

Analysis of Self-gravitational Instability of Partially-ionized Molecular Cloud under Radiative Heat-Loss Function

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Abstract: The problem of Jeans instability of an infinite homogeneous self-gravitating of partially-ionized gaseous plasma carrying a uniform magnetic field in the presence of thermal conductivity and the arbitrary radiative heat-loss function has been investigated. With the help of relevant linearized perturbation MHD equations of the problem using normal mode analysis, a general dispersion relation is obtained and the dispersion relation is discussed for longitudinal propagation and transverse propagation separately. The condition of instability as well as stability of the fluid has been discussed. The stability of the system is discussed by applying Routh-Hurwitz criterion. We found that the effect of collision with neutrals and magnetic field have a stabilizing influence, while thermal conductivity has destabilizing influence on the Jeans instability. In addition, the classical Jeans result regarding the rise of initial break up has been considerably modified due to the arbitrary radiative heat-loss functions.

Keywords: MHD (Magneto-Hydrodynamic), ISM (Inter-Stellar Medium), Thermal conductivity, Arbitrary radiative heat-loss function, Partially-ionized plasma and Collision frequency.

1. Introduction

The gravitational instability is one of the fundamental concepts of modern astrophysical hot plasma gas cloud and it is connected with the fragmentation of interstellar matter in regard to star formation. James Jeans¹ first analyzed this instability problem and suggested that an infinite homogeneous self-gravitating fluid is unstable for all wave numbers k less

than critical Jeans wave number $k_j = \sqrt{\frac{4\pi G \rho}{c^2}}$. Further, Jeans² discussed the condition under which a fluid becomes gravitational unstable under the

action of its own gravity. Now a days, for any relevant length scales in the universe [Stars, galaxies, asteroid, planetary rings, clusters etc.], such as instability has been recognized as the key mechanism to explain the gravitational formation of structure and their evolution, in the linear regime.

The Jeans problem has been extensively studied under varying assumption. A comprehensive account of these investigations has been given by Chandrasekhar³ in monograph on hydrodynamic and hydro magnetic stability. The problem of an isothermal gas sphere subjected to external pressure has been investigated by Ebert⁴ and he found that disturbance of length scale approximately equal to the Jeans length based on the central density, were unstable to gravitational collapse. Since Jeans length $\lambda_j \propto \rho^{-1/2}$ and Jeans mass $M_j \propto \rho \lambda_j^3 \propto \rho^{-1/2}$, this considerably reduced the minimum unstable mass and demonstrated that 'O' star could form in the center of an interstellar cloud. For gravitationally contracting isothermal gas cloud it is observed that perturbations with initial scale of the order of or less than the Jeans length grew less rapidly relative to the back ground density than did perturbation of substantially larger dimension. In this connection, many investigators have discussed the Jeans instability of homogeneous plasma considering the effects of various parameters. [Chhajlani and Vaghela⁵, Herrengger⁶, Sharma and Thakur⁷, Prajapati et al.⁸, and Pensia et al.⁹].

Along with this it is an established fact that magnetic fields play an important role in interstellar gas dynamics. In the interstellar medium (ISM), a large amount of energy is injected by stars, which leads to the formation of shock waves; but when these shocks waves weaken, they become large amplitude hydro magnetic Alfvén waves. Langer¹⁰ studied the importance of magnetic field in the stability of ISM molecular clouds and he suggested that the magnetic fields can provide pressure support and inhibit the contraction and fragmentation of interstellar clouds. The magnetic field interact directly only with the ions, electrons and charged grains in the gas. Collisions of the ions with the predominantly neutral gas in the clouds are responsible for the indirect coupling of the magnetic field to the bulk of the gas. The degree to which the magnetic pressure is important depends upon the field strength and the fractional abundance. In addition to this a cloud which is gravitational unstable along the field lines but not perpendicular to them, would undergo contraction in one dimension until the density increases to a value sufficient to cause collapse the field lines. In this connection, many investigators have discussed the gravitational instability of a homogeneous plasma considering the effect of magnetic field incorporating various

parameters. [Pensia et al.¹¹, Mestel and Spitzer¹², Mestel¹³, Spitzer¹⁴⁻¹⁵, Pensia et al.¹⁶]

