

String Cosmological Models in Alternative Theory of Gravity

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Abstract: In the present communication, we have investigated Bianchi type-III string cosmological models in Brans-Dicke theory (BD) of gravity. For an exact solution of modified Einstein's field equations (EFEs), we have considered that the deceleration parameter (q) as a bilinear function of cosmic time as

$$q(t) = \frac{\alpha(1-t)}{(1+t)}, \text{ here } \alpha \text{ is a non-negative constant. It is analyzed from the}$$

study that the string tension density (λ) is an increasing function of time whereas energy density (ρ) and cosmological constant (Λ) are decreasing with cosmic time and converges to a very small value at present epoch. Physical and geometrical behavior of various parameters have also been discussed.

Keywords: Bianchi type-III space-time, Brans-Dicke gravity, Time varying deceleration parameter, Cosmological constant

Subject classification code: 83F05

1. Introduction

One of the most challenging problems among astrophysicists/cosmologists is to know the structure formulation of our universe. All the existing theories of gravity are either, based on amplification of quantum fluctuation in a scalar field during inflation or upon symmetry breaking phase transition in the early universe. It has been studied in literature that several alternative theories of gravity were proposed as and when, but among all existing alternative theories, the scalar-tensor Brans-Dicke (BD)¹ gravity is simple and widely studied theory that may solve some issues of standard model of cosmology like early time inflation and late time accelerating expansion of the universe. According to BD theory, the gravitational constant G is not

a fundamental constant but it can vary over space and time. The time varying nature of G is equivalent to reciprocal of scalar field ϕ (i.e. $\phi \cong G^{-1}$). The modified EFEs in BD theory of gravity can be written as

(1.1)

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi}{c^4 \phi} T_{\mu\nu} - \frac{\omega}{\phi^2} \left(\phi_{,\mu} \phi_{,\nu} - \frac{1}{2} g_{\mu\nu} \phi^l{}_{,l} \right) - \frac{1}{\phi} (\phi_{;\mu\nu} - g_{\mu\nu} \square \phi),$$

here $\square = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2} = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2$, $\square$ and ∇ are wave operator

and Laplace operator respectively, $R_{\mu\nu}$ is the Ricci curvature tensor, $T_{\mu\nu}$ is the energy momentum tensor, R is the curvature scalar and ω is the Brans-Dicke dimensionless coupling constant. Also the law of conservation of momentum can be expressed as

$$(1.2) \quad T_{;\nu}^{\mu\nu} = 0.$$

Here ‘semicolon’ indicates the co-variant derivative and ‘comma’ indicates the partial derivative. The variation in scalar field ϕ leads to the following equation:

$$(1.3) \quad 2\phi \square \phi - \phi_{,\mu} \phi^{,\mu} = \frac{R}{\omega} \phi^2.$$

Equation (1.3) can be simplified by substituting for R from the contracted form of equation (1.1). We finally get

$$(1.4) \quad \square \phi = \frac{8\pi}{(2\omega + 3)c^4} T,$$

Equation (1.4) leads to the anticipated scalar wave equation for scalar field ϕ with sources in matter. As it contains a scalar field ϕ in addition to the metric tensor $g_{\mu\nu}$. The BD theory is also known as the scalar-tensor theory of gravitation.

It has been studied that the string cosmology attracts the attention of several researchers since 1970s. It is believed that strings play a significant role in the early inflationary phase of the universe. These strings can generate density fluctuations which lead to galaxy formation and can be coupled to gravitational field. Hence, it is interesting to study the gravitaional effects that arise due to presence of strings. In this regards,

Letelier^{2,3} and Stachel⁴ have studied the general relativistic string models of universe.

Recent research findings indicate that viscosity plays an important role in early stage evolution of the universe⁵⁻¹⁰. Also, it is well known that at early stage of the Universe when neutrino decoupling occurred, the matter behaves like viscous fluid¹¹ and coefficient of viscosity (ξ) is decreasing with the evolution of the universe. The viscous string cosmological models have been studied by several authors¹²⁻¹⁹ in the context of general relativity. Many authors²⁰⁻²⁵ have discussed bulk viscous string cosmological models in BD theory. Very recently many cosmologists²⁶⁻³¹ have investigated the several viscous cosmic string cosmological models in BD theory.

