x_{t+\tau_1+\tau_2} - \overline{x_{t+\tau_1+\tau_2}})$$

and $1 \leq \tau_1, \tau_2 \leq \frac{N}{2}$, if N is even and $1 \leq \tau_1, \tau_2 \leq \frac{N-1}{2}$, if N is odd;

where $\overline{x_t}$, $\overline{x_{t+\tau_1}}$ and $\overline{x_{t+\tau_1+\tau_2}}$ are the means of the time series

$\{x_1, x_2, x_3, \dots, x_{N-(\tau_1+\tau_2)}\}$, $\{x_{1+\tau_1}, \dots, x_{N-\tau_2}\}$ and $\{x_{1+\tau_1+\tau_2}, \dots, x_N\}$ respectively.

2.2.2. Same lag repeated

In particular, when $\tau_1 = \tau_2 = \tau$, equation (2) reduces to

$$(3) \quad R_x(\tau) = \frac{\sum_{t=1}^{N-2\tau} \varsigma_t \cdot \varsigma_{t+\tau} \cdot \varsigma_{t+2\tau}}{\sqrt{\sum_{t=1}^{N-2\tau} \varsigma_t^2} \cdot \sqrt{\sum_{t=1}^{N-2\tau} \varsigma_{t+\tau}^2} \cdot \sqrt{\sum_{t=1}^{N-2\tau} \varsigma_{t+2\tau}^2}},$$

$$\text{where } \varsigma_t = (x_t - \overline{x_t}), \varsigma_{t+\tau} = (x_{t+\tau} - \overline{x_{t+\tau}}),$$

$$\varsigma_{t+2\tau} = (x_{t+2\tau} - \overline{x_{t+2\tau}})$$

$$\text{and } 1 \leq \tau \leq \frac{N}{2}, \text{ if } N \text{ is even and } 1 \leq \tau \leq \frac{N-1}{2}, \text{ if } N \text{ is odd;}$$

3. Detection of Independent Coordinates of a Time Series

3.1. Two dimensional correlogram diagram for two and three stages of a signal and its independent coordinates

Since the signal (time series) is non-stationary³⁹ we use equation (1) and equation (3) respectively to determine the generalized auto-correlations $R_x(\tau)$ between two groups $\{x_t\}_{t=1}^{N-\tau}, \{x_{t+\tau}\}_{t=1}^{N-\tau}$ and three groups $\{x_t\}_{t=1}^{N-2\tau}, \{x_{t+\tau}\}_{t=1}^{N-2\tau}, \{x_{t+2\tau}\}_{t=1}^{N-2\tau}$ of the time series $\{x_t\}_{t=1}^N$. In both cases the values of $R_x(\tau)$ are plotted against τ . This plot is called the two dimensional correlogram diagram¹⁸. The proper time-delay τ is then determined from the two dimensional correlogram diagrams, in the sense that it corresponds to that value of τ for which $R_x(\tau)$ comes nearer to zero for the first time. Now the independent coordinates are $x_t, x_{t+\tau}$ and $x_t, x_{t+\tau}, x_{t+2\tau}$ respectively.

3.2. Three dimensional correlogram diagram and independent coordinates of a time series

So far we have seen that the above two dimensional correlogram diagram can detect two and three independent coordinates expressed in terms of the same time-delay. But if we are interested in detecting three independent coordinates expressed in different time-delays, then two dimensional correlogram diagrams is no longer useful. So we generalize the notion of two dimensional correlogram diagrams to three dimensional ones. To do this, the time series $\{x_t\}_{t=1}^N$ is sub-divided into three groups $\{x_t\}_{t=1}^{N-(\tau_1+\tau_2)}, \{x_{t+\tau_1}\}_{t=1}^{N-(\tau_1+\tau_2)}, \{x_{t+(\tau_1+\tau_2)}\}_{t=1}^{N-(\tau_1+\tau_2)}$ and the generalized auto-correlation between three stages, $R_x(\tau_1, \tau_2)$ is calculated by using (2) for different values of τ_1 and τ_2 . Finally $R_x(\tau_1, \tau_2)$ is plotted against τ_1 and τ_2 to obtain a three dimensional diagram, known as three dimensional correlogram diagram. The reconstruction¹⁷⁻²⁹ is carried out with the three independent coordinates as $x_t, x_{t+\tau_1}, x_{t+\tau_1+\tau_2}$ by choosing the suitable values of τ_1 and τ_2 for which

$R_x(\tau_1, \tau_2)$ comes nearer to zero for the first time in the three dimensional correlogram diagram.

4. ECG Signal, Its Characteristics and its Methodology of Collection

ECG (Electrocardiography) is a transthoracic interpretation of the electrical activity of the heart over time captured and is externally recorded by skin electrodes. It is a noninvasive recording produced by an electrocardiographic device. The record is called an Electrocardiogram. An ECG signal consists of waves like P, Q, R, S, and T.

4.1. Characteristics of the signal

1) ECG signal is nonlinear in the sense that its underlying dynamics is nonlinear, which is evident from Surrogate data test⁴².

The result of the Surrogate data test⁴² with 0.01 significant level and the statistical parameter AMI ($\tau = 1$) (Average Mutual Information with time-lag

1) of the ECG signal $\{x(t)\}_{k=1}^{7680}$ is given in figure.1.1.

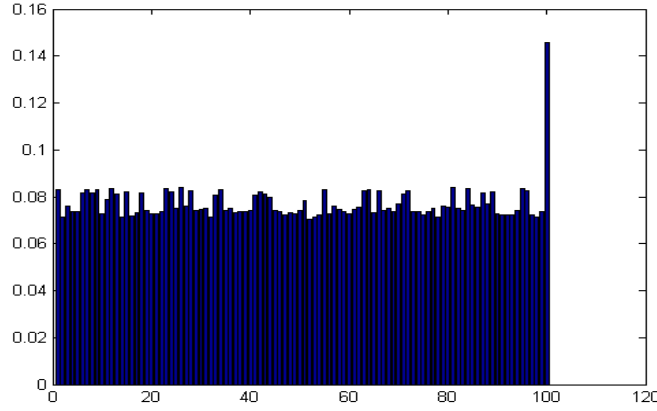


Fig.1. $AMI_{ECG}(\tau = 1) = AMI_{SUR}(\tau = 1)$

In this connection, we take the null hypothesis $H_0 : AMI_{ECG}(\tau = 1) = AMI_{SUR}(\tau = 1)$. If the equality does not hold, we say that null hypothesis fails and alternative H_A holds good. From figure 1, it is seen that the AMI of Surrogate data series is not equal to AMI of the given signal. Hence the null hypothesis $H_0 : AMI_{ECG}(\tau = 1) = AMI_{SUR}(\tau = 1)$ is rejected. Thus, nonlinearity of the signal $\{x(t)\}_{k=1}^{7680}$ is established through Surrogate data test⁴².