A Partially-ionized plasma represents a state which often exists in the universe. The interaction between the natural and the ionized gas components becomes important in the cosmic sphere. The effect of the ion Larmor radius and that of collisions between natural and ionized particles of partially-ionized plasma on the hydro magnetic waves propagating transverse to the main magnetic field has been investigated by Herrengger¹⁶. In this direction, Chhajlani & Parihar¹⁷ investigated this problem incorporating various parameters. In mentioned above studies none of the authors has incorporated the arbitrary radiative heat-loss function.

Along with this, the problem of thermal instability arising owing to various heat mechanism of interstellar matter is of considerable importance in connection with astrophysical condensation and the formation of prominences through condensation of coronal material. In this direction Field¹⁸ investigate the phenomenon of thermal instability arising due to heat-loss mechanisms in dilute plasma. In the recent ISM observations, it has been established as a fact that the radiative heat-loss mechanism plays an important role in the star formation in molecular cloud condensation process in connection with thermal instability. The ISM structure shows that the heat-loss process in major cause for the condensation of large astrophysical compact objects. Basically this radiative heat-loss function shows the decay of heat in an embedded system with respect to local temperature and density. These functions are similar to those of the cooling function considered earlier by Wolfier et al.¹⁹ and Shadmehri & Dib²⁰ for the HII region it leads to several important phenomenon of astrophysics and space plasma physics

From the above studies, we find that thermal conductivity, interaction between two components of partially-ionized plasma and arbitrary radiative heat-loss functions are the important parameters to discuss the gravitational instability of partially-ionized plasma. Thus in present problem, we investigate the effects of arbitrary radiative heat-loss functions and thermal conductivity on the Jeans instability of partially-ionized plasma.

2. Linearized Perturbation Equations

We assume that the two components of the partially-ionized plasma (the ionized fluid and the neutral gas) behave like a continuum fluid and their state velocities are equal. The effects of the magnetic field, field of gravity and the pressure on the neutral components are neglected. Also it is assumed that the frictional force of the neutral gas on the ionized fluid is of the same

order as the pressure gradient of the ionized fluid. Thus, we are considering only the mutual frictional effects between the neutral gas and the ionized fluid. It is assumed that the above medium is permeated with a uniform magnetic field $\vec{H}(0,0,H)$. Thus the linearized perturbation equations governing the motion of hydromagnetic thermally conducting two components of the partially-ionized plasma are given by

$$(2.1) \quad \rho \frac{\partial \vec{u}}{\partial t} = -\vec{\nabla} \delta p + \vec{\nabla} \delta \varphi + \frac{1}{4\pi} (\nabla \times \vec{h}) \times \vec{H} + \rho_d v_c (\vec{u}_d - \vec{u}),$$

$$(2.2) \quad \frac{\partial \vec{v}_d}{\partial t} = -v_c (\vec{u}_d - \vec{u}),$$

$$(2.3) \quad \frac{\partial \delta \rho}{\partial t} = -\rho \vec{\nabla} \cdot \vec{u},$$

$$(2.4) \quad \nabla^2 \delta \varphi = -4\pi G \delta \rho,$$

$$(2.5) \quad \frac{1}{(\gamma-1)} \frac{\partial \delta \rho}{\partial t} - \frac{\gamma}{(\gamma-1)} \frac{p}{\rho} \frac{\partial \delta \rho}{\partial t} + \rho (\mathcal{E}_\rho \delta \rho + \mathcal{E}_T \delta T) - \lambda \nabla^2 \delta T = 0,$$

$$(2.6) \quad \frac{\delta p}{p} = \frac{\delta T}{T} + \frac{\delta \rho}{\rho},$$

