

On Non-invariant Hypersurfaces of a Nearly Sasakian Manifold

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Abstract: The object of the present paper is to study non-invariant hypersurfaces of nearly Sasakian manifolds equipped with (f, g, u, v, λ) -structure. Some properties obeyed by this structure are obtained. The second fundamental form of a non-invariant hypersurface of a nearly Sasakian manifold with (f, g, u, v, λ) -structure has been traced under the condition when f is parallel. The necessary and sufficient conditions also have been obtained for totally umbilical non-invariant hypersurfaces with (f, g, u, v, λ) -structure of nearly Sasakian manifold to be totally geodesic.

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1. Introduction

In 1970, S. I. Goldberg et. al¹ introduced the notion of a non-invariant hypersurface of an almost contact manifold in which the transform of a tangent vector of the hypersurface by the $(1,1)$ structure tensor field ϕ defining the almost contact structure is never tangent to the hypersurface. The notion of (f, g, u, v, λ) -structure was given by K.Yano². It is well known^{3,4} that a hypersurface of an almost contact metric manifold always admits a (f, g, u, v, λ) -structure. Authors¹ proved that there always exists a

(f, g, u, v, λ) - structure on a non-invariant hypersurface of an almost contact metric manifold. They also proved that there does not exist invariant hypersurface of a contact manifold. R. Prasad⁵ studied the non-invariant hypersurfaces of trans-Sasakian manifolds. In the present paper, we study the non-invariant hypersurfaces of nearly Sasakian manifolds.

2. Preliminaries

Let $\tilde{M}$ be an almost contact metric manifold with almost contact metric structure (ϕ, ξ, η, g) where ϕ is (1,1) tensor field, η is 1- form and g is a compatible Riemannian metric such that

$$(2.1) \quad \phi^2 = -I + \eta \otimes \xi, \quad \eta(\xi) = 1, \quad \phi \circ \xi = 0, \quad \eta \circ \phi = 0,$$

$$(2.2) \quad g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y),$$

$$(2.3) \quad g(X, \phi Y) = -g(\phi X, Y), \quad g(X, \xi) = \eta(X),$$

for all $X, Y \in T\tilde{M}$.

An almost contact metric manifold is a nearly Sasakian manifold⁶ if

$$(2.4) \quad (\tilde{\nabla}_X \phi)Y + (\tilde{\nabla}_Y \phi)X = 2g(X, Y)\xi - \eta(Y)X - \eta(X)Y,$$

for all vector field X, Y on $\tilde{M}$, where $\tilde{\nabla}$ is the operator of covariant differentiation with respect to g . From (2.4), we have

$$(2.5) \quad \tilde{\nabla}_X \xi = -\phi X - \phi(\tilde{\nabla}_\xi \phi)X.$$

A hypersurface of an almost contact metric manifold $\tilde{M}(\phi, \xi, \eta, g)$ is called a non-invariant hypersurface, if the transform of a tangent vector of the hypersurface under the action of (1,1) tensor field ϕ defining the contact structure is never tangent to the hypersurface. Let X be a tangent vector on a non-invariant hypersurface of an almost contact metric manifold $\tilde{M}$, then ϕX is never tangent to the hypersurface.

Let M be a non-invariant hypersurface of an almost contact metric manifold. Now, if we define the following

$$(2.6) \quad \phi X = fX + u(X)\hat{N},$$

$$(2.7) \quad \phi \hat{N} = -U,$$

$$(2.8) \quad \xi = V + \lambda \hat{N}, \quad \lambda = \eta(\hat{N});$$

$$(2.9) \quad \eta(X) = v(X),$$

where f is a $(1,1)$ tensor field, u, v are 1-forms, $\hat{N}$ is a unit normal to the hypersurface, $X \in TM$ and $u(X) \neq 0$, then we get an induced (f, g, u, v, λ) -structure on M satisfying the conditions^{2,3}:

$$(2.10) \quad f^2 = -I + u \otimes U + v \otimes V,$$

$$(2.11) \quad fU = -\lambda V, fV = \lambda U,$$

$$(2.12) \quad uof = \lambda v, vof = -\lambda u,$$

$$(2.13) \quad u(U) = 1 - \lambda^2, u(V) = v(U) = 0, v(V) = 1 - \lambda^2,$$

$$(2.14) \quad g(fX, fY) = g(X, Y - u(X)u(Y) - v(X)v(Y)),$$

$$(2.15) \quad g(X, fY) = -g(fX, Y), g(X, U) = u(X),$$

$$g(X, V) = v(X),$$

for all $X, Y \in TM$, where $\lambda = \eta(\hat{N})$.