2) It is continuous non-stationary³⁹. In fact, a signal is stationary³⁹ or nearly stationary³⁹ if each segment of same length of the original signal has same distribution. This means that the signal has same statistics up to some desired accuracy. To test the non-stationarity of the signal, a statistical test viz; Q-Q plot⁴² is used. This plot examines the non-stationarity of the signal by checking whether the distribution of two segments of the signal is same or not. The Q-Q plot⁴³ of (X_1, X_2) for the ECG signal $\{X_k\}_{k=1}^{7680}$ is given by figure 2, where $X_1 = \{X_k\}_{k=1}^{3840}$, $X_2 = \{X_k\}_{k=3841}^{7680}$.

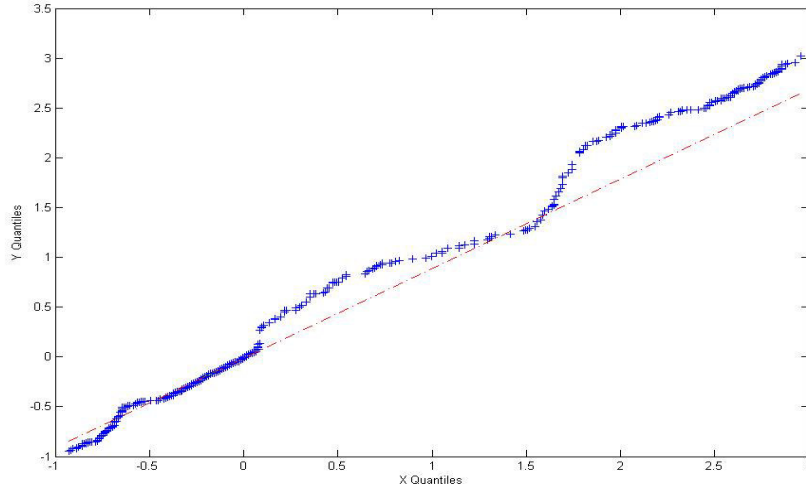


Fig.2 Q-Q plot for the random variable X_1 and X_2

From figure 2, it is observed that most of the points are far from the dotted straight line, i.e., the distribution of X_1 and X_2 are not same. Hence the given ECG signal is a non-stationary signal³⁹.

4.2. Methodology of data collection

Electrocardiogram signal of the controlled and experimental subjects are downloaded from MIT-BIH Normal Sinus Rhythm Database available in Physionet website⁴⁴.

5. Two and Three Dimensional Attractor Reconstruction for ECG Signal

Since ECG signal is nonlinear, we first prove that strange attractor for ECG signal exists by showing that Lyapunov exponent³⁴ $\lambda = 9.0254e - 005 > 0$.

Next we try for attractor reconstruction¹⁷⁻²⁹. To check whether our generalized formula of auto-correlation gives better results compared to those obtained by standard methods, we consider attractor reconstruction¹⁷⁻²⁹ with standard and modified methods separately.

5.1. Attractor reconstruction of ECG signal by Average Mutual Information method

Let us first try for three dimensional attractor reconstruction¹⁷⁻²⁹ of the ECG signal with time-delay τ , where the time-delay is obtained by using the method of Average Mutual Information (AMI)³³. The graph of AMI against the time-delay τ is shown by figure 3.

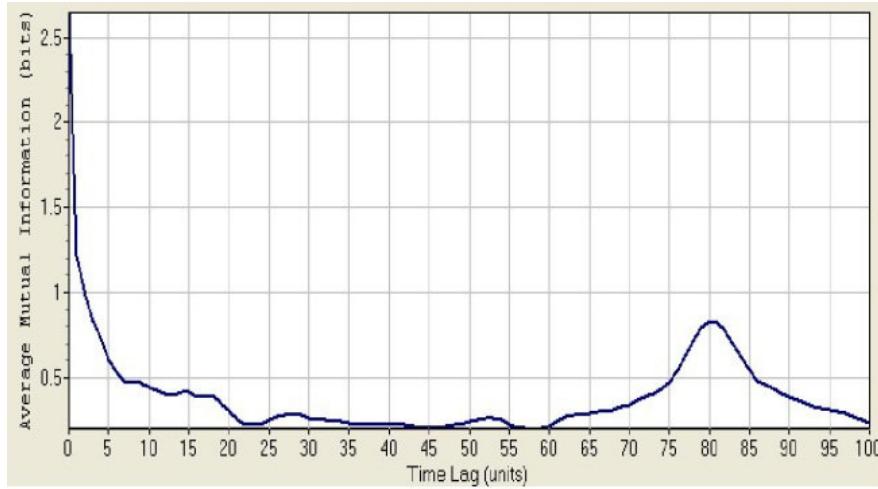


Fig.3 The graph of AMI versus time-delay τ

It is found that AMI comes nearer to zero for the first time, when $\tau = 7$. This is the suitable value of τ for attractor reconstruction of the above ECG signal. Figure.4 shows the two and three dimensional reconstructed attractors for $\tau = 7$.