$$(2.7) \quad \frac{\partial \vec{h}}{\partial t} = \vec{\nabla} \times (\vec{u} \times \vec{H}),$$

$$(2.8) \quad \vec{\nabla} \cdot \vec{h} = 0.$$

The parameters $\varphi, G, \lambda, R, c, \rho, \rho_d, v_c, \gamma, T$, and p denote the gravitational potential, gravitational constant, thermal conductivity, gas constant, velocity of light, density of ionized component, density of neutral components ($\rho \gg \rho_d$), collision frequency between two components, ratio of two specific heats, temperature and pressure, respectively.

The perturbations in fluid velocity, fluid pressure, fluid density, magnetic field, gravitational potential, temperature and the radiative heat-loss function are given as $\vec{u}(u_x, u_y, u_z)$, $\delta p, \delta \rho, h(h_x, h_y, h_z)$, $\delta \phi, \delta T$, and $\delta \mathcal{E}$ respectively.

In equation (2.5), $\mathcal{E}_{\rho, T}$ are the partial derivatives of the density-dependent $(\partial \mathcal{E} / \partial \rho)_T$ and temperature-dependent $(\partial \mathcal{E} / \partial T)_\rho$ heat-loss functions respectively.

3. Dispersion Relation

We assume that all the perturbed quantity vary as

$$(3.1) \quad \text{Exp. } \{i(k_x x + k_z z + \omega t)\},$$

where ω is the frequency of harmonic disturbances, $k_{x,z}$ are wave numbers in x and z direction, respectively, such that $k_x^2 + k_z^2 = k^2$ combining equation (2.5) and (2.6), we obtain the expression for δp as

$$(3.2) \quad \delta p = \left(\frac{\alpha + \sigma c^2}{\sigma + \beta} \right) \delta \rho,$$

where $\sigma = i\omega$, $c = \left(\frac{\gamma p}{\rho} \right)^{1/2}$ is the adiabatic velocity of sound in the medium

$$(3.3) \quad \alpha = (\gamma - 1) \left(\mathcal{E}_T T - \mathcal{E}_\rho + \frac{\lambda k^2 T}{\rho} \right), \quad \beta = (\gamma - 1) \left(\frac{\mathcal{E}_T T \rho}{p} + \frac{\lambda k^2 T}{p} \right).$$

Using equation (2.2) – (3.3) in equation (2.1), we obtain the following algebraic equations for the amplitude components

$$(3.4) \quad \left(\sigma^2 + k_z^2 V^2 + \frac{B v_c}{\sigma + v_c} \sigma^2 \right) u_x + \frac{i k_x}{k^2} \sigma \Omega_T^2 s = 0,$$

$$(3.5) \quad \left(\sigma^2 + k_z^2 V^2 + \frac{B v_c}{\sigma + v_c} \sigma^2 \right) u_y + \frac{i k_x}{k^2} \sigma \Omega_T^2 s = 0,$$

$$(3.6) \quad \left(\sigma + \frac{B v_c}{c + v_c} \right) u_z + \frac{i k_z}{k^2} \Omega_T^2 s = 0,$$

Taking divergence of equation (2.1) with addition of equation (2.2) – (3.3), we obtain

$$(3.7) \quad (ik_x k^2 V^2) u_x - \left(\sigma^3 + \frac{B v_c}{\sigma + v_c} \sigma^3 + \sigma \Omega_T^2 \right) s = 0,$$

where $s = \frac{\delta \rho}{\rho}$ is the condensation of the medium, $V = \frac{H}{(4\pi\rho)^{1/2}}$ is the Alfven velocity, $c^2 = \gamma c'^2$ where c and c' are the adiabatic and isothermal velocities of sound. Also we have assumed the following substitutions

$$B = \frac{\rho d}{\rho}, \quad \Omega_T^2 = \frac{\sigma \Omega_j^2 + \Omega_l^2}{\sigma + \beta}, \quad \Omega_l^2 = k^2 \alpha - 4\pi G \rho, \quad \Omega_j^2 = k^2 c^2 - 4\pi G \rho,$$

$$a = \sigma + \frac{B v_c}{\sigma + v_c} \sigma, \quad M = a \sigma^2 + \sigma \Omega_T^2, \quad E' = a \sigma^2 + k_z^2 V^2,$$

$$R_1 = (1 + B) v_c, \quad \Omega_k = \frac{\gamma \lambda k^2}{\rho c \rho}, \quad E = \sigma a + k^2 V^2.$$