Under the motivation of the research work mentioned above, in this paper, we shall study the Bianchi type-III cosmological model in presence of viscous string. Here, we have obtained the exact solution of modified EFEs with bilinear varying DP. The paper is organized as follows: in section 2, metric and field equations governing the cosmological models are described, the exact solution of modified EFEs has been presented in section 3. In section 4, we describe physical and kinematic behavior of some cosmological parameters of models. Results and discussion are summarized in the section 5. Finally, in section 6, concluding remarks of the study are given.

2. Metric and Field Equations

In present communication we have considered the spatially homogeneous and anisotropic Bianchi type-III space-time as given below

$$(2.1) \quad ds^2 = -dt^2 + A^2(t)dx^2 + e^{-2sx}B^2(t)dy^2 + C^2(t)dz^2$$

Here potential A, B and C are the functions of cosmic time 't' only and s is a positive constant. The energy-momentum tensor $T_{\mu\nu}$ for bulk viscous fluid containing one dimensional cosmic string can be expressed as

$$(2.2) \quad T_{\mu\nu} = (\rho + \bar{p})u_\mu u_\nu + \bar{p}g_{\mu\nu} - \lambda x_\mu x_\nu$$

$$(2.3) \quad \bar{p} = p - 3\xi H$$

Here λ is the string tension density, $\bar{p}$ is effective pressure, ρ is the proper energy density for cloud strings with particles attached to them, u_ν is the

four-velocity vector and x^ν is a unit space-like vector along the direction of string. The vectors u^ν and x^ν satisfy the conditions as

$$(2.4) \quad u_\nu u^\nu = 1 = -x_\nu x^\nu, \quad u^\nu x_\nu = 0$$

The relation between energy density and string tension density of the configuration can be given as

$$(2.5) \quad \rho = \rho_p + \lambda,$$

The string tension density λ may takes positive or negative values. Letelier, Berman and Som^{2,3,32} established that a negative value of λ represents the Universe filled with no string, whereas positive value of λ indicate the universe filled with string particles. For the metric given in equation (2.1) with the help of equations (2.2) and (2.3) the field equation (1.1) yields the set of equations as:

$$(2.6) \quad \begin{aligned} & \frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{s^2}{A^2} + \frac{\ddot{\phi}}{\phi} + \frac{\omega\dot{\phi}^2}{2\phi^2} + \frac{\dot{\phi}}{\phi} \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} \right) \\ & = -8\pi\phi^{-1}(\bar{p} - \lambda) + \Lambda, \end{aligned}$$

$$(2.7) \quad \begin{aligned} & \frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{CA} + \frac{\ddot{\phi}}{\phi} + \frac{\omega\dot{\phi}^2}{2\phi^2} + \frac{\dot{\phi}}{\phi} \left(\frac{\dot{A}}{A} + \frac{\dot{C}}{C} \right) \\ & = -8\pi\phi^{-1}\bar{p} + \Lambda, \end{aligned}$$

$$(2.8) \quad \begin{aligned} & \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} + \frac{\ddot{\phi}}{\phi} + \frac{\omega\dot{\phi}^2}{2\phi^2} + \frac{\dot{\phi}}{\phi} \left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) \\ & = -8\pi\phi^{-1}\bar{p} + \Lambda, \end{aligned}$$

$$(2.9) \quad \begin{aligned} & \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - \frac{s^2}{A^2} - \frac{\omega\dot{\phi}^2}{2\phi^2} + \frac{\dot{\phi}}{\phi} \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) \\ & = -8\pi\phi^{-1}\rho + \Lambda, \end{aligned}$$

$$(2.10) \quad \frac{\dot{A}}{A} - \frac{\dot{B}}{B} = 0,$$

$$(2.11) \quad \ddot{\phi} + \dot{\phi} \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) = \frac{8\pi(3\bar{p} - \rho - \lambda)}{(3 + 2\omega)}.$$

where an over head dot denote derivatives with respect to cosmic time t . Before presenting solution of above cited field equations, we may introduce some cosmological parameters such as the spatial volume (V), the Hubble's parameter (H), the expansion scalar (θ), the deceleration parameter (q), the anisotropy parameter (A_m) and the shear scalar (σ) for the metric (2.1) as,

$$(2.12) \quad V = a(t)^3 = ABC,$$

$$(2.13) \quad \theta = H_1 + H_2 + H_3,$$

Here are the directional Hubble parameters in directions of x, y and z axis respectively.

