

# Cosmological Model with BVDP in $f(R, T)$ Gravity Theory

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**Abstract :** In this communication, we have examined the time dependence of the cosmological constant  $\Lambda$  and gravitational constant  $G$  by considering spatially homogeneous and anisotropic Bianchi type-I space-time in modified  $f(R, T)$  gravity theory along with bilinear varying deceleration parameter )BVDP .(We have observed that anisotropic parameter  $A_m \rightarrow 0$  as  $t \rightarrow \infty$  .This infers that our model of the universe reaches isotropy at late times which is in good agreement with recent cosmological observations .We have also studied the behaviour of various physical parameters along with the help of their graphical representations.

**Keywords :** Bilinear varying deceleration parameter, Variable cosmological constant, Variable gravitational constant.

**AMS Mathematics Subject Classification :** 83F05.

## 1. Introduction

Many experimental and theoretical studies regarding various aspects of the universe have emphasized that our universe is currently in an accelerating phase of its expansion<sup>1-4</sup> .The above study also indicates that dark energy plays a significant role to discuss the dynamic nature of the universe .It is also known that the large negative pressure of dark energy act as an anti-gravity force and helps in speeding up the expansion of universe.

Hubble parameter  $H$  and deceleration parameter) DP ( $q$  are the most significant parameters to understand the various aspects of the universe .The positive sign of the DP indicates the decelerating phase of the expansion of the universe while negative sign of DP indicates accelerating phase .For the universe, which is accelerating at present<sup>1-4</sup> and was decelerating in the past, DP must show flipping in its sign .Under this motivation, many authors have studied cosmological models with time variable DP to discuss the

nature of expansion of the universe<sup>5-9</sup>. In this study, we wish to choose the bilinear form of DP as  $q(t) = \frac{1 - \beta_1 t}{1 + \beta_2 t}$ , where both  $\beta_1$  and  $\beta_2$  are the positive constants having the dimensions as inverse of time. The above bilinear form of variable DP has been motivated from Mishra et al.<sup>10</sup> This BVDP generalizes both constant<sup>11</sup> and linear varying deceleration parameter )LVDP(<sup>12</sup>.

As we know, the term cosmological constant  $\Lambda$  was first introduced by Einstein in his field equations to obtain a static universe. Later, many authors had investigated cosmological models by considering variable cosmological constant  $\Lambda$  and published fruitful results.<sup>13-16</sup> It is now considered as a suitable candidate for dark energy to discuss the current expansion of the universe.

The gravitational constant  $G$  is also an important parameter in cosmology and it is believed that  $G$ -varying cosmologies are relevant to get a possible unification of gravitation and particle physics. Dirac was the first who constructed a cosmology based on varying  $G$ <sup>17</sup>. Thereafter, many authors have studied the behaviour of cosmological models with time dependent  $G$  and  $\Lambda$ <sup>5,16,18-21</sup>.

After publishing several research papers on the subject cited above, we have realized to study the cosmological models in modified theories of gravitation. These theories play a prominent role in understanding the models of the present universe. There are various modified theories of gravitation in literature which includes  $f(R)$  gravity,  $f(G)$  gravity,  $f(T)$  gravity and  $f(R, T)$  gravity. In this paper, we have confined ourselves to construct cosmological model in modified  $f(R, T)$  gravity theory. This modified theory of gravity was formulated by Harko et al.<sup>22</sup> in which usual Ricci Scalar  $R$  in Einstein-Hilbert action is replaced by an arbitrary function of Ricci scalar  $R$  and trace  $T$  of energy momentum tensor  $T_{\mu\nu}$ . This gravity theory can be considered as more suitable theory to address the cosmic evolution of the universe and existence of dark energy.

## 2. Metric and field equations governing the cosmological model

The field equations of  $f(R, T)$  gravity with  $f(R, T) = R + 2f(T)$  and  $f(T) = -\lambda T$   $\lambda$  is an arbitrary constant (can be written as

$$(2.1) \quad R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = -(8\pi G + 2\lambda) T_{\mu\nu} + \lambda(-T - 2p) g_{\mu\nu}$$

Here  $R_{\mu\nu}$  is the Ricci tensor,  $R$  is the Ricci scalar,  $g_{\mu\nu}$  is the metric tensor and  $T$  is the trace of energy-momentum tensor  $T_{\mu\nu}$ . On comparing the above field equations with Einstein's original field equations,

$$(2.2) \quad R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = -8\pi G T_{\mu\nu} + \Lambda g_{\mu\nu}$$

we obtain  $\Lambda = -\lambda(T + 2p)$ .

