

Rotational Impact on Unsteady MHD Double Diffusive Boundary Layer Flow over an Impulsively Emerged Vertical Porous Plate

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Abstract: In this article, an analysis has been performed to study the rotational impact on unsteady MHD double diffusive flow over an impulsively emerged vertical porous plate in fluctuating temperature and mass diffusion in the presence of uniform transversely applied magnetic field. The governing equations involved in the present analysis are solved by using Laplace-transform technique. The expressions for the distributions of velocity, temperature and concentration are computed and with the aid of these the expressions for skin friction, Nusselt number and Sherwood number in non-dimensional form are derived. The effects of various physical parameters on the above flow quantities are considered through graphs and tables.

Keywords: Double diffusive fluid; Rotation; MHD; Porous medium; Radiation and variable temperature and mass diffusion.

MSC Code: 76S05, 76Sxx, 35Q35, 65L10.

1. Introduction

The problem of magnetic hydro dynamic laminar flow in the course of a porous medium has become very significant in latest years because of its possible applications in several branches (Israel-Cookey *et al.*¹, Kim², Raju *et al.*³, Rajput and Kumar⁴). In many processes, in engineering areas, take place at higher temperature and therefore the knowledge of radiation heat transfer becomes a very essential for the design of the related equipment. Also, convective flows with radiation are encountered in many environmental and engineering processes, for example, evaporation from large open reservoirs, heating and cooling chambers, astrophysical flows, solar control technology and space vehicle re-entry. Sharma *et al.*⁵ have found the influence of homogeneous chemical reaction and radiation on unsteady magneto hydrodynamic free convection flow of a viscous incompressible liquid past a heated vertical plate immersed in porous medium in the presence of heat source. Reddy *et al.*⁶ have investigated unsteady MHD radiative and chemically reactive free convection flow near a moving vertical plate in porous medium. Several authors such as Mohamed *et al.*⁷, Singh and Kumar⁸, Muthucumaraswamy and Visalakshi⁹, Saha *et al.*¹⁰, Raju *et al.*¹¹, Hossain and Takhar¹², Muthucumaraswamy and Janakiraman¹³, Hossain *et al.*¹⁴, Prasad *et al.*¹⁵, Manivannan *et al.*¹⁶, Chamkha *et al.*¹⁷ and Pal *et al.*¹⁸ have addressed various issues related to thermal radiation. Double-diffusive convection is an essential process in oceanography and plays a role in mantle convection (magma chambers) and some technological applications. Bakr and Riazahb¹⁹, Mohamed²⁰, Awad *et al.*²¹, Hayat *et al.*²² have concentrated few double diffusive problems.

Rotating fluid flow in hydraulic turbo machines, rotational phase separators and ultra-centrifuges pose a number of challenging problems. Notable contributions^{23–32} related to rotating fluid are reported here. In all the above studies, rotational effect on a double diffusive fluid was not considered. Hence, the above studies motivated us, to investigate rotational effects on unsteady MHD double diffusive flow past an impulsively started vertical porous plate through variable temperature and mass diffusion in the presence of thermal radiation.

1. Formulation of the Problem

We have considered an unsteady flow of an electrically conducting fluid induced by viscous incompressible nature past an impulsively started

vertical porous plate with mass transfer and variable temperature. The fluid and the plate rotate as a rigid body with identical angular velocity Ω^* about y^* -axis in the presence of an imposed uniform magnetic field B_0 normal to the plate. Initially, the concentration and temperature of the fluid near the plate are assumed to be T_∞^* and C_∞^* respectively. At $t^* > 0$ the plate starts moving with a velocity $u^* = u_0$ in its own plane and the temperature and concentration levels near the plate are raised linearly with respect to time. Since the plate occupying the plane $y^* = 0$ is of infinite extent, all the material quantities depend only on y^* and t^* . It is assumed that the induced magnetic field is negligible so that $\vec{B}_0 = (0, 0, B_0)$. Followed by the above assumptions and from the results of Rajput²³, the governing equations in dimensionless form are given below

$$(2.1) \quad \frac{\partial u}{\partial t} - 2\Omega v = Gr\theta + GmC + \frac{\partial^2 u}{\partial y^2} - M_1 u,$$

$$(2.2) \quad \frac{\partial v}{\partial t} + 2\Omega u = \frac{\partial^2 v}{\partial y^2} - M_1 v,$$

$$(2.3) \quad \frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} - N\theta,$$

$$(2.4) \quad \frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2} + S_0 \frac{\partial^2 \theta}{\partial y^2}$$