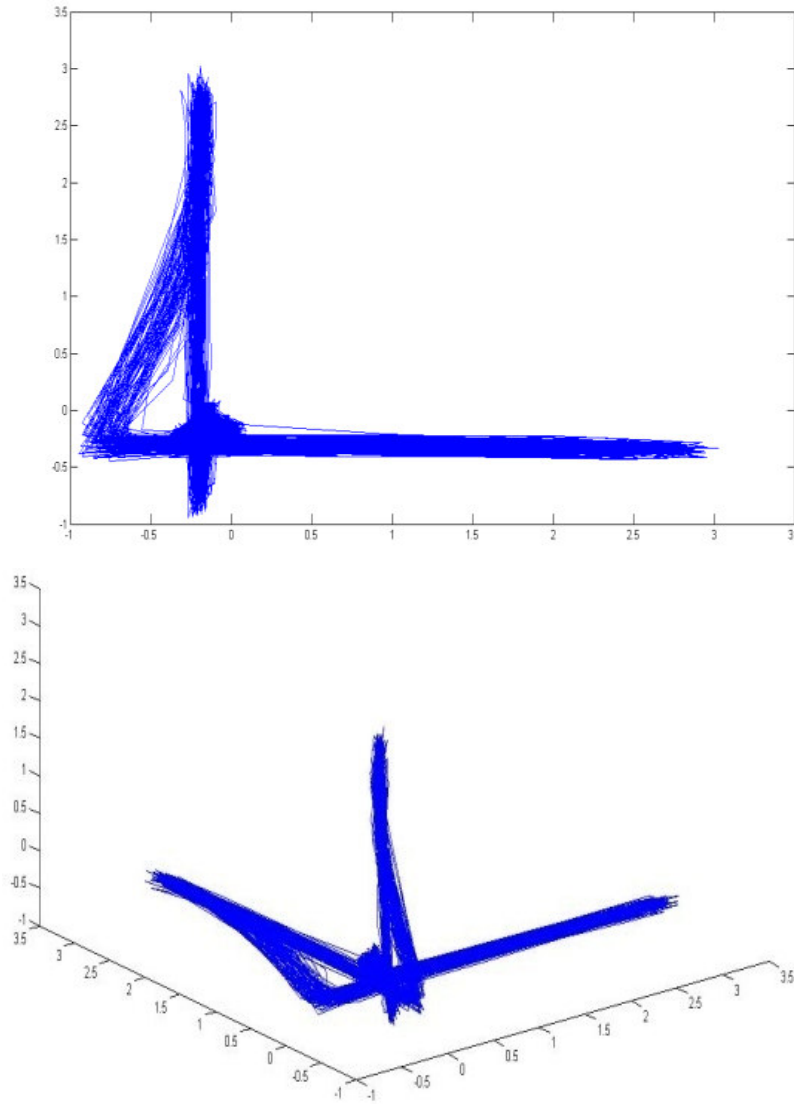


Fig.4. Two and three dimensional reconstructed attractors of ECG signal with $\tau = 7$

5.2. Attractor reconstruction of ECG signal by standard and generalized auto-correlation function between two stages

By using the standard auto-correlation function³⁰⁻³² we first compute the auto-correlation of the ECG signal for different values of the time-delay τ . It is then plotted against τ to obtain the following [figure.5] two dimensional correlogram. It is observed from the graph that auto-correlation function (ACF) attains its positive minimum for the first time, when time-delay $\tau = 3$.

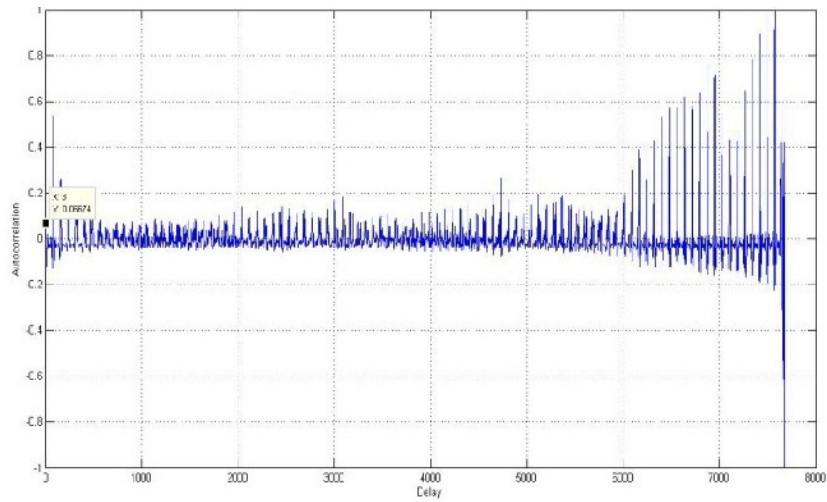


Fig.5 Correlogram of ECG signal under standard auto-correlation

Next we obtain correlogram diagram¹⁸ [figure.6] under generalized auto-correlation function between two stages given by (1) and again we see that time-delay $\tau = 3$.

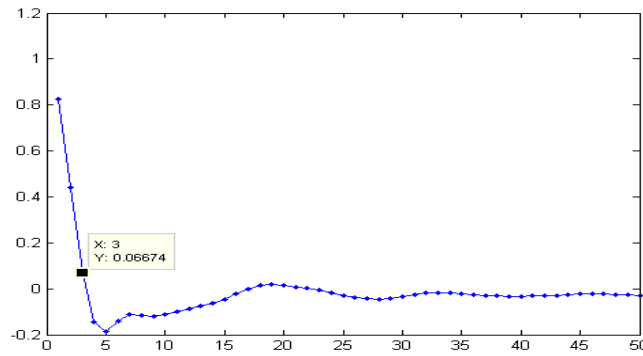


Fig.6. Correlogram of ECG signal under generalized auto-correlation between two stages

As in both cases, we get the same time-delay, so we obtain the same two dimensional attractor and the same three dimensional attractor (with same time- delay repeated) given by figure.7 (i) and 7 (ii) respectively.