Therefore, in  $f(R, T)$  gravity theory, the field equations with time varying  $G$  and  $\Lambda$  can be written as

$$(2.3) \quad R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = -(8\pi G + 2\lambda) T_{\mu\nu} + \Lambda g_{\mu\nu}$$

It can be observed from the eq) .2.3 (that usual energy-conservation law does not hold in  $f(R, T)$  gravity .The left side of eq) .2.3 (has zero divergence through Bianchi identities. Therefore, the right side of the equation must also have zero divergence .

$$(2.4) \quad (-8\pi G + 2\lambda) T_{\mu;\nu}^{\nu} + 8\pi G_{,\mu} (\rho - 3p) + \Lambda_{,\mu} = 0$$

From the above equation, it can be observed that  $T_{\mu;\nu}^{\nu} = 0$  can hold in  $f(R, T)$  gravity theory if we take  $8\pi G_{,\mu} (\rho - 3p) + \Lambda_{,\mu} = 0$ .

We have assumed that the geometry of the universe is described by the spatially homogeneous and totally anisotropic Bianchi type-I space time, given by line element

$$(2.5) \quad ds^2 = dt^2 - (a_1)^2 dx^2 - (a_2)^2 dy^2 - (a_3)^2 dz^2$$

where  $a_1$ ,  $a_2$  and  $a_3$  are the scale factors in the x, y and z direction respectively .

The energy-momentum tensor for a perfect fluid is given by

$$(2.6) \quad T_{\mu\nu} = (p + \rho) u_{\mu} u_{\nu} + p g_{\mu\nu}$$

where  $p$  and  $\rho$  are the isotropic pressure and energy density of the fluid respectively .Here  $u_{\mu}$  is the four-velocity vector of fluid particles satisfying the following relations:

$$(2.7) \quad \mu^\mu \mu_\mu = -1 \text{ and } \mu^\mu \nabla_\nu \mu_\mu = 0$$

The field equations (2.3) (for the Bianchi type-I space-time having perfect fluid reduces to following differential equations :

$$(2.8) \quad \frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_2 \dot{a}_3}{a_2 a_3} = \Lambda - (8\pi G + 2\lambda)p$$

$$(2.9) \quad \frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_3}{a_2} + \frac{\dot{a}_1 \dot{a}_3}{a_1 a_3} = \Lambda - (8\pi G + 2\lambda)p$$

$$(2.10) \quad \frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1 \dot{a}_2}{a_1 a_2} = \Lambda - (8\pi G + 2\lambda)p$$

$$(2.11) \quad \frac{\dot{a}_1 \dot{a}_2}{a_1 a_2} + \frac{\dot{a}_2 \dot{a}_3}{a_2 a_3} + \frac{\dot{a}_1 \dot{a}_3}{a_1 a_3} = \Lambda + (8\pi G + 2\lambda)\rho$$

Since the trace of the energy-momentum tensor of the proposed model is  $3p - \rho$ , therefore, the variable cosmological constant  $\Lambda$  has the form

$$(2.12) \quad \Lambda(t) = \lambda(\rho - 5p)$$

As discussed in introduction, if we take  $8\pi G_{,\mu}(\rho - 3p) + \Lambda_{,\mu} = 0$  then the usual conservation equation  $T_{\mu;\nu}^\mu = 0$  will hold and it yields

$$(2.13) \quad \frac{\dot{\rho}}{\rho} + 9H \left( 1 + \frac{p}{\rho} \right) = 0$$

where  $H$  is the Hubble parameter, which for Bianchi type-I space-time is defined as

$$(2.14) \quad H = \frac{1}{3} \left( \frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3} \right)$$

The spatial volume for the proposed model is given by

$$(2.15) \quad V = a_1 a_2 a_3$$

The expansion scalar  $\theta$ , the shear scalar  $\sigma$  and the anisotropic parameter  $A_m$ , being defined as

$$(2.16) \quad \theta = 3H = \left( \frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3} \right)$$

$$(2.17) \quad \sigma^2 = \frac{1}{2} \left[ \left( \frac{\dot{a}_1}{a_1} \right)^2 + \left( \frac{\dot{a}_2}{a_2} \right)^2 + \left( \frac{\dot{a}_3}{a_3} \right)^2 \right] - \frac{1}{6} \theta^2$$

$$(2.18) \quad A_m = \frac{1}{3} \sum_{\mu=1}^3 \left( \frac{H_\mu - H}{H} \right)^2$$

Here  $H_1 = \frac{\dot{a}_1}{a_1}$ ,  $H_2 = \frac{\dot{a}_2}{a_2}$  and  $H_3 = \frac{\dot{a}_3}{a_3}$  are the directional Hubble parameters in x, y and z direction respectively.