The non-dimensional boundary conditions are also given by

$$(2.5) \quad \begin{cases} t \leq 0 : u = 0, C = 0 \text{ for all the values of } z \\ t > 0 : u = 1, \theta = t, C = t \text{ at } y = 0 \\ \& u \rightarrow 0, \theta \rightarrow 0, C \rightarrow 0 \text{ as } y \rightarrow \infty \end{cases}$$

2. Solution of the Problem

By assuming $u+iv$ and taking Laplace transform of the Equations (2.1)-(2.5), the following equations are obtained

$$(3.1) \quad \frac{d^2 \bar{q}}{dy^2} - (m + s) \bar{q} = -Gr \bar{\theta} - Gm \bar{C},$$

$$(3.2) \quad \frac{d^2 \bar{\theta}}{dy^2} - (N + s) \bar{\theta} = 0,$$

$$(3.3) \quad \frac{d^2 \bar{C}}{dy^2} - sSc \bar{C} = -S_0 Sc \frac{d^2 \bar{\theta}}{dy^2},$$

with the following boundary conditions

$$(3.4) \quad \begin{cases} \bar{u} = \frac{1}{s}, \bar{\theta} = \frac{1}{s^2}, \bar{C} = \frac{1}{s^2} \text{ at } y = 0 \\ \& \bar{u} \rightarrow 0, \bar{\theta} \rightarrow 0, \bar{C} \rightarrow 0 \text{ as } y \rightarrow \infty. \end{cases}$$

Taking the inverse Laplace transform as explained by Hetnarski²⁹, Equations (3.1)-(3.3) along with the boundary conditions (3.4), the representative velocity, temperature and concentration fields are obtained.

Skin- Friction: The Skin friction components τ_x and τ_y are obtained as

$$\tau_x + i\tau_y = - \left(\frac{\partial q}{\partial y} \right)_{y=0}.$$

Rate of Heat Transfer: The rate of heat transfer at the plate in terms of

$$\text{Nusselt number, } Nu = - \left(\frac{\partial \theta}{\partial y} \right)_{y=0}.$$

Rate of mass transfer: The mass transfer coefficient at the plate in terms of Sherwood number is

$$Sh = - \left(\frac{\partial C}{\partial y} \right)_{y=0}.$$

4. Results and Discussions

In order to find the physical insight of the problem, numerical computations have been conceded for the distributions of temperature, concentration, velocity, Skin friction, Nusselt number, and Sherwood number and the effects of various physical parameters on flow quantities are considered through graphs. The variations in temperature distribution under the influence of Prandtl number and radiation parameter N are shown in figures 1 and 2. From these figures, it is observed that temperature decreases with an increase in Prandtl number and radiation parameter N . Prandtl number is the ratio of kinematic viscosity and thermal diffusivity. If other parameters remain constant, an increase in kinematic viscosity, momentum diffusivity leads to enhance the velocity boundary layer thickness. This is due to the smaller values of Prandtl number which increases the thermal conductivities, and therefore, heat is able to be different away from the heated surface more rapidly than for higher values of Prandtl number. Thus higher Prandtl number fluid causes lower thermal diffusivity and hence reduces the temperature at all points. This observation coincides among the result of Mishra *et al.*³⁰. It is evident that as radiation parameter N increases the thermal boundary layer comes to decrease. The effect of Soret number S_0 on concentration distribution is displayed in figure 3. From this figure, it is evident that concentration distribution asymptotically decreases in the presence of heavier species. Soret number describes the effect of temperature gradient inducing imperative mass diffusion effect and therefore concentration decreases with the increasing values of Soret number S_0 .