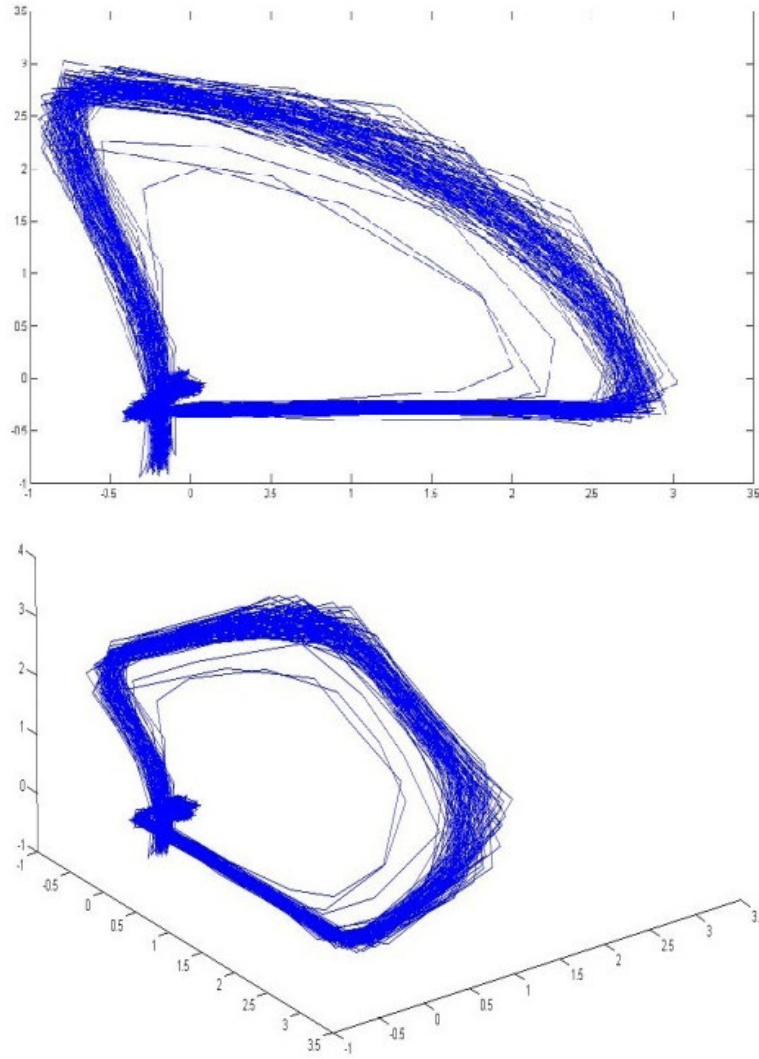


Fig.7. Reconstructed (i) two dimensional and (ii) three dimensional attractors of the ECG signal for time-delay $\tau = 3$

5.3. Attractor reconstruction of the ECG signal under generalized notion of auto-correlation amongst three stages (same time-delay repeated)

The suitable value of τ is obtained from the two dimensional correlogram¹⁸, where $R_x(\tau)$'s for different values of τ are calculated by using generalized auto-correlation given by (3). This is shown in figure.8.

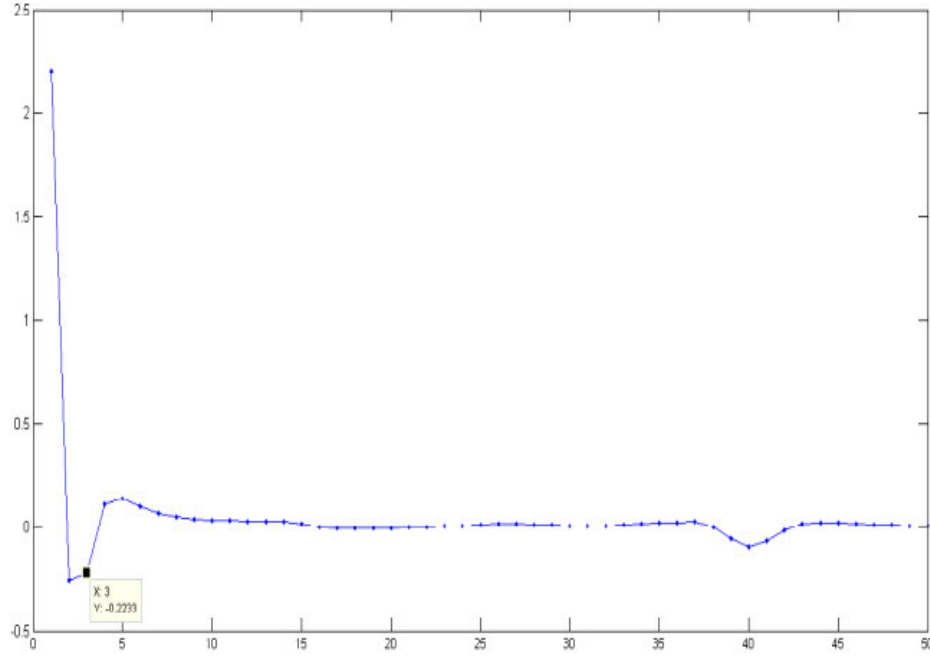


Fig.8. Two dimensional Correlogram for the ECG signal under generalized auto-correlation amongst three stages given by (3)

It is evident from the above figure that the generalized auto-correlation given by (3) comes nearer to zero for the first time, when $\tau = 3$. We next reconstruct the attractor¹⁷⁻²⁹ of the above ECG signal in three dimensional spaces with a time-delay $\tau = 3$ for which $(x_t, x_{t+\tau}, x_{t+2\tau})$, $t = 1, 2, 3, \dots, N - 2\tau$ are independent. Naturally, the attractor reconstructed [17-29], with this value of τ is same as that shown by figure.7 (ii).

5.4. Attractor reconstruction of the ECG signal under generalized notion of auto-correlation amongst three stages (under different time-delays)

Finally, we reconstruct the attractor of the aforesaid ECG signal with two different time-delays τ_1 and τ_2 . For this purpose, the generalized auto-correlation between three stages given by (2) is used. The values of $R_x(\tau_1, \tau_2)$, thus obtained for different values of τ_1 and τ_2 are then plotted against τ_1 and τ_2 to obtain the three dimensional correlogram, shown by figure9.