The field equations )2.8 (to )2.11 (and eq )2.13 (involve six unknowns, viz.,  $a_1$ ,  $a_2$ ,  $a_3$ ,  $p$ ,  $\rho$ ,  $G$  and  $\Lambda$ . To solve the field equations explicitly, we require two additional physical constraints which we shall consider in our next section.

### 3. Solution of field equations

As discussed earlier, we have assumed the bilinear form of DP i.e.

$$(3.1) \quad q(t) = \frac{1 - \beta_1 t}{1 + \beta_2 t}$$

where both  $\beta_1$  and  $\beta_2$  are the positive constants having the dimensions as inverse of time.

From eq )2.19(, it is clear that  $q > 0$  for  $t < \frac{1}{\beta_1}$  and  $q < 0$  for  $t > \frac{1}{\beta_1}$ . This

behaviour of DP indicates that our model of universe is evolving from an early deceleration stage to the present acceleration stage which is in good agreement with the recent cosmological observations<sup>1-4</sup>. The similar behaviour of DP can be visualized through fig.1 .

We also assume that the fluid obeys the equation of the state )EoS (of the form

$$(3.2) \quad p = \gamma\rho, 0 \leq \gamma \leq 1$$

where  $\gamma$  is a constant and is called as EoS parameter .

The variable deceleration parameter is defined as:

$$(3.3) \quad q(t) = \frac{d}{dt} \left( \frac{1}{H} - t \right)$$

On substituting the value of  $q(t)$  from eq) .2.19 (in eq) .2.21(, we obtain the following expression for  $H$  :

$$(3.4) \quad H = \frac{1}{\beta_3 t + \beta_4 \log(1 + \beta_2 t) + c_1}$$

$$\text{where } \beta_3 = \left( 1 - \frac{\beta_1}{\beta_2} \right) \text{ and } \beta_4 = \left[ \frac{1}{\beta_2} + \frac{\beta_1}{(\beta_2)^2} \right]$$

Since the rate of expansion is very large at the time of inflation, therefore  $c_1 = 0$  .Eq) .2.22 (can be re-written as

$$(3.5) \quad H = \frac{1}{\beta_3 t + \beta_4 \log(1 + \beta_2 t)}$$

On simplifying the above expression of  $H$  , the expression for spatial volume  $V$  can be obtained as:

$$(3.6) \quad V = a_1 a_2 a_3 = \left( c_2 \sqrt{t} \right)^3 \exp \{ 3K_1(t) \}$$

where  $K_1(t) = k_1 t + \frac{k_2 t^2}{2} + \frac{k_3 t^3}{3} + \frac{k_4 t^4}{4} + O(t^5)$  and  $c_2$  is an integrating constant.

The constants  $k_1$  ,  $k_2$  ,  $k_3$  and  $k_4$  are obtained as

$$k_1 = \frac{\beta_1 + \beta_2}{8}$$

$$k_2 = \frac{(\beta_1 + \beta_2)(3\beta_1 - 5\beta_2)}{96}$$

$$k_3 = \frac{\beta_1 + \beta_2}{4} \left[ \frac{(\beta_1 + \beta_2)^2}{32} - \frac{\beta_2(\beta_1 + \beta_2)}{6} + \frac{(\beta_2)^2}{4} \right]$$

$$k_4 = \frac{\beta_1 + \beta_2}{4} \left[ \frac{(\beta_1 + \beta_2)^3}{128} - \frac{\beta_2(\beta_1 + \beta_2)^2}{16} + \frac{13(\beta_2)^2(\beta_1 + \beta_2)}{72} - \frac{(\beta_2)^3}{5} \right]$$