Primary and secondary velocity profiles are displayed from figures 4 -8 respectively. Fig.4 reveal that velocity decreases with an increase in magnetic parameter (M). This is due to application of magnetic field which retards the flow that is known as Lorentz force. Thus, the Lorentz force decelerates the fluid flow accordingly resulted in thinning of momentum boundary layer. Our result is in conformity with the result of Das *et al.*³¹ and Miashra *et al.*³⁰, whereas an exact opposite reaction is noticed in the case of secondary velocity in the presence of magnetic parameter. Fig 5 depicts the primary velocity profiles with the variations in S_0 . From this figure, it is observed that velocity decreases with an increase in Soret number. Opposite reaction is noticed in the case of secondary velocity. Figs 6 and 8 witness the variations in the velocity when Ω changes. Both the profiles of velocity i.e. Primary velocity and secondary velocity are noticed to decrease when the rotation parameter increases.

Table 1 depicts the effects of various physical parameters on Skin friction. From this table, it is noticed that τ_x and τ_y increase with an increase in Grashof number Gr , modified Grashof number Gm , dimensionless rotation parameter Ω , Schmidt number Sc , and time t . An increase in magnetic parameter M shows a decrease in τ_x and τ_y , whereas radiation parameter N increases and τ_x increases but τ_y decreases. Whereas Soret number S_0 increases then τ_x decreases but τ_y increases. Therefore, our observation confirms that an increase in modified Grashof number Gm , Grashof number Gr , dimensionless rotational parameter Ω , Schmidt number Sc and time t lead to exert greater Skin friction on the boundary, whereas magnetic parameter M reduces it. Thus, the flow under consideration is stable for heavier species with larger magnetic field strength. This coincides with the result of Khan and Sanjayanand³³. From table 2, it is seen that an increase in Prandtl number Pr leads to an increase in Nusselt number. Whereas radiation parameter shows reverse effect on Nusselt number. It is clear that from table 3, as Schmidt number Sc increases the magnitude of Sherwood number increases whereas, an increase in Soret number S_0 decreases the magnitude of Sherwood number.

5. Conclusions

In this paper a theoretical analysis has been carried out to study the rotational effects on unsteady MHD double diffusive flow past an impulsively started vertical porous plate. The following are the significant conclusions:

Temperature distribution is observed to increase with decrease in Prandtl number Pr and radiation parameter N . Concentration increases with decrease in Soret number S_0 . Primary velocity decrease with the increase in magnetic parameter, Soret number S_0 , and dimensionless rotational parameter Ω . Secondary velocity increases with the increase in magnetic parameter M , Soret number S_0 , and decreases with increase in dimensionless rotational parameter Ω . Skin friction τ_x and τ_y increases with an increase in Grashof number Gr , modified Grashof number Gm , dimensionless rotational parameter Ω , Schmidt number Sc , time t , but magnetic parameter M increases, τ_x and τ_y decrease, whereas radiation parameter N increases, τ_x increases and τ_y decreases, and Soret number S_0 increases, then τ_x decreases and τ_y increases. As Sc increases the magnitude of Sherwood number increases whereas S_0 increases as the magnitude of Sherwood number decreases.