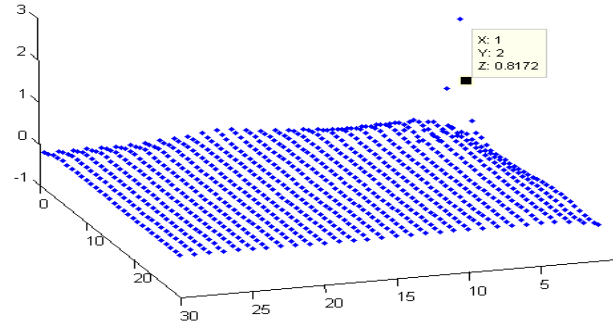


Fig.9. Three dimensional Correlogram for the ECG signal under generalized auto-correlation with different time-delays given by (2)

Figure.9 clearly shows that the value of the generalized auto-correlation $R_x(\tau_1, \tau_2)$ comes nearer to zero for the first time when $\tau_1 = 1$ and $\tau_2 = 2$. The three dimensional attractor, reconstructed¹⁷⁻²⁹ with the independent coordinates $(x_t, x_{t+\tau_1}, x_{t+\tau_1+\tau_2})$, $t=1,2,3,\dots,N-(\tau_1+\tau_2)$ for these values of τ_1 and τ_2 is given by figure.10.

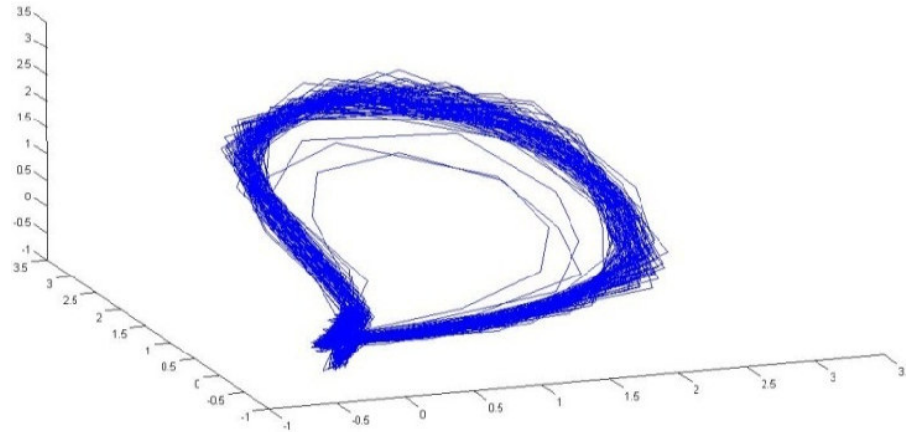


Fig.10 Three dimensional reconstructed attractor of the ECG signal with $\tau_1 = 1, \tau_2 = 2$

6. A Music Signal, its Characteristics and its Methodology of Collection

6.1. Characteristics of the signal

1) The present music signal of raga 'Bhairavi' is nonlinear in the sense that its underlying dynamics is nonlinear. This is evident from Surrogate data test. The result of the Surrogate data test⁴² with 0.01 significant level and the

statistical parameter AMI ($\tau=1$) (Average Mutual Information with time-lag 1) of this music signal $\{x(t)\}_{t=1}^{5000}$ is given in figure 11.

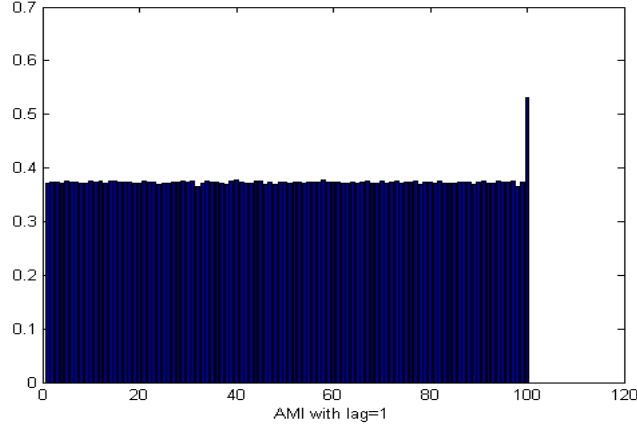


Fig.11 $AMI_{\text{Bhairavi}}(\tau=1) = AMI_{\text{SUR}}(\tau=1)$

From figure.11, it is seen that the AMI of Surrogate data series is not equal to AMI of the given signal and hence the null hypothesis $H_0 : AMI_{\text{Bhairavi}}(\tau=1) = AMI_{\text{SUR}}(\tau=1)$ is rejected. Thus, nonlinearity of the signal $\{x(t)\}_{t=1}^{5000}$ is established through Surrogate data test ⁴².

2) It is a continuous non-stationary signal ³⁹. This is shown by the Q-Q plot ⁴³ of (X_1, X_2) for the aforesaid music signal $\{X_k\}_{k=1}^{5000}$, where $X_1 = \{X_k\}_{k=1}^{1000}$, $X_2 = \{X_k\}_{k=1001}^{2000}$ be two shorter segment of the given music signal [figure.12].

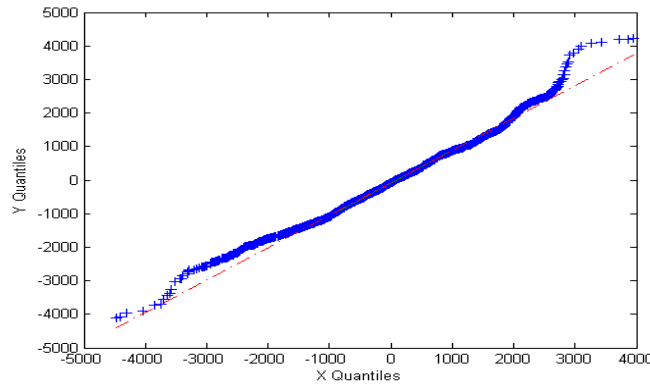


Fig.12. Q-Q plot of (X_1, X_2) for the music signal of raga 'Bhairavi'

It is evident from figure.12 that many points are far from the dotted line i.e., the distribution of X_1 and X_2 are not same. Thus the given music signal of raga ‘Bhairavi’ is a non-stationary signal³⁹.

6.2. Method of collecting data

Source- Recorded north Indian Classical Music (Sarod)

Artist- Ustad. Amjad Ali Khan

Raga- Bhairavi

Software- Adobe Audition 1.5

Samples: 5000

Bits per Samples: 16

Sample rate: 44100

Normalized: False.