The Gauss and Weingarten formulae are given by

$$(2.16) \quad \tilde{\nabla}_X Y = \nabla_X Y + \sigma(X, Y)\hat{N},$$

$$(2.17) \quad \tilde{\nabla}_X \hat{N} = -A_{\hat{N}} X,$$

for all $X, Y \in TM$, where $\tilde{\nabla}$ and ∇ are the Riemannian and induced Riemannian connections on $\tilde{M}$ and M respectively and $\hat{N}$ is the unit normal vector in the normal bundle $T^\perp M$. In this formula σ is the second fundamental form on M related to $A_{\hat{N}}$ by

$$(2.18) \quad \sigma(X, Y) = g(A_{\hat{N}}X, Y), \quad \text{for } X, Y \in TM.$$

3. Non-invariant Hypersurfaces

Lemma 3.1: *Let M be a non-invariant hypersurface with (f, g, u, v, λ) -structure of a nearly Sasakian manifold $\tilde{M}$, then*

$$(3.1) \quad (\tilde{\nabla}_X \phi)Y + (\tilde{\nabla}_Y \phi)X = ((\nabla_X u)Y + (\nabla_Y u)X + \sigma(X, fY) + (\sigma(Y, fX))\hat{N} + (\nabla_X f)Y + (\nabla_Y f)X + 2\sigma(X, Y)U - u(Y)A_{\hat{N}}Y),$$

$$(3.2) \quad (\tilde{\nabla}_X \eta)Y + (\tilde{\nabla}_Y \eta)X = (\nabla_X u)Y + (\nabla_Y u)X - 2\lambda\sigma(X, Y),$$

$$(3.3) \quad \tilde{\nabla}_X \xi = (\nabla_X V - \lambda A_{\hat{N}}X) + (\sigma(X, V) + X\lambda)\hat{N}.$$

Proof: Consider

$$\begin{aligned} (\tilde{\nabla}_X \phi)Y &= (\tilde{\nabla}_X Y) - \phi(\tilde{\nabla}_X Y) \\ &= \tilde{\nabla}_X(fY + u(Y)\hat{N}) - \phi(\nabla_X Y + \sigma(X, Y)\hat{N}) \\ &= \tilde{\nabla}_X fY + \tilde{\nabla}_X(u(Y)\hat{N}) - f(\nabla_X Y) - u(\nabla_X Y)\hat{N} - \sigma(X, Y)(-U) \\ &= \nabla_X fY + \sigma(X, fY)\hat{N} + u(Y)(-A_{\hat{N}}X) + (\nabla_X u(Y))\hat{N} - f(\nabla_X Y) \\ &\quad - u(\nabla_X Y)\hat{N} + \sigma(X, Y)U, \end{aligned}$$

which gives

$$(3.4) \quad (\tilde{\nabla}_X \phi)Y = (\nabla_X f)Y - u(Y)A_{\hat{N}}X + \sigma(X, Y)U + ((\nabla_X u)Y + \sigma(X, fY))\hat{N}.$$

Similarly, we have

$$(3.5) \quad (\tilde{\nabla}_X \phi)X = (\nabla_Y f)X - u(X)A_{\hat{N}}Y + \sigma(X, Y)U + ((\nabla_Y u)X + \sigma(Y, fX))\hat{N},$$

From (3.4) and (3.5), we have

$$(\tilde{\nabla}_X \phi)Y + (\tilde{\nabla}_Y \phi)X = ((\nabla_X u)Y + (\nabla_Y u)X + \sigma(X, fY) + (\sigma(Y, fX))\hat{N}$$

$$+(\nabla_X f)Y + (\nabla_Y f)X + 2\sigma(X, Y)U \\ - u(Y)A_{\hat{N}}X - u(X)A_{\hat{N}}Y$$

Also,

$$(\tilde{\nabla}_X \eta)Y = \tilde{\nabla}_X \eta(Y) - \eta(\tilde{\nabla}_X Y) \\ = \nabla_X (v(Y)) - v(\nabla_X Y) - \sigma(X, Y)\eta(\hat{N});$$

therefore

$$(3.6) \quad (\tilde{\nabla}_X \eta)Y = (\nabla_X v)Y - \lambda\sigma(X, Y),$$

Similarly

$$(3.7) \quad (\tilde{\nabla}_Y \eta)X = (\nabla_Y v)X - \lambda\sigma(X, Y),$$

Adding (3.6) and (3.7), we get

$$(\tilde{\nabla}_X \eta)Y + (\tilde{\nabla}_Y \eta)X = (\nabla_X v)Y + (\nabla_Y v)X - 2\lambda\sigma(X, Y).$$

