

Mixed Block Domination in Graphs

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(Received December 22, 2010)

Abstract. Let $B(G)$ denotes the set of all blocks of a graph G . A vertex $v \in V$ and a block $b \in B(G)$ is said to block dominate (b-dominate) each other if v is in the block b . A set $D \subseteq V$ is said to be a *vertex block dominating set* (VBD set) if every block in G is b-dominated by some vertex in D . The *vertex - block domination number* $\gamma_{vb} = \gamma_{vb}(G)$ is the minimum cardinality of a block dominating set of G . Similarly a set $F \subseteq B(G)$ is said to be a *block - vertex dominating set* (BVD set) if every vertex in G is b-dominated by some block in F . The *block vertex domination number* $\gamma_{bv} = \gamma_{bv}(G)$ is the minimum cardinality of a block vertex dominating set of G . In this paper we initiate a study of these new varieties of mixed block domination and obtain few bounds on the new parameter.

Key words: Vertex block domination, block vertex domination, Block tree, Block degree.

2010 AMS Subject Classification No.: 05C69

1. Introduction

For any undefined terminologies we refer^{1,2}. By a graph $G(p, q)$ we mean a connected finite simple graph with p vertices and q edges. A maximal connected nonseparable subgraph is called a block of G . A maximal complete subgraph is a *clique*. Let $B(G)$ denotes the set of all blocks of G . Since edges of any graph can be partitioned into blocks of G , the study of graphs through their blocks of G is quite natural and was initiated in the early ages since the origin of graph theory. We find several characterization of graphs in literature using blocks of a graph. In³ we

observe that a block behaves like an edge with multiple vertices. A vertex $v \in V$ is a cut point if $G - v$ is disconnected. Let $C(G)$ denotes the set of all cutpoints of G . If a block $b \in B(G)$ contains a cutpoint $c \in C(G)$ then we say that b and c are incident to each other. Two blocks b_1 and b_2 are adjacent if there is a common cutpoint incident with b_1 and b_2 . Again two cut points c_1 and c_2 are adjacent if there is a common block incident with c_1 and c_2 . A block graph $B_G(G)$ is a graph with vertex set $B(G)$ and two blocks b_1 and b_2 are adjacent in $B_G(G)$ if and only if there is a common cut point incident with b_1 and b_2 . A cut point graph $C_G(G)$ is a graph with vertex set $C(G)$ and any two cut points c_1 and c_2 are adjacent in $C_G(G)$ if and only if there is a common block incident with c_1 and c_2 . It is observed that $B_G(B_G(G)) = C_G(G)$. Further a *block cut point graph* $BC(G)$ is a bigraph with vertex set $B(G) \cup C(G)$ and a cut point $c \in C(G)$ and a block $b \in B(G)$ are adjacent if and only if c is incident with the block b . A block b is a *pendant block* if it is incident with only one cutpoint, otherwise b is a *nonpendant block*.

Remark 1. *Number of cutpoints in G = number of blocks in $B_G(G)$ = number of vertices in $C_G(G)$.*

To acquaint the new terminologies and definitions, we give some preliminary results on blocks discussed³.

1.1 Block walks and Block Paths and Block Trees

A *block-walk* (B-walk) is a sequence of blocks and cutpoints say, $b_1, c_2, b_3, c_4, b_5, \dots, b_{m-2}, c_{m-1}, b_m$ beginning and ending with blocks in which each cutpoint c_i is incident with the blocks b_{i-1}, b_{i+1} . The *length* of a block walk is the number of cut points in a block walk. If all the cut points are distinct (and hence all the blocks are distinct) in a block walk, then such a walk is called a *block path*. A block path with m blocks is denoted as B_{pm} . Any block path B_{pm} contains $m-1$ cut points.