7. Two and Three Dimensional Attractor Reconstruction for the Music Signal of Raga ‘Bhairavi’

Since the music signal of raga ‘Bhairavi’ is nonlinear, we first show that strange attractor for the aforesaid music signal exists by showing that Lyapunov exponent³⁴ $\lambda = 7.1335 \times 10^{-5} > 0$. So, we try for attractor reconstruction¹⁷⁻²⁹. Again, for the establishment of the supremacy of our generalized formula of auto-correlation over the standard formula for two stage auto-correlation³⁰⁻³² and Average Mutual Information³³, we consider attractor reconstruction¹⁷⁻²⁹ both with the standard and modified methods separately.

7.1. Attractor reconstruction of music signal based on raga ‘Bhairavi’ by Average Mutual Information method

We first try for three dimensional attractor reconstruction¹⁷⁻²⁹ of the aforesaid music signal of raga ‘Bhairavi’ with time-delay τ , where the time-delay τ is obtained by using the method of Average Mutual Information (AMI)³³. The graph of AMI against the time-delay τ is shown by figure.13.

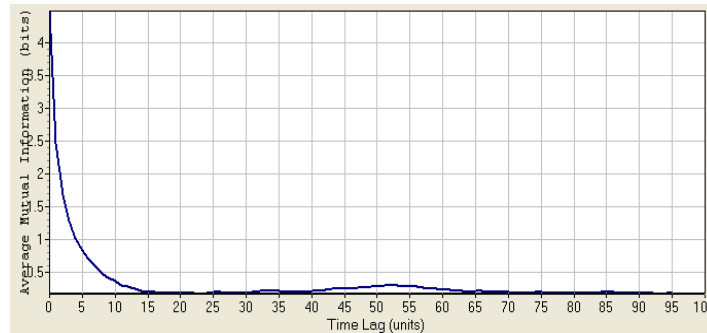


Fig.13. The graph of AMI versus time-delay τ

It is found that $\tau=18$ is the suitable value of τ for attractor reconstruction¹⁷⁻²⁹ of the present music signal. Figure.14 shows the two and three dimensional reconstructed attractors for $\tau=18$.

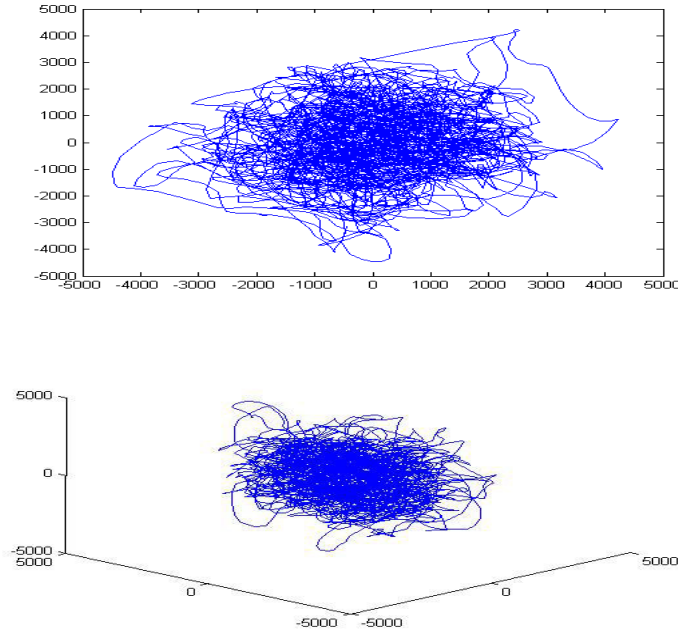


Fig.14. Two and three dimensional reconstructed attractors of the music signal of raga 'Bhairavi' with $\tau = 18$

7.2. Attractor reconstruction of the music signal based on raga Bhairavi' by standard and generalized auto-correlation function between two stages

The graph of standard auto-correlation versus time-delay is given by figure.15.

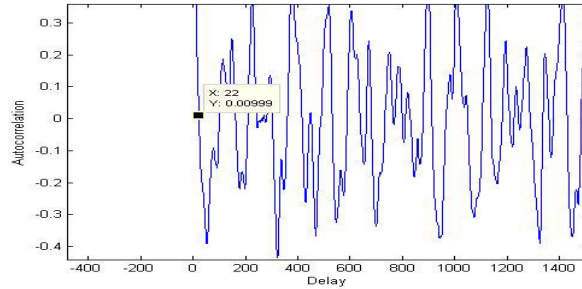


Fig.15 Two dimensional Correlogram of the music signal of raga 'Bhairavi' under standard auto-correlation

The notion of generalized auto-correlation between two stages given by (1) is now used to find the proper time-delay for attractor reconstruction¹⁷⁻²⁹. The two dimensional correlogram¹⁸ thus obtained is given by figure 16.

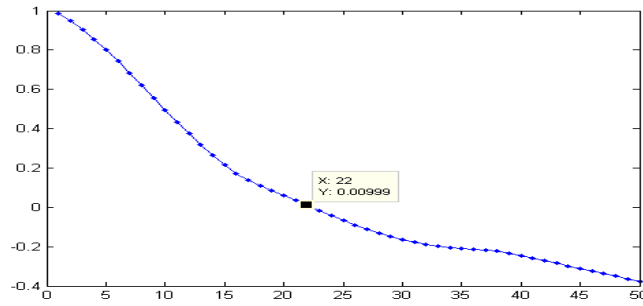
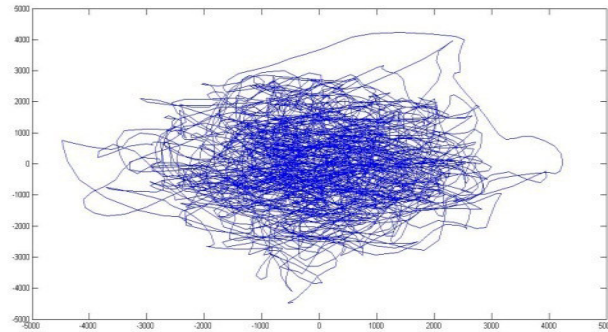
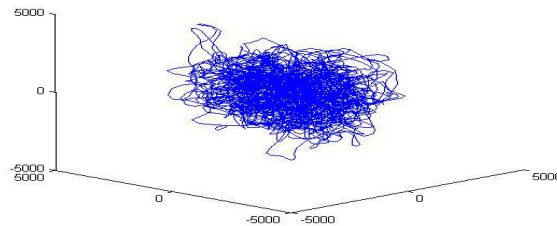


Fig.16. Two dimensional Correlogram for the music signal of raga 'Bhairavi' under generalized auto-correlation between two stages

It is found from both the figures [figure.15 and figure.16] that the suitable value of the time-delay τ is 22. So the attractors of the music signal of raga 'Bhairavi' under both standard auto-correlation and generalized auto-correlation between two stages are same [figure.17].