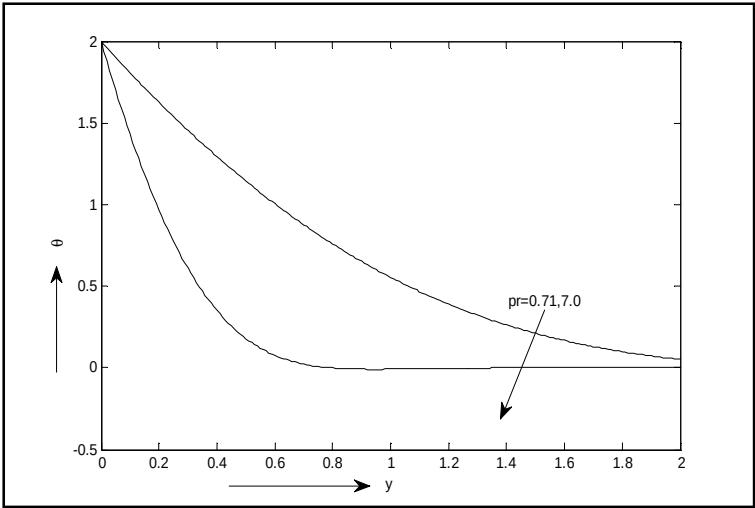


Fig.1. Effect of Pron Temperature

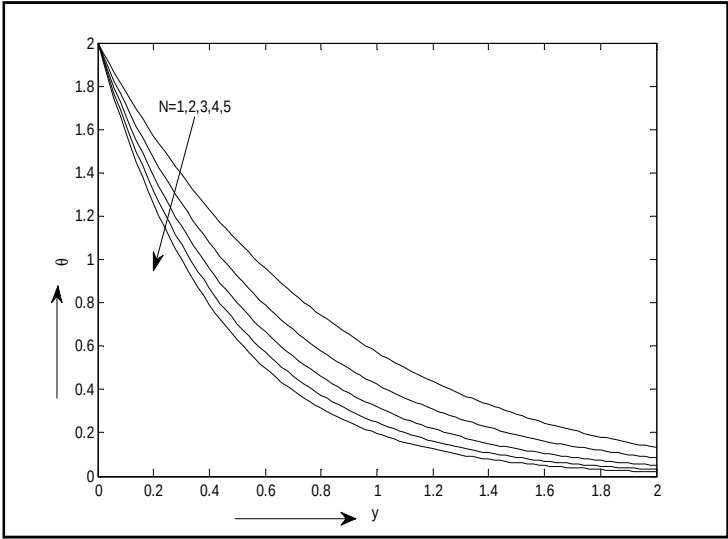
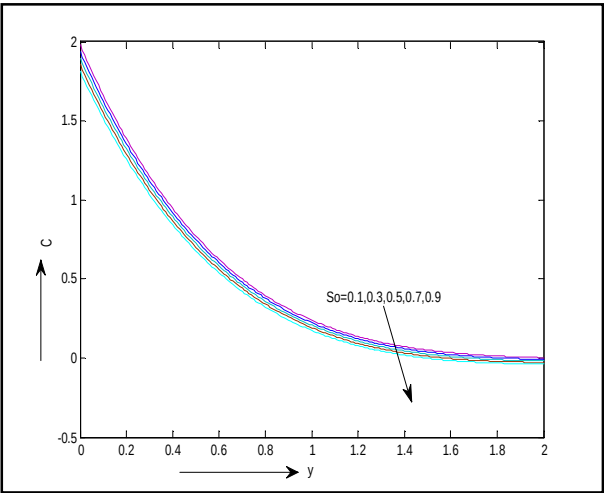
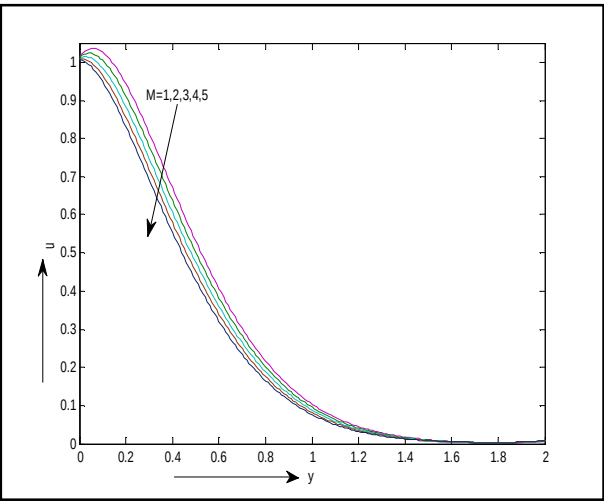


Fig.2. Effect of N on Temperature



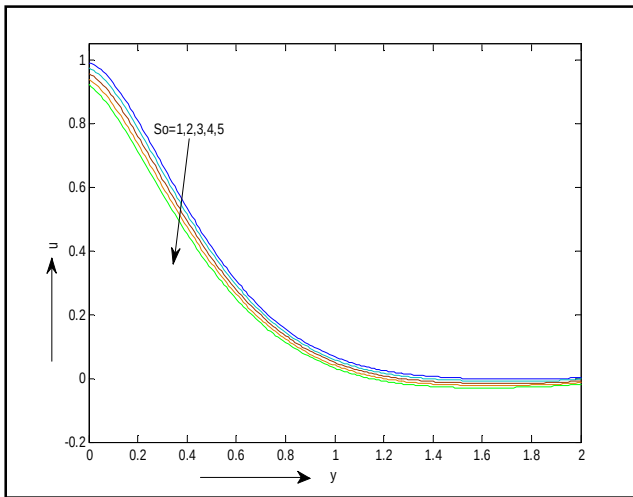
$t=2$; $Gm=5$;
 $K=0.1$; $N=0.1$;
 $M=5$; $Gr=5$;
 $Sc=0.22$;
 $\Omega=0.5$;