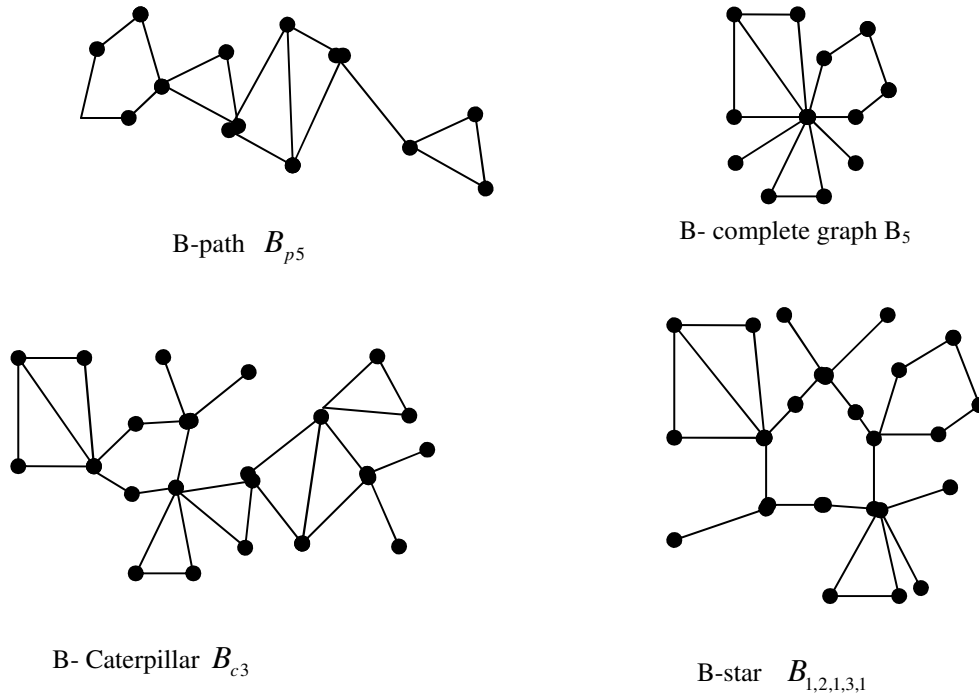


Fig. 1.1

Let V_{nc} be the non-cutpoints in the block b and X_b be the edge set of the block $b \in B(G)$. Then $G - b$ is a graph with vertex set $V - V_{nc}$ and the edge set $X - X_b$. Note that by our definition of block removal, if $b = x$ is a pendant edge then $G - b \neq G - x$. A graph G is said to be block connected if there is a block path between any two blocks of G . We observe that if b is a nonpendant block of G then $G - b$ is a disconnected graph. If d is a pendant block, then $G - d$ remains connected.

A graph G is a *B-Caterpillar* B_{cm} , if removal of all pendant blocks results in a B-path B_{pm} where B_{pm} is some times called *B-spine* of G . A graph G with m blocks is *B-complete* graph B_m if any two blocks of G are adjacent. In particular, a star $K_{1,m} = B_m$ is a B-complete graph. A graph G is a *B-star* denoted by $B_{m_1, m_2, \dots, m_k}$ if there exists a block b with k cutpoints and i^{th} cutpoint is incident with $m_i + 1$ blocks, $m_i \in N, 1 \leq i \leq k$. Therefore any B-star has $\left(1 + \sum_{i=1}^k m_i\right)$ blocks. The block b is then called the central block. B-star is a *flower* if the central block does not contain any

noncutpoint. B-path with 5 blocks, B_{P_5} B-complete graph B_5 , B-star $B_{1,2,1,3,1}$ and B- caterpillar B_{c_3} are shown in the Fig.1.1

A graph G is said to be a *block tree* if there exists a unique block path between any two blocks of a graph.