The non-trial solution of the determinant of the matrix obtained from equation (3.4) to (3.7) with u_x, u_y, u_z, S having various coefficients that should vanish is to give the following dispersion relation:

$$(3.8) \quad EE' a M + E' a k_x^2 V^2 \sigma \Omega_T^2 = 0.$$

The dispersion relation (3.8) shows the combined influence of thermal conductivity and arbitrary radiative heat-loss functions on the self-gravitational instability of a two components of the partially-ionized plasmas we find that in this dispersion relation the terms due to the arbitrary radiative heat-loss function with thermal conductivity have entered through the factor Ω_T^2 .

4. Analysis of the dispersion relation

4.1. Longitudinal mode of propagation (K||H)

For this case we assume all the perturbations longitudinal to the direction of the magnetic field i.e. $(k_z = k, k_x = 0)$. Thus the dispersion relation reduces in the simple form to give

$$(4.1) \quad \left(\sigma + \frac{Bv_c}{\sigma + v_c} \sigma \right) \left(\sigma^3 + \frac{Bv_c \sigma^3}{\sigma + v_c} + \sigma \Omega_T^2 \right) \left(\sigma^2 + \frac{Bv_c \sigma^2}{\sigma + v_c} + k^2 V^2 \right) = 0.$$

we find that in the longitudinal mode of propagation the dispersion relation is modified due to the pressure of neutral particles, thermal conductivity and arbitrary radiative heat-loss functions. This dispersion relation has three independent factors, each represents the mode of propagation incorporating different parameters. The first factor of this dispersion relation equating to zero, we get

$$(4.2) \quad \sigma + (1 + B)v_c = 0.$$

Equation (4.2) cannot have a real positive root hence it satisfies the necessary and sufficient condition of stability. Thus equation (4.2) represents stable damped mode due to collision frequency. It is evident from the equation (4.2) that this mode is independent of the magnetic field, thermal conductivity, self gravitation and arbitrary radiative heat-loss function.

The second factor of equation (4.1) equating to zero, we obtain following dispersion relations

$$(4.3) \quad \sigma^4 + \sigma^3(R_1 + \beta) + \sigma^2[(1 + B)\beta v_c + \Omega_j^2] + \sigma[v_c \Omega_j^2 + \Omega_I^2] + v_c \Omega_I^2 = 0.$$

This dispersion relation for self-gravitating fluid incorporated effect of neutral particles, thermal conductivity and arbitrary radiative heat-loss function. It is evident from equation (4.3) that the condition of instability is independent of magnetic field. The dispersion relation (4.3) is a fourth degree equation which may be reduced to particular cases so that the effect of each parameter is analyzed separately.

For thermally non-conducting, non-radiating, fully ionized fluid we have $\alpha = \beta = v_c = 0$ the dispersion relation (4.3) reduces to

$$(4.4) \quad \sigma^2 + \Omega_j^2 = 0,$$

It is clear from equation (4.4) that when $\Omega_j^2 < 0$, the product of the root of at least one root of σ is positive. Hence, the system is unstable. Thus for the cases of equation (4.4) the condition of instability is

$$(4.5) \quad \begin{aligned} \Omega_j^2 &= (c^2 k^2 - 4\pi G \rho) < 0, \\ k &< k_j = \left(\frac{4\pi G \rho}{c^2} \right)^{1/2}, \end{aligned}$$

where k_j is the Jeans wave number. Equation (4.5) is original Jeans expression for instability. The Jeans length is given as

$$(4.6) \quad \lambda_j = c \left(\frac{\pi}{G\rho} \right)^{1/2}.$$

The fluid is unstable for all Jeans length $\lambda > \lambda_j$, of Jeans wave number $k < k_j$. It is evident from equation (4.5) that Jeans criterion of instability remains unchanged in the presence of neutral particles.