$$(2.14) \quad q = -\frac{a\ddot{a}}{\dot{a}^2} = -1 + \frac{d}{dt} \left(\frac{1}{H} \right).$$

$$(2.15) \quad H_1 = \frac{\dot{A}}{A}, H_2 = \frac{\dot{B}}{B}, H_3 = \frac{\dot{C}}{C}$$

$$(2.16) \quad \sigma^2 = \frac{1}{2} \left(\sum_{i=1}^3 H_i^2 - 3H^2 \right),$$

Here $\Delta H_i = H_i - H, i = 1, 2, 3$. Now we required the solution of above stated field equations (2.6)-(2.11).

3. Solution of the Field Equations

On integrating equation (2.10), we get

$$(3.1) \quad A = k_0 B,$$

Here k_0 is a constant of integration, for the convenience and avoid complicity it can be taken as unity, hence

$$(3.2) \quad A = B.$$

On putting $A = B$ into set of field equations (2.6)-(2.9) and (2.11), we have new set of field equations as,

$$(3.3) \quad 2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} - \frac{s^2}{A^2} + \frac{\ddot{\phi}}{\phi} + \frac{\omega\dot{\phi}^2}{2\phi^2} + 2\frac{\dot{A}}{A}\frac{\dot{\phi}}{\phi} \\ = -8\pi\phi^{-1}(\bar{p} - \lambda) + \Lambda,$$

$$(3.4) \quad \frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} + \frac{\ddot{\phi}}{\phi} + \frac{\omega\dot{\phi}^2}{2\phi^2} + \frac{\dot{\phi}}{\phi}\left(\frac{\dot{A}}{A} + \frac{\dot{C}}{C}\right) \\ = -8\pi\phi^{-1}\bar{p} + \Lambda,$$

$$(3.5) \quad \frac{\dot{A}^2}{A^2} + 2\frac{\dot{A}\dot{C}}{AC} - \frac{s^2}{A^2} - \frac{\omega\dot{\phi}^2}{2\phi^2} + \frac{\dot{\phi}}{\phi}\left(2\frac{\dot{A}}{A} + \frac{\dot{C}}{C}\right) \\ = -8\pi\phi^{-1}\rho + \Lambda,$$

$$(3.6) \quad \ddot{\phi} + \dot{\phi}\left(2\frac{\dot{A}}{A} + \frac{\dot{C}}{C}\right) = \frac{8\pi(3\bar{p} - \rho - \lambda)}{(3 + 2\omega)}.$$

The field equations (3.3)-(3.6) is a system of four equations along with seven parameters $A, C, \lambda, \rho, \bar{p}, \phi$ and Λ therefore for explicit solution of set of equations, we need at least three more constraints related to these parameters. Hence, we may apply following assumptions:

i) It is assumed that the bulk viscosity function $\xi(t)$ is proportional to some power of energy density ρ as proposed by Murphy and Borrow^{33,34},

$$(3.7) \quad \xi = \xi_0\rho^\beta,$$

here ξ_0 is proportional constant and β is ordinary constant ($\beta = 1$ for small density which correspond to a radiative fluid). For specification of ξ , we assume that the fluid obeys the barotropic equation of state of the form

$$(3.8) \quad p = \gamma\rho$$

Where γ is constant, known as equation of state parameter and $\gamma(0 \leq \gamma \leq 1)$. Now with the help of equations (2.3), (3.4), (3.5) and (3.8) we get,

$$\begin{aligned}
 (3.9) \quad & \frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\ddot{\phi}}{\phi} + \frac{\dot{A}\dot{C}}{AC} + \frac{\omega\dot{\phi}^2}{2\phi^2} + \frac{\dot{\phi}}{\phi} \left(\frac{\dot{A}}{A} + \frac{\dot{C}}{C} \right) \\
 & = -8\pi\phi^{-1}[\gamma\rho - \xi_0\rho^\beta\theta] + \Lambda,
 \end{aligned}$$