Theorem³ 1.1. *Every connected graph is a block tree.*

1.2 Block degree and Cutpoint degree

The different types of block degrees are defined³. *vb-degree* (vertex block-degree) $d_{vb}(u)$ of a vertex u , is the number of blocks incident on u . For any noncutpoint $w \in V$, $d_{vb}(w) = 1$ and for any cut point $c \in C(G)$, $d_{vb}(c) \geq 2$. Similarly, *bv-degree* (block vertex-degree) of a block f , $d_{bv}(f)$ is the number of vertices in the block f and *be-degree* (block edge-degree) of a block f , $d_{be}(f)$ is the number of edges in the block f . *bb-degree* (block block-degree) of a block h , $d_{bb}(h)$ is the number of blocks adjacent to h . Finally, *cutpoint-degree* of a block b , $d_c(b)$ is the number of cutpoints incident on b . If b is a *pendant block* then $d_c(b) = 1$. For any nonpendant block $d_c(b) \geq 2$. Let $\Delta_{vb}(G)$ and $\delta_{vb}(G)$ denote the maximum and minimum *vb-degrees* of G . Then $\Delta_{bv}(G), \delta_{bv}(G), \Delta_{be}(G), \delta_{be}(G), \Delta_{bb}(G), \delta_{bb}(G), \Delta_c(G)$ and $\delta_c(G)$ are similarly defined. One can find a study of several kinds of degree concepts in graphs^{4,5,6}. The following results appear^{7,8}. Those results in our terminologies read as follows.

Proposition 1.2.1 . *For any graph G , with m blocks and n cut points,*

$$(1.1) \quad \sum_{u \in V} (d_{vb}(u) - 1) = m - 1 \quad \text{Harary}^8$$

$$(1.2) \quad \sum_{h \in B(G)} (d_c(h) - 1) = c - 1 \quad \text{Gallai}^7$$

Surekha and P. G. Bhat³ proved that sum of *vb-degree* of all cutpoints and sum of *cutpoint degree* of all blocks are equal.

Proposition³ 1.2.2. *For any graph G , with m blocks and n cut points,*

$$(1.3) \quad \sum_{c \in C(G)} d_{vb}(c) = \sum_{h \in B(G)} d_c(h) = m + n - 1.$$

After defining B-path it is natural to define block distance between two blocks. We present some results on Block distance.

2. B-distance between Two Blocks

The *length* of a B-path is the number of cutpoints in it. For any two blocks $b_1, b_2 \in B(G)$, the *B-distance* $d(b_1, b_2)$ is the length of the B-path from b_1 to b_2 . Further, $d(b_1, b_2)$ has the following properties:

- (i) $d(b_1, b_2) \geq 0$ and $d(b_1, b_2) = 0$ if and only if $b_1 = b_2$.
- (ii) $d(b_1, b_2) = d(b_2, b_1)$,
- (iii) $d(b_1, b_2) \leq d(b_1, b_3) + d(b_3, b_2)$.

Using B-distance we can define the central tendencies of a block, like B-radius and B-diameter of a block in a graph. The *B-eccentricity* of a block $h \in B(G)$ is defined as $e(h) = \max_{g \in B(G)} d(h, g)$. Then *B-diameter* $d_b(G) = \max_{h, g \in B(G)} e(h)$ and *B-radius* $r_b(G) = \min_{h \in B(G)} e(h)$. The set of blocks with minimum eccentricity is called *B-center* $C_b(G)$ of the graph. Immediately these parameters yield a lower bound for the usual diameter $d(G)$ and radius $r(G)$ of a graph. Hence we have $d_b(G) \leq d(G)$ and $r_b(G) \leq r(G)$ and equality holds if G is a tree.

Proposition 2.1. *Let G be a graph with B-diameter $d_b(G)$ and B-radius $r_b(G)$. Let $B_G(G)$ and $C_G(G)$ be the block graph and cutpoint graph of G . Then*

$$(2.1) \quad r_b(G) \leq d_b(G) \leq 2r_b(G),$$

$$(2.2) \quad d_b(G) = d_b(B_G(G)) + 1,$$

$$(2.3) \quad d_b(G) = d_b(C_G(G)) + 2,$$

$$(2.4) \quad r_b(G) \leq r_b(B_G(G)) + 1,$$

$$(2.5) \quad r_b(G) = r_b(C_G(G)) + 1.$$

Proof. Equation (2.1) follows from the fact that $r_b(G) = \left\lceil \frac{d_b(G)}{2} \right\rceil$.

To prove (2.2). Consider the diametrical B-path in G . Then

$$\begin{aligned} d_b(G) &= \text{number of cutpoints in the diametrical B-path in } G \\ &= \text{number of blocks in the diametrical B-path in } B_G(G) \text{ (using Remark 1)} \\ &= \text{number of cutpoints in the diametrical B-path in } B_G(G) + 1 = d_b(B_G(G)) + 1. \end{aligned}$$

Since $B_G(B_G(G)) = C_G(G)$ equation (2.3) follows from (2.2).