Fig.3. Effect of SoConcentration



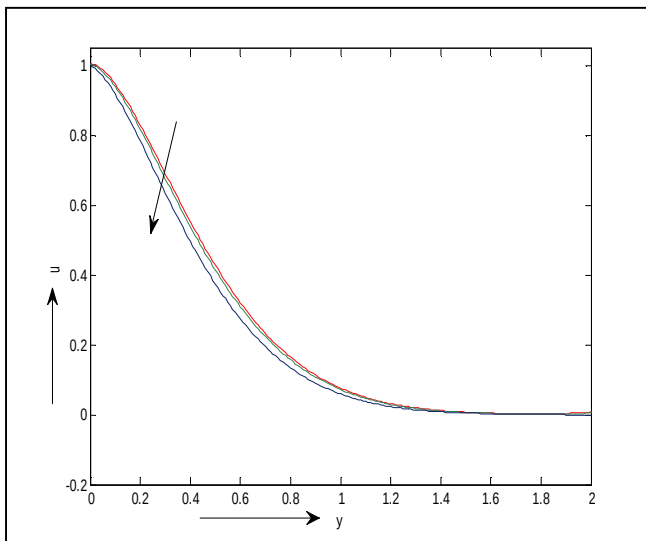
$t=2$; $Gm=5$;
 $K=0.1$; $N=0.1$;
 $Gr=5$;
 $Sc=0.22$;
 $\Omega=0.5$;
 $Pr=0.71$

Fig.4. Effect of M on Primary velocity



$t=2$; $Gm=5$;
 $K=0.1$;
 $N=0.1$;
 $M=5$; $Gr=5$;
 $Sc=0.22$;
 $\Omega=0.5$;
 $Pr=0.71$;

Fig.5. Effect of So Primary Velocity



$t=2$; $Gr=5$;
 $K=0.1$; $Gm=5$;
 $M=5$; $S_0=0.1$;
 $Sc=0.22$;
 $Pr=0.71$;