Model with linear bulk viscosity coefficient of ρ i.e. $\xi = \xi_0\rho$ when $\beta = 1$:

$$(3.10) \quad 8\pi\rho = \frac{\phi}{[\gamma - 1 - \xi_0\theta]} \left[\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\ddot{\phi}}{\phi} - \frac{\dot{A}^2}{A^2} - \frac{\dot{A}\dot{C}}{AC} + \frac{\omega\dot{\phi}^2}{\phi^2} - \frac{\dot{A}\dot{\phi}}{A\phi} + \frac{s^2}{A^2} \right]$$

$$\begin{aligned}
 (3.11) \quad \Lambda = & \frac{1}{[\gamma - 1 - \xi_0\theta]} \left[\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\ddot{\phi}}{\phi} - \frac{\dot{A}^2}{A^2} - \frac{\dot{A}\dot{C}}{AC} + \frac{\omega\dot{\phi}^2}{\phi^2} - \frac{\dot{A}\dot{\phi}}{A\phi} + \frac{s^2}{A^2} \right] \\
 & + \left(\frac{\dot{A}}{A} \right)^2 + 2 \frac{\dot{A}\dot{C}}{AC} - \frac{s^2}{A^2} - \frac{\omega\dot{\phi}^2}{2\phi^2} + \frac{\dot{\phi}}{\phi} \left(\frac{2\dot{A}}{A} + \frac{\dot{C}}{C} \right)
 \end{aligned}$$

(ii) The objective of this study to investigate the dynamic nature of the universe with the help of variable DP. As we knew the sign of q , measured the expansion rate of the universe (either may speeding up or slowing down). On the basis of sign of H and q , we may classify the expansion nature of the universe as follows: if $H > 0$ and $q > 0$, the universe is expanding with decelerating rate, if $H > 0$ and $q < 0$, the universe is expanding with accelerating rate and if $H > 0$ and $q = 0$, the universe is expanding with constant rate. As recent cosmic observations confirmed that our universe expanding with an acceleration rate^{35,36}. Therefore the DP is a not a constant at all, but it is a time function. To consider the time varying nature of the q , our research group Mishra et al. had already published several results³⁷⁻⁴⁰.

Motivated from above mention work, we again considered time varying DP as a special bilinear function of cosmic time as suggested by Mishra et al.³⁹, i.e.

$$(3.12) \quad q = \frac{\alpha(1-t)}{1+t}.$$

Now solving the equation (2.14) with the help of equation (3.14), we obtain the Hubble parameter as

$$H = \frac{1}{\int (1 + \frac{\alpha(1-t)}{1+t}) dt + k_1}$$

Here k_1 is a constant of integration. For physical viability of the solution we may taken the constant of integration unity i.e. $k_1 = 0$, as $t \rightarrow 0, H \rightarrow \infty$. As at early inflationary stage the expansion rate is very-very large.

$$(3.13) \quad H = \frac{1}{(1-\alpha)t + 2\alpha \log(1+t)}$$

$$(3.14) \quad a(t) = a(t_0)t^{\frac{1}{1+\alpha}}.e^{F(t)},$$

Here

$$F(t) = \frac{\alpha}{(1+\alpha)^2}t + \frac{-2\alpha + \alpha^2}{6(1+\alpha)^3}t^2 + \frac{3\alpha - 2\alpha^2 + \alpha^3}{18(1+\alpha)^4}t^3 + \frac{-18\alpha + 11\alpha^2 - 14\alpha^3 + 2\alpha^4}{180(1+\alpha)^5}t^4 + O(t^4)$$

(iii) The shear scalar is proportional to scalar expansion as suggested by Collins et al.⁴¹ i.e.

$$(3.15) \quad A = C^m,$$

where $m \neq 0$, is non zero constant, which takes care of the anisotropy of the space.