(i)



(ii)

Fig.17. Reconstructed (i) two dimensional and (ii) three dimensional attractor of the music signal of raga 'Bhairavi' for time-delay $\tau = 22$

7.3. Reconstruction of three dimensional attractor under generalized auto-correlation between three stages (same time-delay repeated)

The two dimensional correlogram diagram of the generalized auto-correlation R_x given by (3) against time-delay is given by figure 18.

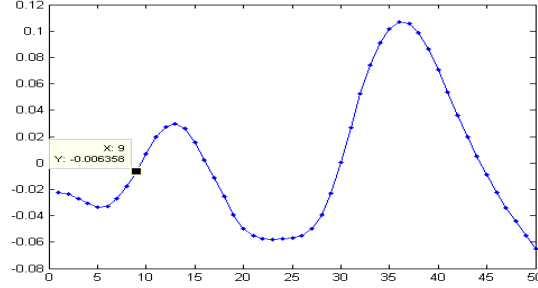


Fig.18. Two dimensional Correlogram for the above music signal of raga 'Bhairavi' under generalized auto-correlation given by (3)

From the above figure the suitable time-delay τ is found to be 9. The reconstructed attractor of the above music signal of raga 'Bhairavi' with this value of τ is given by figure.19.

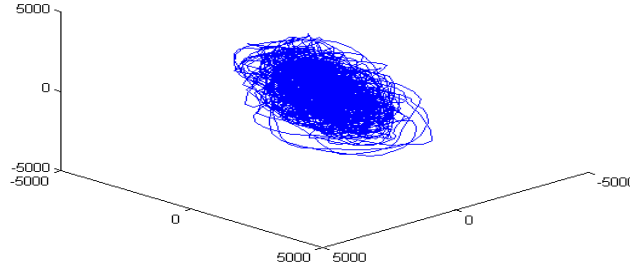


Fig.19. Three dimensional reconstructed attractor of the music signal (raga 'Bhairavi') with $\tau=9$ repeated

7.4. Reconstruction of three dimensional attractor under generalized auto-correlation between three stages (under different time-delays)

The generalized auto-correlation $R_x(\tau_1, \tau_2)$ with different time-delays τ_1 and τ_2 obtained by (2) for different values of τ_1 and τ_2 are plotted against τ_1 and τ_2 to obtain the three dimensional correlogram shown by figure.20.

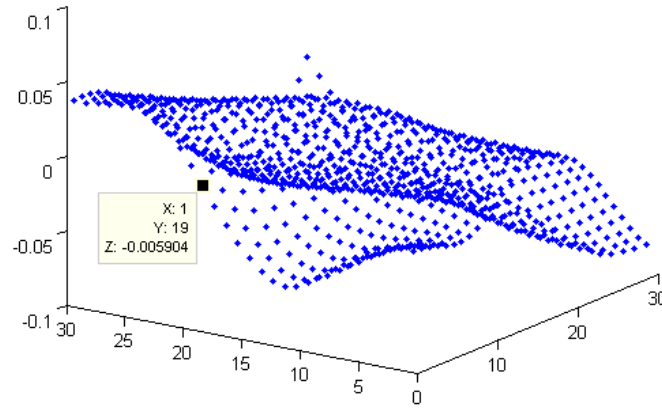


Fig.20 Three dimensional Correlogram for the above music signal of raga 'Bhairavi' under generalized auto-correlation with different time-delays given by (2)

From the above figure, the suitable values of the time-delays are found to be $\tau_1 = 1$ and $\tau_2 = 19$. The three dimensional reconstructed attractor is shown by figure.21.

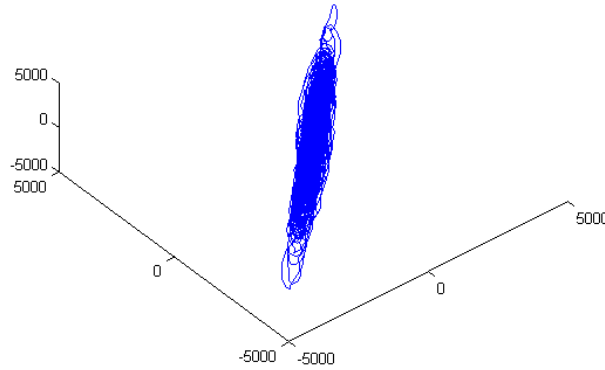


Fig.21. Three dimensional reconstructed attractor of the music signal raga 'Bhairavi' with $\tau_1 = 1$ and $\tau_2 = 19$.

8. Conclusions

(i) The value of time-delay for ECG signal, as determined under standard notion of auto-correlation³⁰⁻³² and generalized notion of auto-correlation between two stages is always found to be same, as the small time-delay can keep the statistical parameters almost same. Naturally, we get the same two and three dimensional (with same time-delay repeated) attractors given by figure.7 (i) and (ii) respectively. For similar reasons two and three dimensional attractors (with same time-delay repeated) of music signal based on raga 'Bhairavi' given by figure.17 (i) and (ii) respectively are also the same. Among these attractors, only the three dimensional attractor for the above ECG signal reconstructed under standard auto-correlation³⁰⁻³² and

generalized auto-correlation between two stages can be considered as approximate attractor for it in the proper sense of the term. Also, the two and three dimensional attractors reconstructed under the notion of Average Mutual Information³³ for ECG and the music signal based on raga ‘Bhairavi’ given by figure.4 and figure.14 respectively are not at all attractors in the proper sense of the term. These necessitate the consideration of three dimensional attractors under generalized notion of auto-correlation amongst three stages only (with same time-delay repeated or different time-delays).