To prove (2.4). Again from (2.1) and (2.2), we have

$$r_b(G) = \left\lceil \frac{d_b(G)}{2} \right\rceil = \left\lceil \frac{(d_b(B_G(G)) + 1)}{2} \right\rceil \leq \left\lceil \frac{d_b(B_G(G))}{2} \right\rceil + 1 = r_b(B_G(G)) + 1$$

as desired.

Also from (2.3) we have, $d_b(G) = d_b(C_G(G)) + 2$.

$$\text{Hence } r_b(G) = \left\lceil \frac{d_b(G)}{2} \right\rceil = \left\lceil \frac{d_b(C_G(G))}{2} \right\rceil + 1 = r_b(C_G(G)) + 1 \text{ which yields (2.5).}$$

A *skeleton* of G is obtained by deleting all the pendant blocks from G . An i^{th} skeleton $S_i(G)$ is obtained iteratively by getting skeleton of G , i times. Therefore $G = S_0(G)$. If a vertex v is a cutpoint both in G and $S_1(G)$ then v is a *nonend-cutpoint* of G , otherwise v is an *end-cutpoint* of G .

Proposition 2.2. *For any graph G with m blocks the block center $C_b(G)$ is B-complete.*

Proof. *Case 1.* Suppose $m = 1$. Then $d_b(G) = r_b(G) = 0$. Hence $C_b(G) = B_1$ and the result follows.

Case 2. Suppose $m > 1$ and $G = B_m$. Then $d_b(G) = r_b(G) = 1$. Hence $C_b(G) = B_m$ and the result follows.

Case 3. Suppose $m > 1$ and G is not a B-complete graph.

Claim: *G has atleast two pendant blocks and two end-cutpoints.*

Consider the diametrical B-path in G . Then the two blocks with maximum B-eccentricity are the required two pendant blocks and the cutpoint incident on these two blocks are the required two end-cutpoints. Hence our claim. Now obtain the skeleton $S_1(G)$. Then all the blocks are at B-distance one less than the B-distance in the original graph. Also the blocks which were in the block center in the original graph remain same in the new graph. Hence $C_b(G) = C_b(S_1(G))$. Let k_1 be the number of blocks in $S_1(G)$. Check whether $S_1(G)$ is B-complete and if so, from case 1 and case 2, the result follows and we stop. If $S_1(G)$ is not B-complete, we get $S_2(G)$. We repeat the process of getting skeleton $S_{i+1}(G)$ from $S_i(G)$ iteratively, if and only if $S_i(G)$ is not B-complete. After say n steps we get $S_n(G)$ which is B-complete with k_n blocks and $C_b(G) = C_b(S_n(G)) = S_n(G)$ which is B-complete. Then by case 1 and case 2, the result follows. This completes the proof.

From the above proof we can get an algorithm to get B-centre of G which eventually gives the B-radius of G . Note that if we take skeleton of G , i times implies that block radius of G is $i + 1$.

$i = 0$

$count = m$

$k = count$

begin

while $(S_i(G) \neq B_k)$

$S_i(G) = S_{i+1}(G)$

$i = i + 1$

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count = number of blocks in  $S_{i+1}(G)$ 
k = count
end while
 $C_b(G) = S_i(G)$ 
block radius of  $G = i + 1$ 
end

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3. Mixed Block Domination

The study of mixed domination was initiated by Sampathkumar and S. S. Kamath⁹. A vertex v , m -dominates an edge x in G if $x \in \langle N[v] \rangle$. Similarly an edge x m -dominates a vertex v if $v \in N[x]$. A set $D \subseteq V$ is said to be a *vertex edge dominating set* (VED set) if every edge in G is m -dominated by some vertex in D . The *vertex - edge domination number* $\gamma_{ve} = \gamma_{ve}(G)$ is the minimum cardinality of a VED set of G . Another parameter *cutfree domination number* $\gamma_b(G)$ defined as minimum number of vertices in a dominating set such that $\langle S \rangle$ is block of G is introduced by Paulraja and Armugam¹⁰. The minimal number of cliques that cover all the vertices of a graph is well known in graph theory as *partition number* $\theta(G)$ introduced by Berge¹¹ and has been celebrated in Berge's conjecture on perfect graphs. This number is also called as point clique covering number by Choudam et al¹². They studied its edge analogue *line clique covering number* $\theta_l(G)$ defined as the minimum number of cliques that cover all the lines of a graph. Parthasarathy and Choudam¹³ studied the same concepts extending it to product graphs. These definitions motivated to ask the question what is the minimum number of blocks (instead of cliques) required to cover all the vertices? Answer to this question lead to define different block domination parameters.