(iv) Without any loss of generality, we shall assume that the scalar field(ϕ) is some power of the scale factor i.e. the power law relation between scale factor and scalar field has been suggested by Johri and Desikan⁴² can be expressed as,

$$(3.16) \quad \phi = \phi_0 a^b(t),$$

where ϕ_0 is a proportionality constant and b is an arbitrary constant. From equations (2.12), (3.14) and (3.15), we have following expression for the metric potentials A, B and C,

$$(3.17) \quad A = B = [a(t_0)t^{\frac{1}{1+\alpha}}.e^{F(t)}]^{\frac{3m}{2m+1}},$$

$$(3.18) \quad C = [a(t_0)t^{\frac{1}{1+\alpha}}.e^{F(t)}]^{\frac{3}{2m+1}},$$

The directional Hubble parameters may be expressed as

$$(3.19) \quad H_1 = H_2 = \frac{3m}{(2m+1)\{(1-\alpha)t + 2\alpha \log(1+t)\}},$$

$$(3.20) \quad H_3 = \frac{3}{(2m+1)\{(1-\alpha)t + 2\alpha \log(1+t)\}},$$

After substituting the value of A, B and C metric equation (2.1) can be written as

$$(3.21) \quad ds^2 = -dt^2 + [a(t_0)t^{\frac{1}{1+\alpha}}.e^{F(t)}]^{\frac{6m}{2m+1}}(dx^2 + e^{-2x}dy^2) \\ + [a(t_0)t^{\frac{1}{1+\alpha}}.e^{F(t)}]^{\frac{6}{2m+1}}dz^2$$

Now, we calculated some physical parameters such as spatial volume, expansion scalar (θ) , shear scalar σ and anisotropy parameter A_m are expressed as

$$(3.22) \quad V = ABC = a^3(t_0)t^{\frac{3}{1+\alpha}}.e^{3F(t)},$$

$$(3.23) \quad \theta = \frac{3}{(1-\alpha)t + 2\alpha \log(1+t)}$$

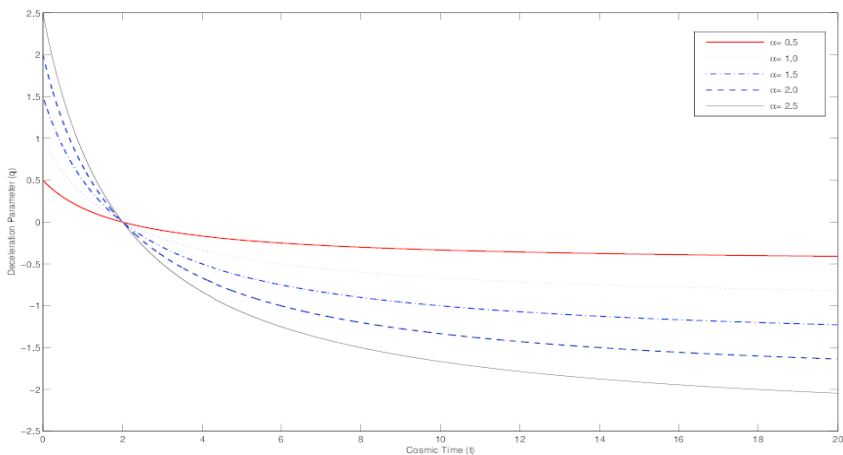


Figure 1: Plot of deceleration parameter q versus cosmic time t .

From equation (3.22), It is evident that as $t \rightarrow 0$ then $\theta \rightarrow \infty$. i.e. the of universe was expanded with an infinity rate after the Big-Bang. In Figure 1, we depicts the variation between DP with cosmic time t . It is clear from this figure that Universe expansion is accelerating for different choice of $\alpha > 0$ and also show transitional phase (i.e. early time deceleration to late time acceleration. This type behaviour of q indicates that the Universe has decelerated in past and at present time undergoes an accelerating phase, which is good agreement with observational findings^{35,36}.

$$(3.24) \quad \sigma^2 = \frac{3(m-1)^2}{[(2m+1)\{(1-\alpha)t + 2\alpha \log(1+t)\}]^2}$$

$$(3.25) \quad A_m = 2\left[\frac{m-1}{(2m+1)}\right]^2, m \neq \frac{1}{2}$$

Above equation (2.41) indicates that A_m is constant throughout evolution of the Universe. This means universe never approaches to isotropy. Also $\frac{\sigma^2}{\theta^2} = \frac{(m-1)^2}{(2m+1)^2}(\text{constant})$, and if $m \rightarrow 1$ then $\frac{\sigma^2}{\theta^2} \rightarrow 0$ universe shows isotropy which agreed with results of Collins et al.⁴¹.