Further, consider

$$\tilde{\nabla}_X \xi = \nabla_X \xi + \sigma(X, \xi)\hat{N} \\ = \nabla_X V + \nabla_X \lambda\hat{N} + \sigma(X, V)\hat{N} \\ = \nabla_X V + \lambda\nabla_X \hat{N} + (X\lambda)\hat{N} + \sigma(X, V)\hat{N}$$

or

$$\tilde{\nabla}_X \xi = (\nabla_X V + \lambda A_{\hat{N}}X) + (\sigma(X, V) + X\lambda)\hat{N}.$$

Theorem 3.2: Let M be a non-invariant hypersurface with (f, g, u, v, λ) -structure of a nearly Sasakian manifold $\tilde{M}$, then

$$(3.8) \quad \sigma(X, \xi)U = f^2 X - u(X)U + f^2((\tilde{\nabla}_\xi \phi)X) - u((\tilde{\nabla}_\xi \phi)X)U + f(\tilde{\nabla}_X \xi),$$

$$(3.9) \quad u(\nabla_X \xi) = -u(fX) - u(f(\tilde{\nabla}_\xi \phi)X).$$

Proof: Consider

$$(\tilde{\nabla}_X \phi)\xi = \tilde{\nabla}_X \phi\xi - \phi(\tilde{\nabla}_X \xi)$$

$$= -\phi\left(-\phi X - \phi\left((\tilde{\nabla}_\xi \phi)X\right)\right)$$

or

$$(3.10) \quad (\tilde{\nabla}_X \phi)\xi = -\phi\left(-fX - u(X)\hat{N} - f\left((\tilde{\nabla}_\xi \phi)X\right) - u\left((\tilde{\nabla}_\xi \phi)X\right)\hat{N}\right).$$

Since we know the relation

$$(3.11) \quad (\tilde{\nabla}_X \phi)\xi = -\phi(\nabla_X \xi) + \sigma(X, \xi)U.$$

Equation (3.10) and (3.11), gives

$$\begin{aligned} -\phi(\nabla_X \xi) + \sigma(X, \xi)U &= -\phi\left(-fX - u(X)\hat{N} - f\left((\tilde{\nabla}_\xi \phi)X\right) - u\left((\tilde{\nabla}_\xi \phi)X\right)\hat{N}\right) \\ &= \phi(fX) - u(X)U + \phi\left(f\left((\tilde{\nabla}_\xi \phi)X\right) - u\left((\tilde{\nabla}_\xi \phi)X\right)\hat{N}\right)U. \end{aligned}$$

Now from (2.6) and (2.7), we have

$$\begin{aligned} -f(\nabla_X \xi) - u(\nabla_X \xi)\hat{N} + \sigma(X, \xi)U &= f(fX) + u(fX)\hat{N} - u(X)U \\ &\quad + f\left(f\left((\tilde{\nabla}_\xi \phi)X\right) + u\left(f\left((\tilde{\nabla}_\xi \phi)X\right)\hat{N}\right) - u\left((\tilde{\nabla}_\xi \phi)X\right)U. \end{aligned}$$

Equating tangential and normal parts, we have

$$\sigma(X, \xi)U = f^2 X - u(X)U + f^2\left((\tilde{\nabla}_\xi \phi)X\right) - u\left((\tilde{\nabla}_\xi \phi)X\right)U + f(\tilde{\nabla}_X \xi).$$

and

$$u(\nabla_X \xi) = -u(fX) - u\left(f\left((\tilde{\nabla}_\xi \phi)X\right)\right).$$

Theorem 3.3: Let M be a non-invariant hypersurface with (f, g, u, v, λ) -structure of a nearly Sasakian manifold, $\tilde{M}$ then

$$(3.12) \quad (\nabla_X f)Y + (\nabla_Y f)X = 2g(X, Y)V - v(Y)X - v(X)Y + 2\sigma(X, Y)U$$

$$+ u(Y)A_{\hat{N}}X - u(X)A_{\hat{N}}Y,$$

$$(3.13) \quad (\nabla_X u)Y + (\nabla_Y u)X = 2\lambda g(X, Y) - \sigma(X, \lambda Y) - \sigma(Y, fX).$$

Proof: In view of (3.1) and (2.4), we have

$$\begin{aligned}
& (\nabla_X f)Y + (\nabla_Y f)X - u(Y)A_{\hat{N}}X - u(X)A_{\hat{N}}Y + 2\sigma(X, Y)U \\
& + \{(\nabla_X u)Y + (\nabla_Y u)X + \sigma(X, fY) + \sigma(Y, fX)\}\hat{N} \\
& = 2g(X, Y)V - \eta(Y)X - \eta(X)Y + 2\lambda g(X, Y).
\end{aligned}$$

Equating tangential and normal components of above, we can obtain (3.18) and (3.19) respectively.