For non-arbitrary radiative heat-loss function but thermally conducting and self-gravitating fluid having neutral particles, the dispersion relation (4.3) reduces to

$$(4.7) \quad \sigma^4 + \sigma^3(R_1 + \Omega_k) + \sigma^2[R_1 \Omega_k + \Omega_{j1}^2] \sigma \{ \Omega_{j1}^2 (v_c + \Omega_k) \} + v_c \Omega_k \Omega_j^2 = 0,$$

where $c' = (p/\rho)^{1/2}$ is the isothermal velocity of the sound.

$$\Omega_{j1}^2 = c'^2 k^2 - 4\pi G\rho \quad \text{and} \quad \Omega_k = \frac{\lambda \gamma k^2}{\rho c \rho}.$$

It is clear from the constant term of equation (4.7) that the system leads to instability if $\Omega_{j1}^2 < 0$, which gives

$$c'^2 k^2 - 4\pi G\rho < 0 \quad \text{or}$$

$$(4.8) \quad k < k_{j1} = \left(\frac{4\pi G\rho}{c'^2} \right)^{1/2},$$

where k_{j1} is the modified Jeans wave number for thermally conducting system and the corresponding modified Jeans length is given as

$$(4.9) \quad \lambda_{j1} = c' \left(\frac{\pi}{G\rho} \right)^{1/2}.$$

Comparing equation (4.5) and (4.8) we see that owing to thermal conduction, the sonic velocity is changed from adiabatic to isothermal one is Jeans expression. From equation (4.6) and (4.9) we obtain

$$(4.10) \quad \lambda_{j1} = \lambda_j \left(\frac{1}{\gamma} \right)^{1/2}.$$

It is clear from equation (4.10) that the Jeans length is reduced due to thermal conduction [as $\gamma > 1$], thus the system is destabilized. If we consider self-gravitating and thermally non-conducting plasma incorporated with neutral particles, arbitrary radiative heat-loss function then the condition of instability is given as

$$k, k_{j_2} = k_j \left(\frac{\gamma \mathfrak{E}_T}{\mathfrak{E}_T - \rho \frac{\mathfrak{E}_\rho}{T}} \right)^{1/2},$$

where k_{j_2} is the modified critical wave number due to inclusion of arbitrary radiative heat-loss function. Hence, for wave number $k < k_{j_2}$, the system is unstable, therefore, the system begins to break in to fragments of size comparable to

$$\lambda_{j_2} \left(= \frac{2\pi}{k_{j_2}} \right).$$

Comparing equation (4.5) and (4.11) we find that the critical wave number k_{j_2} is very much different from the original Jeans wave number k_j and k_{j_2} depends on derivatives of the arbitrary radiative heat-loss function with respect to local temperature $\mathfrak{E}_T$ and local density $\mathfrak{E}_\rho$ in the configuration. It is clear from equation (4.11) that when the arbitrary radiative heat-loss function is independent of density of the configuration (i.e. $\mathfrak{E}_\rho = 0$), then $k_{j_2} = k_j$ i.e., critical wave number remains unaffected and if the arbitrary radiative heat-loss function is independent of temperature ($\mathfrak{E}_T = 0$), then k_{j_2} vanishes.