Using eqs) .2.8 (–) 2.10(, we obtain

$$(3.7) \quad \frac{a_1}{a_2} = m_1 \exp \left( n_1 \int \frac{dt}{V} \right)$$

$$(3.8) \quad \frac{a_1}{a_3} = m_2 \exp \left( n_2 \int \frac{dt}{V} \right)$$

$$(3.9) \quad \frac{a_2}{a_3} = m_3 \exp \left( n_3 \int \frac{dt}{V} \right)$$

Here  $m_1$ ,  $n_1$ ,  $m_2$ ,  $n_2$ ,  $m_3$  and  $n_3$  are the constants of integration. Further, using eqs) .2.25) - (2.27(, we get the following expressions for scale factors:

$$(3.10) \quad a_1(t) = r_1 c_2 \sqrt{t} \exp \{ K_1(t) + s_1 K_2(t) \}$$

$$(3.11) \quad a_2(t) = r_2 c_2 \sqrt{t} \exp \{ K_1(t) + s_2 K_2(t) \}$$

$$(3.12) \quad a_3(t) = r_3 c_2 \sqrt{t} \exp \{ K_1(t) + s_3 K_2(t) \}$$

$$\text{where } K_2(t) = \int \frac{dt}{(c_2 \sqrt{t})^3 \exp \{ 3K_1(t) \}}, \quad s_1 = \frac{n_1 + n_2}{3}, \quad s_2 = \frac{n_3 - n_1}{3},$$

$$s_3 = \frac{-(n_2 + n_3)}{3}, \quad r_1 = \sqrt[3]{m_1 m_2}, \quad r_2 = \sqrt[3]{(m_1)^{-1} m_3} \quad \text{and} \quad r_3 = \sqrt[3]{(m_2 m_3)^{-1}}$$

The constants  $r_1, r_2, r_3, S_1, S_2$  and  $s_3$  satisfy the following relations:

$$(3.13) \quad r_1 r_2 r_3 = 1 \text{ and } s_1 + s_2 + s_3 = 0$$

#### 4. Physical and geometrical properties of the model

For the proposed model, the various physical parameters such as directional Hubble parameters  $H_\mu$ , expansion scalar  $\theta$  and shear scalar  $\sigma$  are obtained as

$$(4.1) \quad H_\mu = \frac{1}{\beta_3 t + \beta_4 \log(1 + \beta_2 t)} + \frac{s_\mu}{(c_2 \sqrt{t})^3 \exp\{3K_1(t)\}}; \mu = 1, 2, 3$$

$$(4.2) \quad \theta = \frac{3}{\beta_3 t + \beta_4 \log(1 + \beta_2 t)}$$

$$(4.3) \quad \sigma^2 = -\frac{S}{(c_2 \sqrt{t})^6 \exp\{6K_1(t)\}}$$

where  $S = s_1 s_2 + s_2 s_3 + s_3 s_1 = -\frac{1}{2}(s_1^2 + s_2^2 + s_3^2)$

From above equations, it is clear that  $H_\mu, \theta$  and  $\sigma$  approach to zero as  $t$  tends to infinity. All these parameters diverge at  $t = 0$ .

The anisotropic parameter  $A_m$  is obtained as

$$(4.5) \quad A_m = -\frac{2}{3} \left[ \frac{S\{\beta_3 t + \beta_4 \log(1 + \beta_2 t)\}^2}{(c_2 \sqrt{t})^6 \exp\{6K_1(t)\}} \right]$$

From eq) .2.35(, it is observed that the anisotropic parameter  $A_m = 0$  for  $S = 0$  which means the proposed model of the universe is isotropic throughout the cosmic evolution. However, for  $S \neq 0$ ,  $A_m \rightarrow 0$  as  $t \rightarrow \infty$ . This infers that our model attains isotropy at late times.

From eqs. (2.8) - (2.13), the values of energy density  $\rho$ , isotropic pressure  $p$ , gravitational constant  $G$  and cosmological constant  $\Lambda$  are obtained as:

$$(4.6) \quad \rho = c_3 T^{-9(1+\omega)}$$

$$\text{where } T = c_2 \sqrt{t} \exp\{K_1(t)\}$$

$$(4.7) \quad p = \omega c_3 T^{-9(1+\omega)}$$

$$(4.8) \quad G = \frac{1}{8\pi c_3} \left[ \frac{3T^{9(1+\omega)}}{\{\beta_3 t + \beta_4 \log(1 + \beta_2 t)\}^2} + ST^{3(1+3\omega)} + \lambda c_3 (5\omega - 3) \right]$$