A vertex $v \in V$ and a block $b \in B(G)$ is said to block dominate (b-dominate) each other if v is in the block b . A set $D \subseteq V$ is said to be a *vertex- block dominating set* (VBD set) if every block in G is b-dominated by some vertex in D . The *vertex - block domination number* $\gamma_{vb} = \gamma_{vb}(G)$ is the minimum cardinality of a block dominating set of G . Similarly a set $F \subseteq B(G)$ is said to be a *block - vertex dominating set* (BVD set) if every vertex in G is b-dominated by some block in F . The *block-vertex*

domination number $\gamma_{bv} = \gamma_{bv}(G)$ is the minimum cardinality of a block vertex dominating set of G . In this paper we initiate a study of these new varieties of mixed block domination.

We now give γ_{vb} and γ_{bv} of some standard graphs. Since the results are straightforward we omit the proof.

Proposition 3.1. (i) For any B-complete graph B_m with m blocks,

$$\gamma_{vb}(B_m) = 1 \text{ and } \gamma_{bv}(B_m) = m.$$

(ii) For any B-star $B_{m_1, m_2, \dots, m_k}$ with k cutpoints,

$$\gamma_{vb}(B_{m_1, m_2, \dots, m_k}) = k \text{ and } \gamma_{bv}(B_{m_1, m_2, \dots, m_k}) \leq 1 + \sum_{i=1}^k m_i$$

(iii) For any flower $\gamma_{bv}(B_{m_1, m_2, \dots, m_k}) = \sum_{i=1}^k m_i$.

Proposition 3.2. For any B-path B_{pm} with m blocks,

$$\gamma_{vb}(B_{pm}) = \frac{m}{2} \quad \text{and} \quad \gamma_{bv}(B_{pm}) \geq \frac{m+1}{2}.$$

Proof. Since each cut point b-dominates two blocks in a B-path in order to b-dominate all the m blocks we need $\frac{m}{2}$ cutpoints. In a B-path with m blocks there will be at least $m+1$ vertices, and each block b-dominates at least two vertices, we need atmost $\frac{m+1}{2}$ blocks to b-dominate all the vertices.

3.1 VB independence number

Vertex covering number $\alpha_0(G)$ is defined as the minimum number of vertices needed to cover all the edges of G . *Edge covering number* $\alpha_1(G)$ is similarly defined. The *independence number* $\beta_0(G)$ (*edge independence number* $\beta_1(G)$) is the maximum number of vertices (edges) such that no two vertices (edges) are adjacent in G . The four parameters defined above are related by classical theorem now known as Gallai's Theorem. The strong (weak) coverings and strong (weak) independent sets are studied by S. S. Kamath and R. S. Bhat⁶.

Theorem⁷ 3.1.1. *For any isolate free graph $G = (V, X)$ with p vertices,*

$$(3.1) \quad \alpha_0 + \beta_0 = p,$$

$$(3.2) \quad \alpha_1 + \beta_1 = p.$$

We now define a new type of independent sets called vertex block independent (abbreviated as VB independent) sets. A set $S \subseteq V$ is said to be *VB independent* if $\langle S \rangle$ the subgraph induced by the set S does not induce any block of G . The *VB independence number* $\beta_{vb}(G)$ is the maximum number of vertices in a VB independent set of G . Since $V - C(G)$ does not induce any block of G , we have the set $V - C(G)$ is a VB independent set and hence $\beta_{vb}(G) \geq p - n$. Note that for any tree T , by our definition $\gamma_{vb}(T) = \alpha_0(T)$, $\beta_{vb}(T) = \beta_0(T)$ and $\gamma_{bv}(T) = \alpha_1(T)$.