(ii) In three stages with same time-delay repeated, the same three dimensional attractor is obtained for the ECG signal [figure.7 (ii)] as the time-delay obtained in this case is same as that obtained by standard auto-correlation³⁰⁻³² and by the notion of generalized auto-correlation between two stages. On the other hand, the quality of the attractor improves for the music signal of raga ‘Bhairavi’ (figure.19) in the sense that it contains dense orbits except for some outliers. So, for further improvement of the reconstructed three dimensional attractor for the above two signals, we consider the last alternative namely “reconstruction of three dimensional attractor under three stage auto-correlation with different time-delays”.

(iii) With different time-delays, the attractors in three dimensions significantly improve for both the signals viz., ECG signal (figure.10) and music signal of raga ‘Bhairavi’ (figure.21).

Thus it may be concluded that for better attractor reconstruction for continuous non-stationary signals (time series)³⁹ in three dimensions, our newly introduced method of generalized auto-correlation is highly suitable especially under the choice of different time-delays.

References

1. E. N. Lorenz, *J. Atmos. Sci.*, **20** (1963) 130–141.
2. N. M. Donald and R. R. Whitehead, *Eur. J. Phys.*, **6**(1985) 143–147.
3. H. N. N. Yepez, A. L. S. Brito, C. A. Vargas and L. A. Vicente, *Eur. J. Phys.*, **10**(1989)99–105.
4. G. J. Rodgers, *Phys. Educ.*, **27**(1992) 14–17.
5. R. C. Gonzalez, H. N. N. Yepez and A. L. S. Brito, *Eur. J. Phys.*, **15**(1994)139–148.
6. M. Barrientos, A. Perez and A. F. Ranada, *Eur. J. Phys.*, **16**(1995)106–112.
7. P. H. Borchers, *Phys. Educ.*, **30**(1995) 372–381.
8. P. R. Hobson and A. N. Lansbury, *Phys. Educ.*, **31**(1996) 39–43.
9. T. Schmidt and M. Marhl, *Eur. J. Phys.*, **18**(1997)377–383.
10. S. J. Martin and P. J. Ford, *Phys. Educ.*, **36** (2001)108–114.

11. M. Gitterman, *Eur. J. Phys.*, **23**(2002)119–122.
12. A. Tamaševičius, Mykolaitis, V. Pyragas and K. Pyragas, *Eur. J. Phys.*, **26** (2005)61–63.
13. M. Perc, *Eur. J. Phys.*, **26** (2005)525–534.
14. M. Perc, *Eur. J. Phys.*, **26** (2005)579.
15. S. Kodba, M. Perc and M. Marhl, *Eur. J. Phys.*, **26**(2005) 205–215.
16. S. H. Strogatz, *Non Linear Dynamics and chaos- with application to Physics, Biology, Chemistry and Engineering, Advance Book Program*, Persueus Books, Cambridge, Massachusetts 1994.
17. I. H. D. Abarbanel, *Analysis of Observed Chaotic Data*, Springer, New York 1996.
18. H. sKantz and T .Schreiber, *Nonlinear Time Series Analysis*, Cambridge University Press 1997.
19. J. C. Sprott, *Chaos and Time-Series Analysis*, Oxford University Press 2003.
20. D.T. Kaplan and L. Glass, *Understanding Nonlinear Dynamics*, Springer, New York 1995
21. H. D. I. Abarbanel, *Analysis of Observed Chaotic Data*, Springer-Verlag, New-York 1997.
22. D. T. Kaplan, *Signal Processing, warwick- exercises.tex.*, (2004) 5-9.
23. D. J. Christini, F. M. Bennett, K. R. Lutchén, H. M. Ahmed, J. M. Hausdorff and N. Oriol, *IEEE Trans. Biomed. Eng.* **42**(1995) 411–415.
24. J. P. Eckmann and D. Ruelle, *Rev. Mod. Phys.*, **57** (1985) 617–656.
25. L. Glass and M. C. Mackey *From clocks to chaos: the rhythms of life*, Princeton University Press, Princeton, NJ 1988
26. F. Takens, In *Dynamical systems and turbulence*, **898** (1981) 366–381.
27. E. Ott, *Chaos in dynamical systems*, Cambridge University Press 1993.
28. D. Ruelle, *Chaotic Evolution and Strange Attractors*, Cambridge University Press 1989.
29. G.W. Frank, T. Lookman and M. A. H. Nerenberg, *Physica*, **D 46** (1990) 427–438.
30. G. P. Williams, *Chaos Theory Tamed*, Joseph Henry Press, Washington, D.C. 1997.
31. E. Nichols and I. Knopke, *31st International Conference*, London, U. K. (2007) 25–27.
32. O. Fojt and J. Holcik, *IEEE Eng. Med. Biol. Mag.*, **17** (1998) 96-101.
33. W. Zhao; F. U. Chong, *Journal of Communication and Computer*, **4(8)** (2007) 33.
34. A. Wolf, J. B. Swift, H. L. Swinney and J. A. Vastano, *Physica*, **D(16)**(1985) 285-317.
35. H. G. Schuster, *Deterministic Chaos*, Weinheim, VCH 1989.
36. E. Ott, *Chaos in Dynamical Systems*, Cambridge University Press 1993.
37. S. H. Strogatz, *Nonlinear Dynamics and Chaos*, Addison-Wesley 1994.
38. D. T. Kaplan and L. Glass, *Understanding Nonlinear Dynamics*, Springer, New York, 1995.
39. J. A. Sills, *Non-stationary signal modeling, filtering, and parameterization*, School of Electrical and Computer Engineering, Georgia Institute of Technology, 1995.

40. V. Anishchenko, A. Neiman, T. Vadivasova, V. Astakhov, and L. Shimansky-Geier, *Dynamics of Chaotic and Stochastic Systems*, 2nd ed., Springer-Verlag, 2007.
41. G. L. Baker and J. P. Gollub, *Chaotic Dynamics: An Introduction*, Cambridge University Press 1998.
42. D. L. Guarín López, A. A.O. Gutierrez, E. D. Trejos, A new surrogate data method for non-stationary time series, *Nonlinear Analysis: Real World Applications*, (2010).
43. W. L. Martinez and A. R. Martinez, *Computational Statistics Handbook with Matlab*, Chapman & Hall/CRC 2002.
44. <http://www.physionet.org>