$$(4.9) \quad \Lambda = \lambda c_3 (1 - 5\omega) T^{-9(1+\omega)}$$

For the present universe i.e.  $t_0 = 13.8 \text{ Gy}$  with  $q_0 = -0.73^{23}$ , eq. 2.19 (yields the following relationship between  $\beta_1$  and  $\beta_2$  :

$$(4.10) \quad 13.8\beta_1 - 10.07\beta_2 - 1.73 = 0$$

In the graphical representations of various physical parameters, we have chosen the values of  $\beta_1$  and  $\beta_2$  so that above relation 2.40 (is satisfied).

## 5. Concluding remarks

The present study deals with investigation of Bianchi-type I cosmological model in modified  $f(R, T)$  gravity theory with time variable gravitational constant  $G$  and cosmological constant  $\Lambda$ . To get the solution of field equations, we have assumed BVDP i.e.  $q(t) = \frac{1 - \beta_1 t}{1 + \beta_2 t}$ , where  $\beta_1$  and

$\beta_2$  are the positive constants having the dimensions as inverse of time. It is also assumed that the fluid obeys linear equation of state i.e.  $p = \gamma \rho$ , where  $\gamma$ )  $0 \leq \gamma \leq 1$  (is known as EoS parameter). Based on above study, the main features of the model are discussed below:

- Proposed bilinear form of DP shows signature flipping which indicates that our model of universe was decelerating in the past and is in accelerating stage of the expansion at present time. Such nature of the expansion of the universe validates with current cosmological observations.

- From eq) .2.24(, it is observed that spatial volume  $V = 0$  at  $t = 0$  and  $V \rightarrow \infty$  as  $t \rightarrow \infty$  .This means our model of the universe has initial big-bang singularity and thereafter, it is expanding continuously with the passage of time .
- Fig .2 represents the behaviour of Hubble parameter against time  $t$  .It shows that  $H$  diverges at  $t = 0$  and  $H \rightarrow 0$  as  $t \rightarrow \infty$  .
- Fig .3 represents the time-varying nature of energy density  $\rho$  for different values of  $\omega$  .It is observed that  $\rho$  remains positive throughout the evolution of universe and is decreasing to zero as  $t \rightarrow \infty$  .
- Fig .4 represents the time-varying behaviour of the gravitational constant  $G$  for different values of  $\omega$  .The graph indicates that  $G$  is positively increasing throughout the evolution of the universe.

The time-dependence of the cosmological constant  $\Lambda$  has been shown in the fig.5 .In all the three cases, the present value of  $\Lambda$  is near about zero at present epoch which is good harmony with the recent observational data.