The condition of instability of the system, when combined effect of all the parameters represented by the original dispersion relation (19) is given as

$$(4.12) \quad \Omega_1^2 = \left[\frac{k^2(\gamma-1) \left(T \mathfrak{E}_T - \rho \mathfrak{E}_\rho \frac{\lambda k^2 T}{\rho} \right) - 4\pi G \rho (\gamma-1)}{\left(\frac{T \rho \mathfrak{E}_T}{p} + \frac{T \lambda k^2}{p} \right)} \right] < 0,$$

It is evident from condition of instability of instability (4.10) that the Jeans criterion of instability is modified due to inclusion of thermal conductivity and arbitrary radiative heat-loss function. The condition (4.12)

is same as equation (4.6) earlier obtained by Bora and Talwar²¹ for infinitely conducting medium.

From inequality (4.12) the range of critical wave number k_{j3} is given as

$$(4.13) k_{j3} = \frac{1}{2^{1/2}} \left[\left\{ \frac{4\pi G \rho}{c'^2} + \frac{\rho^2 \mathfrak{E}_\rho}{\lambda T} - \frac{\rho \mathfrak{E}_T}{\lambda} \right\} \pm \left\{ \left(\frac{4\pi G \rho}{c'^2} + \frac{\rho^2 \mathfrak{E}_\rho}{\lambda T} - \frac{\rho \mathfrak{E}_T}{\lambda} \right)^2 + \frac{16\pi G \rho^2 \mathfrak{E}_T}{\lambda c'^2} \right\}^{1/2} \right]^{1/2},$$

If inequalities (4.13) is applied for a purely temperature dependent arbitrary radiative heat-loss function ($\mathfrak{E}_\rho = 0$), increases with temperature ($\mathfrak{E}_T = 0$), then the condition of monotonic instability is given as $k < k_{j1}$.

However, if instead the arbitrary radiative heat-loss function decreases with temperature ($\mathfrak{E}_T < 0$), the instability arises for k^2 lying between the values

$$[\mathfrak{E}_T/\lambda] \text{ and } \left[\frac{4\pi G \rho}{c'^2} \right] \text{ for parallel propagation.}$$

Furthermore, if it is considered that the arbitrary radiative heat-loss function is purely density-dependent ($\mathfrak{E}_T = 0$) then the condition of instability is given as

$$(4.14) \quad k, k_{j4} = \left(\frac{4\pi G \rho}{c'^2} + \frac{\rho^2 \mathfrak{E}_\rho}{\lambda T} \right)^{1/2}.$$

It is evident from inequality (4.14) that the critical wave number is increased or decreased, depending on whether the arbitrary radiative heat-loss function is an increasing or decreasing function of the density.

Now we examine the dynamical stability of the fluid by applying the Routh-Hurwitz criterion on the dispersion relation (4.3). If $\Omega_l^2 > 0$ and $\Omega_j^2 > 0$, then all the coefficient of the equation (4.3) are positive, which is a necessary condition for the stability of the system. To obtain the sufficient condition, the principal diagonal minors of the Hurwitz must be positive and obtain as

$$\Delta_1 = (R_1 + \beta) > 0 \quad \text{as} \quad \gamma > 1,$$

$$\Delta_2 = [\beta R_1 (R_1 + \beta) + B \Omega_j^2 v_c + k^2 \alpha] > 0,$$

$$\begin{aligned}\Delta_3 = & \{v_c \Omega_j^2\} \{k^2 \alpha + v_c \beta \Omega_j^2\} \\ & + \Omega_j^2 \{(1+B)k^2 \alpha\} \{B\beta v_c\} \{R_1 + \beta\} \{Bv_c + \beta\} + k^2 \alpha \\ & + v_c \beta \Omega_j^2 + v_c^2 [R_1 + \beta] > 0,\end{aligned}$$

$$\Delta_4 = v_c \Omega_I^2 \Delta_3 > 0.$$

These all Δ 's are positive, thereby, satisfying the Routh-Hurwitz criterion, according to which equation (4.3) will not include any positive real root of $\sigma(i\omega)$ or a complex root whose real part is positive. Hence, the system expressed by equation (4.3) is stable if $\Omega_j^2 > 0$ and $\Omega_I^2 > 0$.