4. Physical and Kinematical Properties of the Model

In this section we discuss the some physical and kinematic properties of the model represented by equation (41). The string tension density (λ), the energy density (ρ), the particle density (ρ_p) and the cosmological constant (Λ) for $\beta = 0, 1$ may be expressed as

$$(4.1) \quad 8\pi\lambda\phi^{-1} = \frac{3}{(2m+1)^2}[(2m^2 + m - 1)\{1 + \alpha + (1 - \alpha)t + 3b\} + 12m^2 - 9m - 3]H^2 - \frac{s^2}{A^2}$$

$$(4.2) \quad \lambda = \frac{\phi}{8\pi} \left[\frac{3(2m^2 + m - 1)\{1 + \alpha + (1 - \alpha)t + 3b\} + 36m^2 - 27m - 9}{[(2m+1)\{(1-\alpha)t + 2\alpha \log(1+t)\}]^2} - s^2 \left[a(t_0)t^{\frac{1}{1+\alpha}} e^{\frac{-6m}{F(t)}} \right]^{\frac{-6m}{2m+1}} \right]$$

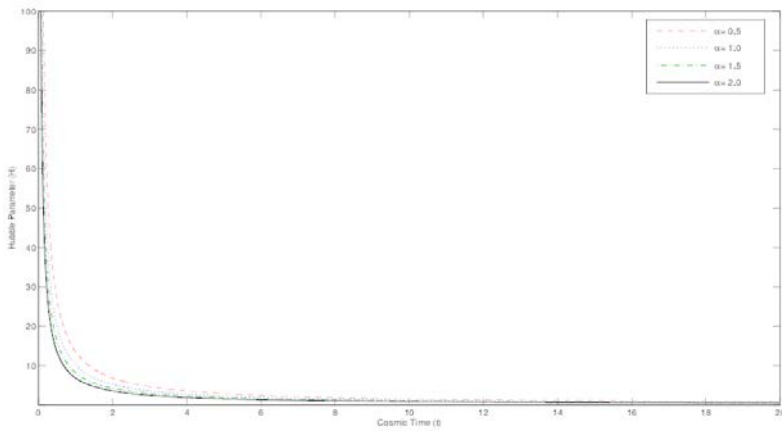


Figure 2: Plot of Hubble parameter versus time t .

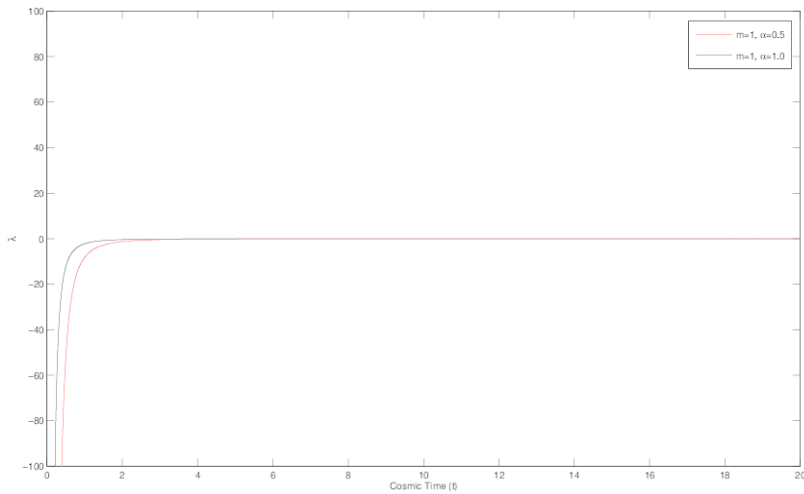


Figure 3: Plot of string tension density λ versus time t . For $\omega = 1$, $b = -1$, $\phi_0 = \xi_0 = 1$.

Here we discussed particular case if $\beta = 1$ i.e. bulk viscosity coefficient is the linear function of energy density ($\xi = \xi_0 \rho$)

$$\begin{aligned}
 8\pi\rho = & \frac{\phi}{\{1-\gamma-\xi_0\theta\}} \left[\frac{3}{(2m+1)^2} \{12m^2 + 9m + 3 - 3mb(2m+1)\} \right. \\
 (4.3) \quad & + (1+\omega)(2m+1)^2 b^2 + (6m^2 + 5m + 2) \{1 + \alpha + (1-\alpha)t\} H^2 \\
 & \left. + s^2 \left[a(t_0) t^{\frac{1}{1+\alpha}} e^{F(t)} \right]^{\frac{-6m}{2m+1}} \right],
 \end{aligned}$$