Proposition 3.1.2. *Let G be a (p, q) graph and $S \subseteq V$, then S is a VBD set of G if and only if $V - S$ is VB independent set of G .*

Proof. Let S be a VBD set of G . Then, since every block is b-dominated by some vertex in S , at least one vertex of every block is in S . Hence $\langle V - S \rangle$ cannot induce any block of G . Hence $V - S$ is VB independent.

Conversely, let S be a VB independent set of G . Suppose that $V - S$ is not a VBD set. Then there exists at least one block $b \in G$ which is not b-dominated by any vertex in $V - S$. This implies all the vertices of b are in S . Then $\langle S \rangle$ contains the block b which contradicts the fact that S is VB independent set of G .

3.2 Gallai's type Theorem

Proposition 3.2.1. *For any graph G with p vertices,*

$$(3.3) \quad \gamma_{vb}(G) + \beta_{vb}(G) = p.$$

Proof. Let S_1 be a γ_{vb} -set of G . Then from Proposition 3.1.2, $V - S_1$ is a VB independent set of G . Hence $\beta_{vb}(G) \geq |V - S_1|$. Therefore

$$(i) \quad \gamma_{vb}(G) + \beta_{vb}(G) \geq p.$$

To prove the reverse in equality, suppose that S_2 is a β_{vb} -set of G , then again from Proposition 3.2, we have $V - S_2$ is a VBD set of G . Hence $|V - S_2| \leq \gamma_{vb}(G)$. Therefore

$$(ii) \quad \gamma_{vb}(G) + \beta_{vb}(G) \leq p.$$

Then the result follows from (i) and (ii).

4. Bounds on Vertex Block Domination Number

Proposition 4.1. *For any block Caterpillar B_{cm} with block diameter $d_b(B_{cm})$ and block radius $r_b(B_{cm})$*

$$(4.1) \quad r_b(B_{cm}) \leq \gamma_{vb}(B_{cm}) \leq d_b(B_{cm}).$$

Proof. Deleting all the pendant blocks from the block caterpillar B_{cm} we get a block spine B_{pm} which is a B-path with m blocks. Hence there exists a longest B-path B_{pm+2} with $m+2$ blocks in the B-caterpillar B_{cm} . Then $r_b(B_c) = \frac{m+2}{2} = \gamma_{vb}(B_{pm+2}) \leq \gamma_{vb}(B_{cm})$. Now suppose every cut point of B_{pm+2} has a pendant block in B_{cm} so that $\gamma_{vb}(B_{cm}) = m+1 = d_b(B_{cm})$. Therefore for any B_{cm} we have $\gamma_{vb}(B_{cm}) \leq d_b(B_{cm})$.

A cutpoint c is called a *support cutpoint* if c is incident with a pendant block. Similarly a nonpendant block which is adjacent to a pendant block is called a support block.

Proposition 4.2. *Let G be any graph with s support cutpoints, then*

$$(4.2) \quad s \leq \gamma_{vb}(G)$$

and equality holds if and only if every nonpendant block of G is a support block of G .

Proof. In order to b-dominate all the pendant blocks every support cut point is essential in a VBD set. Hence the set S of all support cutpoints is a subset of all VBD set of G . Therefore $s \leq \gamma_{vb}(G)$.

Suppose $s = \gamma_{vb}(G)$, then the set S of all support cutpoints is a minimum VBD set of G . If possible there exists a nonpendant block b in G which is not a support block in G . Then all the cutpoints of the block b are not support cut points hence are not in S . But then b is not dominated by any vertex in S . Hence S is not a VBD set of G - a contradiction.

Conversely, if every non pendant block of G is a support block of G . Then we already have $s \leq \gamma_{vb}(G)$. Since every block is a support block, each block has at least one support cutpoint and hence all these blocks are b-dominated by the corresponding support cutpoint in S , and hence no more vertices are needed to b-dominate any block. Thus we conclude that $s = \gamma_{vb}(G)$.