Now equating zero the third factor of equation (4.1) and after solving we obtain dispersion relation as

$$(4.15) \quad \sigma^6 + \sigma^5 A_1 + \sigma^4 A_2 + \sigma^3 A_3 + \sigma^2 A_4 + \sigma A_5 + A_6 = 0.$$

$$A_1 = 2R_1,$$

$$A_2 = 2V^2 k^2 + R_1^2,$$

$$A_3 = 2V^2 k^2 (R_1^2 + v_c),$$

where

$$A_4 = V^4 k^4 + R_1 [4V^2 k^2 v_c],$$

$$A_5 = V^4 k^4 v_c,$$

$$A_6 = V^4 k^4 v_c^2.$$

This dispersion relation shows the combined influence of magnetic field and the effect of the neutral particles, but this mode is independent of thermal conductivity, arbitrary radiative heat-loss function and self gravitation. Since the coefficient of the equation (4.15) are all positive including the constant term, therefore, this equation cannot have a positive roots which means that system represented by equation (4.15) is stable. Thus, this equation gives Alfvén mode modified by the dissipative effect of the neutral particles.

In the absence of neutral particles, the dispersion relation (4.15) reduces to

$$(4.16) \quad \sigma^4 + 2\sigma^2 V^2 k^2 + V^4 k^4 = 0.$$

It is evident from equation (32) that

$$\begin{aligned}
 \sigma_{1,2}^2 &= -V^2 k^2 - \omega_{1,2}^2 \\
 (4.17) \quad &= -V^2 k^2 \omega_{1,2} \\
 &= \pm V k.
 \end{aligned}$$

Thus, we see in equation (4.17), the two Alfven waves moving in opposite direction.

4.2 Transverse propagation

For this case we assume all the perturbations are propagating perpendicular to the direction of the magnetic field, for, our convenience, we take $k_x = k$, and $k_z = 0$, the general dispersion relation (3.8) reduces to

$$(4.18) \quad a^2[\sigma^2 a(\sigma a + k^2 V^2) + \sigma a \Omega_T^2] = 0.$$

This is the general dispersion relation for transverse propagation shows the combined influence of magnetic field, self-gravitation, thermal conductivity, presence of neutral particles and arbitrary radiative heat-loss function on the self-gravitational instability of a two components of the partially-ionized plasmas.

Dispersion relation (4.18) has two distinct factors, each represents different mode of propagation when equated to zero, separately. The first mode is same as discussed in the dispersion relation (4.2). The second factor of equation (4.18) equating zero and substituting the values of $\mathbf{a}$ and Ω_T^2 we get

$$\begin{aligned}
 (4.19) \quad &\sigma^5 + \sigma^4[2R_1 + \beta] + \sigma^3[\Omega_j^2 + k^2 V^2 + 2\beta R_1 + R_1^2] + \sigma^2[\Omega_T^2 + 2v_c R_1 \Omega_j^2 \\
 &+ k^2 V^2 (R_1 + v_c + \beta) + R_1 \beta (R_1 + v_c)] + \sigma[\Omega_T^2 (R_1 + v_c) + v_c R_1 \Omega_j^2 \\
 &+ k^2 V^2 R_1 v_c + \beta k^2 V^2 (R_1 + v_c)] + R_1 v_c (\beta k^2 V^2 + \Omega_T^2) = 0.
 \end{aligned}$$

This dispersion relation for transverse propagation shows the combined influence of thermal conductivity, arbitrary radiative heat-loss function on the self-gravitational instability of two components partially-ionized plasmas with the effect of neutral particles. The condition of instability is obtained from dispersion relation (4.19) as

$$(4.20) \quad \beta k^2 V^2 + k^2 \alpha - 4\pi G \rho \beta < 0.$$

This is the modified Jeans condition of instability due to magnetic field, thermal conductivity and the arbitrary radiative heat-loss function.