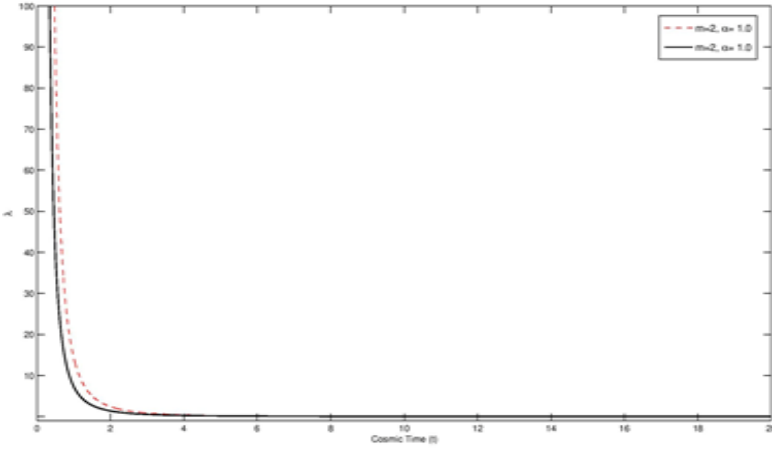


Figure 4: Plot of string tension density λ versus time. For $\omega = 1$, $b = -1$, $\phi_0 = \xi_0 = 1$.

$$\begin{aligned}
 8\pi\rho_p = & \frac{\phi}{\{1-\gamma-\xi_0\theta\}} \left[\frac{3}{(2m+1)^2} \{12m^2+9m+3-3mb(2m+1)\} \right. \\
 (4.4) \quad & + (1+\omega)(2m+1)^2 b^2 + (6m^2+5m+2)\{1+\alpha+(1-\alpha)t\} H^2 \\
 & \left. + s^2 [a(t_0)t^{\frac{1}{1+\alpha}} e^{F(t)}]^{-\frac{6m}{2m+1}} \right] - 8\pi\lambda,
 \end{aligned}$$

$$\begin{aligned}
 (4.5) \quad \Lambda = & \frac{1}{(\gamma-1-3\xi_0 H)} \left[\frac{3}{(2m+1)^2} \cdot \{(2m^2+3m+1+b(2m+1)^2)(1+\alpha+(1-\alpha)t) \right. \\
 & - 6m^2+3+b^2(2m+1)^2 \\
 & - 9m+\omega b^2(2m+1)^2-3mb(2m+1))H^2\} + s^2 [a(t_0)t^{\frac{1}{1+\alpha}} e^{F(t)}]^{-\frac{6m}{2m+1}} \\
 & + \frac{36m+(6b-\omega b^2)(2m+1)^2}{2\{(2m+1)((1-\alpha)t+2\alpha\log(1+t))\}^2} \\
 & \left. - s^2 [a(t_0)t^{\frac{1}{1+\alpha}} e^{F(t)}]^{-\frac{6m}{2m+1}} \right]
 \end{aligned}$$

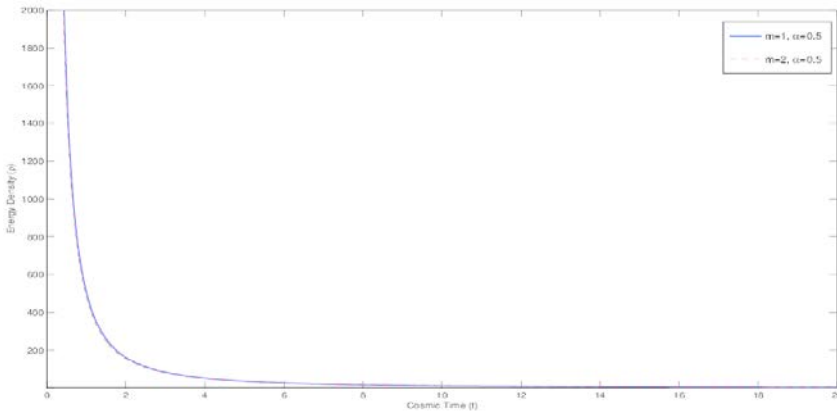


Figure 5: Plot of energy density ρ versus time t . For $\omega = 1$, $b = -1$, $\phi_0 = \xi_0 = 1$.