Proposition 4.3. *Let G be any graph with n cut points, then $\gamma_{vb}(G) \leq n$ and equality holds if and only if every cut point of G is a support cut point.*

Proof. Since the set of all cut points is a VBD set of G we have $\gamma_{vb}(G) \leq n$. Suppose $\gamma_{vb}(G) = n$. Then $C(G)$ the set of all cut points of G forms a minimum VBD set of G . If possible there exists a cut point c which is not a support. Then c is incident with at least two nonpendant blocks say b_1 and b_2 which has two support cut points c_1 and c_2 . Hence $V - \{c\}$ is a VBD set of G - a contradiction. Converse is straightforward and hence we omit the proof.

In the next proposition we get a bound for γ_{vb} in terms of maximum VB degree Δ_{vb} .

Proposition 4.4. *Let G be any graph with m blocks and maximum VB degree Δ_{vb} and minimum CVB degree δ_{cvb} , then*

$$(4.3) \quad \left\lceil \frac{m}{\Delta_{vb}} \right\rceil \leq \gamma_{vb},$$

$$(4.4) \quad \left\lceil \frac{p}{\Delta_{bv}} \right\rceil \leq \gamma_{bv}.$$

Further these bounds are sharp.

Proof. Since any cutpoint can b-dominate atmost Δ_{vb} blocks, we need atleast $\left\lceil \frac{m}{\Delta_{vb}} \right\rceil$ vertices to b-dominate all the blocks of G . This yields the bound in (4.3). Again a block can b-dominate atmost Δ_{bv} vertices, we need atleast $\left\lceil \frac{p}{\Delta_{bv}} \right\rceil$ blocks to b-dominate all the vertices of G . Any B-complete graph attains the bound in [4.3] any B-path in which every block has same number of vertices attains the bound in [4.4].

References

1. F. Harary, *Graph Theory*, Addison Wiley, 1969.
2. D. B. West, *Introduction to Graph Theory*, Prentice hall, 1996.
3. Surekha and P. G. Bhat, *Sum of block degrees and B - regular graphs*, submitted.
4. S. S. Kamath and R. S. Bhat, Strong/ Weak Neighbourhood Number of a graph (K3-coverings), *Proc. Int. Conf. on Disc. Math., Narosa Pub., New Delhi, India*, (2004) 142-150.
5. S. S. Kamath and R. S. Bhat, Some new degeree concepts in graphs, *Proceedings of the ICDM*, (2006) 237-243.
6. S. S. Kamath and R. S. Bhat, Strong (weak) independence and covering numbers of a graph, *Disc. Math*, **307** (2007) 1136 – 1145.
7. Gallai, Elementare relationen bezüglich der glieder und trennenden punkte von graphen. *Magyar Tud.Akad.Mat. Kutato Int. Kozl.*, **9** (1964) 235 -236.
8. F. Harary, An elementary theorem on graphs. *Amer. Math. Monthly*, **66** (1959) 405 – 407.
9. E. Sampathkumar and S. B. Chikkodimath, Semitotal graphs of a graph - I, *J. of Karnataka Univ.Sci*, **18** (1973) 274 – 280.
10. Paulraja Joseph and A. Armugam, On connected cutfree domination in graphs, *Ind. Jour. Of Pure and appl. Math.*, **23 (9)** (1992) 643-647.
11. C. Berge, *Theory of graphs and its applications*, Methuen, London.
12. S. A. Choudam , K. R. Parthasarathy and G. Ravindra, Line-clique cover number of a graph, *Proceedings of INSA*, **41 (A) (3)** (1975) 289-293.
13. K. R. Parthasarathy and S. A. Choudam , Edge-clique cover number of product graphs, *J. Math. and Phy. Sci.*, **10(3)** (1976) 255-261.
14. Behzad M., A criterion for planarity of total graphs, *Proc. Camb. Phil. Soc.* **63**, (1967) 679-681
15. V. R. Kulli, The block point tree of a graph, *Indian J. Pure and Applied Math.*, **7** (1976) 10 -14.
16. E. Sampathkumar and S. S. Kamath, Mixed domination in graphs, *Sankhya*, **Spl. Vol. 54** (1992) 392-402.