If the fluid expressed by equation (4.19) does not contain arbitrary radiative heat-loss function then the critical wave number below which the

system is unstable is obtained from the constant terms of equation (4.19) and is given by

$$(4.21) \quad k_{j5}^2 = \frac{\gamma k_j^2}{\left(1 + \frac{u^2}{c^2}\right)},$$

If the arbitrary radiative heat-loss functions are included in a thermally non-conducting configuration, the corresponding value of critical wave number is given by

$$(4.22) \quad k_{j6}^2 = \gamma k_j^2 \left[\frac{T \mathfrak{L}_T}{T \mathfrak{L}_T \left(1 + \frac{u^2}{c^2}\right) - \mathfrak{L}_\rho \rho} \right],$$

The disturbances with a wave number $k < k_{j6}$ are unstable, where for $k > k_{j6}$, the disturbances are stable. Thus it is clear that the configurations becomes unstable for all perturbation wave numbers if

$$(4.23) \quad T \mathfrak{L}_T > \frac{\rho \mathfrak{L}_\rho}{\left(1 + \frac{u^2}{c^2}\right)},$$

It is clear from equation (4.22) that the critical wave number vanishes if the arbitrary radiative heat-loss functions are independent of temperature $\mathfrak{L}_T = 0$, and if the arbitrary radiative heat-loss functions are independent of density of the medium then the Jeans critical wave number is given by equation (4.21). In the general case when both the arbitrary radiative heat-loss functions and thermally conductive effect are present simultaneously then it can be seen by a little simplification of equation (4.20) that the critical Jeans wave number is given by

$$(4.24) \quad 2 \left(1 + \frac{u^2}{c^2}\right) k_{j7}^2 = \left[\left(\frac{4\pi G \rho}{c^2} + \frac{\rho^2 \mathfrak{L}_\rho}{\lambda T} - \frac{\rho \mathfrak{L}_T}{\lambda} \left(1 + \frac{u^2}{c^2}\right) \right) \pm \left\{ \left(\frac{4\pi G \rho}{c^2} + \frac{\rho^2 \mathfrak{L}_\rho}{\lambda T} - \frac{\rho \mathfrak{L}_T}{\lambda} \left(1 + \frac{u^2}{c^2}\right) \right)^2 + \frac{16\pi G \rho}{\lambda c^2} \left(1 + \frac{u^2}{c^2}\right) \right\} \right]^{1/2}.$$

The medium is unstable for wave number $k < k_{j7}$. It may be noted here that the critical wave number involves, derivative of temperature-dependent and density-dependent arbitrary radiative heat-loss function, thermal conductivity of the medium and the magnetic field.

5. Conclusion

We have developed a model of wave propagation in an infinite self-gravitating magnetized two components of partially-ionized system incorporating thermal conductivity and arbitrary radiative heat-loss functions. The general dispersion relation is obtained, which is modified due to the presence of these parameters. This dispersion relation is reduced for longitudinal and transverse modes of propagation. We find that the Jeans condition remains valid but the expression of the critical Jeans wave number is modified. Owing to the inclusion of the thermal conductivity, the isothermal sound velocity is replaced by the adiabatic velocity of sound. The effect of the collision with neutrals does not affect the instability condition of the system in both the longitudinal and transverse mode of propagation.

In the case of longitudinal propagation, we obtain Alfven mode modified by the collisions with the neutrals. The thermal conductivity has a destabilizing influence. It is also found that the density-dependent heat-loss function has a destabilizing influence on the instability of the system.

In the transverse mode of propagation, we find a gravitating thermal mode influenced by thermal conductivity and arbitrary radiative heat-loss functions. We also found the condition of instability and the expression of critical Jeans wave number both are modified due to the presence of magnetic field, thermal conductivity and arbitrary radiative heat-loss function. It is found that radiative critical wave number is same as original Alfven critical wave number when the arbitrary radiative heat-loss functions are independent of density of the medium.

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