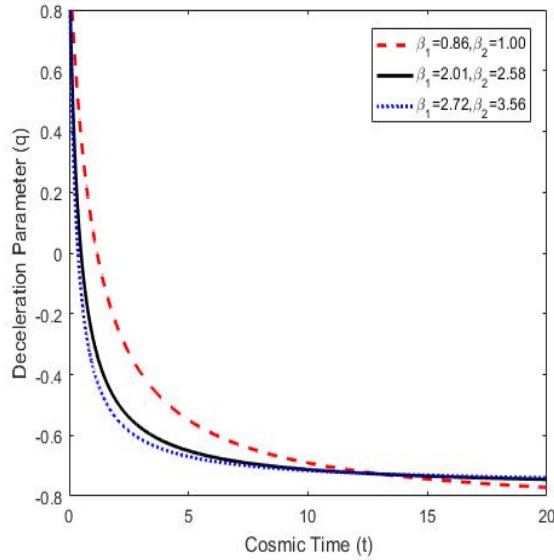


**Figure 1:** Time-varying behaviour of deceleration parameter  $q$

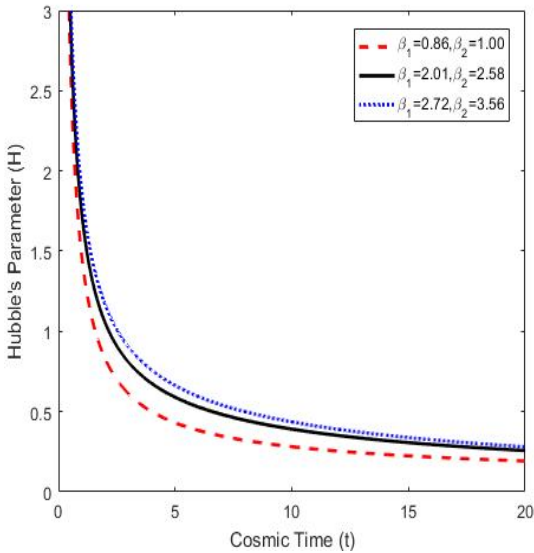


**Figure 2:** Time-varying behaviour of Hubble parameter  $H$

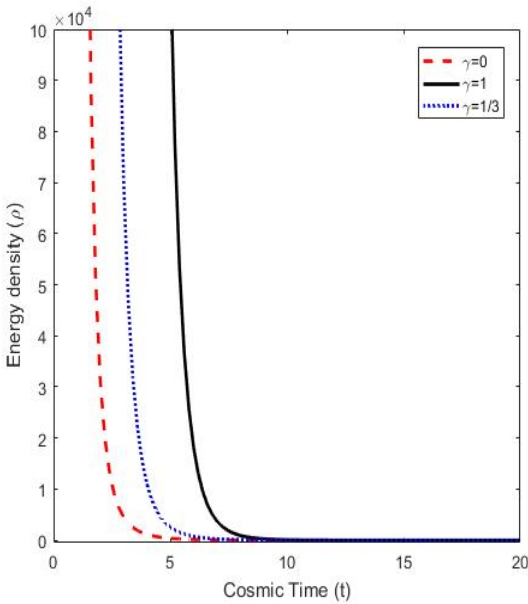


**Figure 3 :**Time-varying behaviour of energy density  $\rho$  for  $c_2 = 1$ ,  
 $c_3 = 1.22$ ,  $\beta_1 = 0.86$  and  $\beta_2 = 1.00$

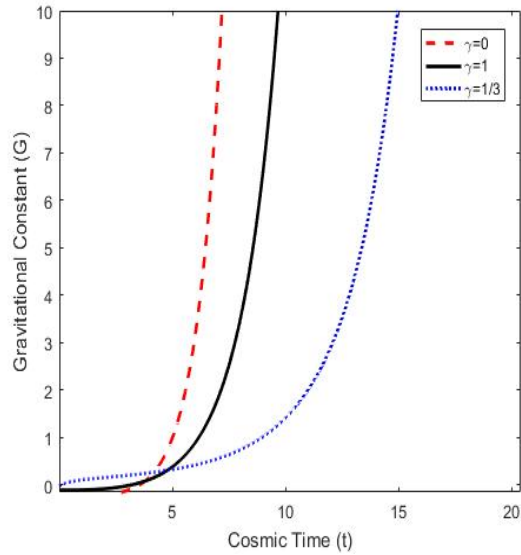


**Figure 4:** Time-varying behaviour of gravitational constant  $G$  for  $c_2 = 1$ ,  $c_3 = 1.22$ ,  $\beta_1 = 0.86$ ,  $\beta_2 = 1.00$ ,  $S = -1$  and  $\lambda = 1.751$

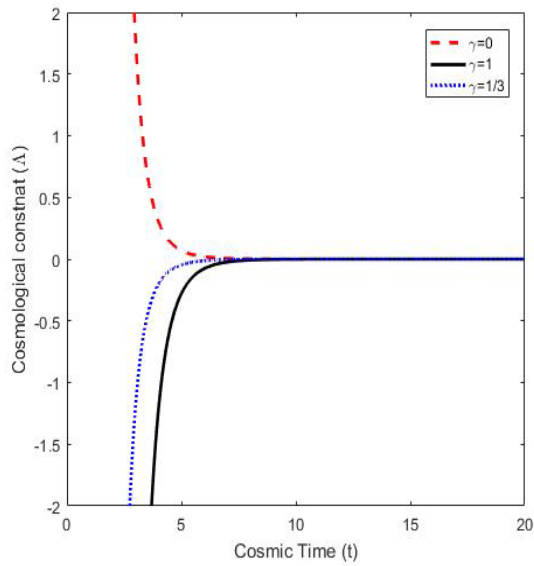


**Figure 5:** Time-varying behaviour of cosmological constant  $\Lambda$  for  $c_2 = 1$ ,  $c_3 = 1.22$ ,  $\beta_1 = 0.86$ ,  $\beta_2 = 1.00$  and  $\lambda = 1.751$

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