Fig.6. Effect of Ω on Primary Velocity

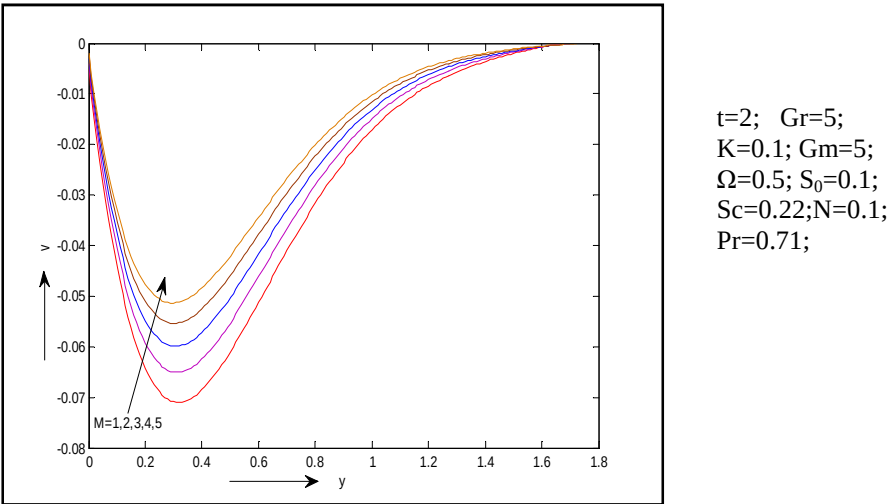


Fig. 7. Effect of M on secondary velocity

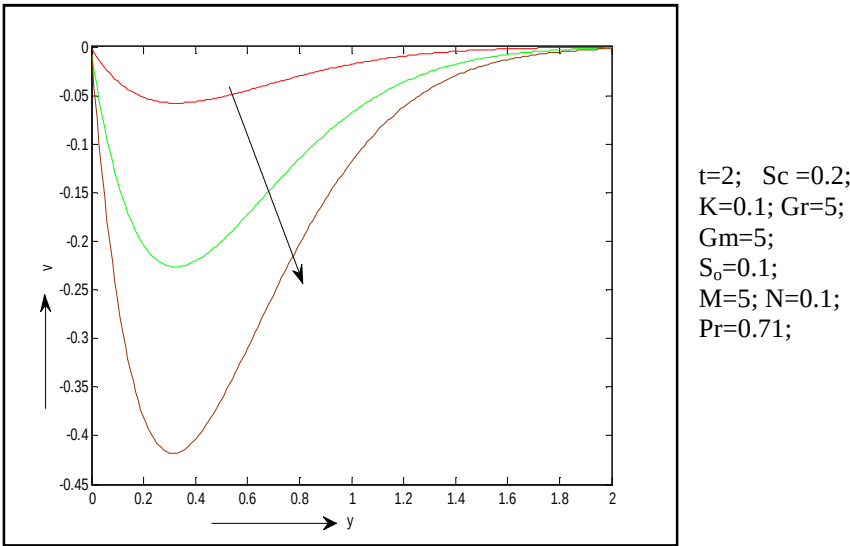


Fig.8. Effect of Ω Secondary Velocity

Table1: Skin friction for different parameters

Gm	Gr	N	Ω	Sc	S_0	M	T	K	τ_x	τ_y
1	5	0.1	0.5	0.22	0.1	1	2	1	0.0726	5.1481
5	5	0.1	0.5	0.22	0.1	1	2	1	0.1597	14.6243
1	10	0.1	0.5	0.22	0.1	1	2	1	2.1321	8.5342
1	5	0.5	0.5	0.22	0.1	1	2	1	2.1302	2.9073
1	5	0.1	1.0	0.22	0.1	1	2	1	0.8197	0.3462
1	5	0.1	0.5	0.68	0.1	1	2	1	0.1690	3.8865
1	5	0.1	0.5	0.22	0.2	1	2	1	0.0678	5.1489
1	5	0.1	0.5	0.22	0.3	1	2	1	0.0600	5.1500
1	5	0.1	0.5	0.22	0.1	2	2	1	1.9354	1.2260
1	5	0.1	0.5	0.22	0.1	1	4	1	2.3271	5.8941
1	5	0.1	0.5	0.22	0.1	1	2	2	0.0726	5.1481

Table 2: Nusselt number (Nu)			Table 3: Sherwood Number (Sh)		
Pr	N	Nu	Sc	So	Sh
0.71	1	2.4484	0.1	1	2.6215
7.0	1	4.7226	0.2	1	2.7498
0.71	0.1	2.6629	0.3	1	2.8932
0.71	0.2	2.4860	0.4	1	3.2615
0.71	0.3	2.3754	0.2	2	2.5483
0.71	0.4	2.3141	0.2	3	2.3214
0.71	0.5	2.2893	0.2	4	2.0952

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NOMENCLATURE

C_{∞}^* concentration in the fluid far away from the plate	t^* time
D mass diffusion coefficient	t dimensionless time
D_1 thermal diffusivity coefficient	T^* temperature of the plate
$(e_{\lambda h})$ Plank function	T_{∞}^* Temperature of the fluid far away
g acceleration due to gravity	u^* primary velocity of the fluid
Gr Grashof number	u dimensionless velocity along x - axis
Gm modified Grashof number	u_0 velocity of the fluid
$(K_{\lambda})_w$ absorption coefficient	V^* secondary velocity of the fluid
M magnetic parameter	V dimensionless velocity along y-axis
q_r Radiative heat flux	y^* coordinate axis normal to the plate
Sc Schmidt number	
Greek symbols	
β_c coefficient of volume expansion with concentration	Ω dimensionless rotation parameter
β_T coefficient of volume expansion	σ Stefan-Boltzmann constant
$\mathcal{K}$ Thermal conductivity of the fluid	ρ density of the fluid
ν Kinematic viscosity	Ω^* dimensionless rotational parameter