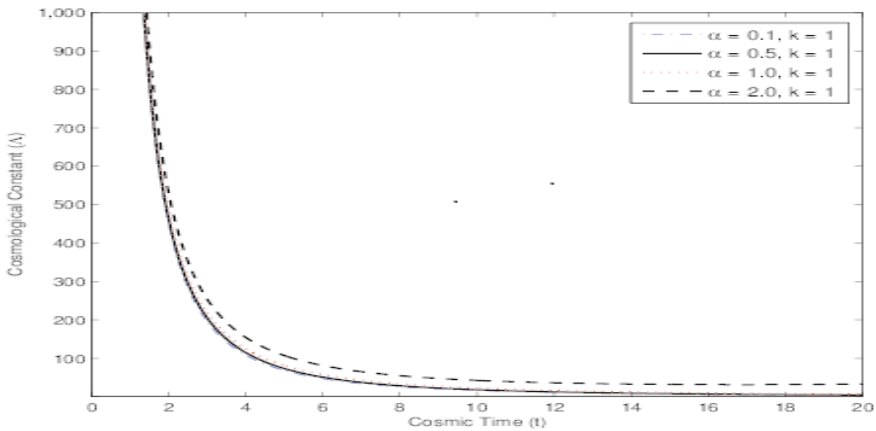


Figure 6: Plot of cosmological constant Λ versus time t . For $\omega = 1$, $b = -1$, $\phi_0 = \xi_0 = 1$.

5. Results and Discussions

In this paper, we have studied Bianchi type-III viscous string cosmological models in Brans-Dicke gravity along with BVDP. The main features of the constructed model is presented as

The universe is expanding with decelerating rate ($q > 0$) when $0 < t < 1$ and $\alpha > 0$, expanding with constant rate when $t = 1$ and $\alpha > 0$, expanding exponentially ($-1 < q < 0$) when $t > 1$, $0 < \alpha < 1$ and super exponential acceleration ($q < -1$) when $t > 1, \alpha > 1$.

•Fig. 1 shows the variation of q with cosmic time t . The behaviour of DP shows transitional phase of the universe (i.e. from early time deceleration to present acceleration phase).

•Fig. 2 depicts the variation of Hubble's parameter H with cosmic time t . It is clear from the figure that H is decreasing function of time and approaches to very small value at late time.

•It is evident from equation (3.21) that at $t = 0$ the spatial volume V is zero and thereafter it is increasing exponentially with time. Such type of the nature models suggested that there is a singularity as $t = 0$. Hence we can say that the Universe grow up from zero singularity.

•From Fig. 3, we observe that the string tension density λ is an increasing function of time for $m = 1$, which is always negative and approaches to zero at late time, whereas in Fig. 4 it is notice that the string tension density λ positive decreasing with time for $m = 2$ and approaches to zero at late time. As suggested by Letelier^{2,3} the string tension density λ may have positive or negative values, corresponding to $\lambda > 0$ the string dominant over particle whereas in case of $\lambda < 0$ the string disappear from universe. In our case the particles density dominate over the string tension density at present epoch.

•It is self exploratory from Fig. 5 that energy density ρ is decreasing function of time .

•In Fig. 6, we have plotted cosmological constant Λ with cosmic time t . We noticed that cosmological constant Λ is decreasing function of time and approaches to small value at late time. This type of behavior of Λ is good agreement with recent cosmic observations.

On the basis of above study we may write the concluding remarks presented below.

6. Concluding Remarks

In this paper Bianchi-III space-time viscous string cosmological models have been investigated in scalar-tensor BD theory of gravity with time dependent q and cosmological constant Λ . The exact solution of EFE have been obtained by assuming viscosity is some power function of energy density ρ and DP is taken as bilinear function of cosmic time along with

suitable assumptions $q(t) = \frac{\alpha(1-t)}{1+t}$, here α is positive constants. As we

observed from above mention study that the universe is growing up form initial singularity and expand with an accelerating rate at present time. We have also observed that the viscous string play an significant role in the

evolution of the universe. The string may be dominant at inflationary phase but disappears from the universe at present accelerating phase.

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