Theorem 3.4: *Let M be a noninvariant hypersurface with (f, g, u, v, λ) -structure of a nearly Sasakian manifold $\tilde{M}$, then*

$$\begin{aligned}
(3.14) \quad & (\tilde{\nabla}_X \phi)Y + (\tilde{\nabla}_Y \phi)X = 2\lambda g(X, Y) + 2g(X, Y)V - v(Y)X \\
& - u(X)Y + 4\sigma(X, Y)U.
\end{aligned}$$

Proof: Consider

$$\begin{aligned}
& (\tilde{\nabla}_X \phi)Y = (\tilde{\nabla}_X \phi)Y - \phi(\tilde{\nabla}_X Y) \\
& = \tilde{\nabla}_X fY + \tilde{\nabla}_X (u(Y)\hat{N}) - f(\nabla_X Y) - u(\nabla_X Y)\hat{N},
\end{aligned}$$

or

$$(3.15) \quad (\tilde{\nabla}_Y \phi)Y = (\nabla_X f)Y - u(Y)A_{\hat{N}}X + \sigma(X, Y)U + \left(\begin{array}{c} (\nabla_X u)Y \\ +\sigma(X, fY) \end{array} \right) \hat{N},$$

Similarly

$$(3.16) \quad (\tilde{\nabla}_Y \phi)X = (\nabla_X f)X - u(X)A_{\hat{N}}Y + \sigma(X, Y)U + \left(\begin{array}{c} (\nabla_X u)X \\ +\sigma(Y, fX) \end{array} \right) \hat{N},$$

Using (3.15) and (3.16), we have

$$\begin{aligned}
(3.17) \quad & (\tilde{\nabla}_X \phi)Y + (\tilde{\nabla}_Y \phi)X = ((\nabla_X u)Y + (\nabla_Y u)X + \sigma(X, fY) \\
& + \sigma(Y, fX))\hat{N} + (\nabla_X f)Y + (\nabla_Y f)X \\
& + 2\sigma(X, Y)U - u(Y)A_{\hat{N}}X - u(X)A_{\hat{N}}Y.
\end{aligned}$$

Now from (3.17), we have

$$(3.18) \quad (\tilde{\nabla}_X \phi)Y + (\tilde{\nabla}_Y \phi)X = 2g(X, Y)V - \eta(Y)X - \eta(X)Y + 2\lambda g(X, Y).$$

From (3.17) and (3.18), we have

$$\begin{aligned}
(3.19) \quad & (\nabla_X f)Y + (\nabla_Y f)X = 2g(X, Y)V - v(Y)X - v(X)Y + 2\sigma(X, Y)U \\
& + u(Y)A_{\hat{N}}X - u(X)A_{\hat{N}}Y,
\end{aligned}$$

and

$$(3.20) \quad (\nabla_X u)Y + (\nabla_Y u)X = 2\lambda g(X, Y) - \sigma(X, fY) - \sigma(Y, fX),$$

now equation (3.17), (3.19), (3.20) enables us to deduce (3.14).

Now we proceed for some interesting results on totally geodesic noninvariant hypersurfaces.

Theorem 3.5: *Let M be a totally umbilical noninvariant hypersurface with (f, g, u, v, λ) -structure of a nearly Sasakian manifold. Then it is totally geodesic if and only if*

$$(3.21) \quad u(X) + u\left(\left(\tilde{\nabla}_\xi \phi\right)X\right) + X\lambda = 0,$$

and, in particular, if nearly Sasakian manifold admits a contact structure, then (3.21) can be expressed as

$$(3.22) \quad \frac{1}{\lambda}u + d(\log \lambda) = 0.$$

Proof: Consider

$$\begin{aligned} \tilde{\nabla}_X \xi &= \nabla_X \xi + \sigma(X, \xi)\hat{N} \\ &= \nabla_X (V + \lambda\hat{N}) + \sigma(X, V)\hat{N} \\ &= \nabla_X V + \nabla_X \lambda\hat{N} + \sigma(X, V)\hat{N} \\ &= \nabla_X V + \lambda\nabla_X \hat{N} + (X\lambda)\hat{N} + \sigma(X, V)\hat{N} \end{aligned}$$

or

$$(3.23) \quad \tilde{\nabla}_X \xi = (\nabla_X V - \lambda A_{\hat{N}} X) + (\sigma(X, V) + X\lambda)\hat{N},$$

From (2.5) and (3.23), we calculate

$$\begin{aligned} &-fX - u(X)\hat{N} - f\left(\left(\tilde{\nabla}_\xi \phi\right)X\right) - u\left(\left(\tilde{\nabla}_\xi \phi\right)X\right)\hat{N} \\ &= (\nabla_X V - \lambda A_{\hat{N}} X) + (\sigma(X, V) + X\lambda)\hat{N}, \end{aligned}$$

Equating normal part, we have

$$(3.24) \quad \sigma(X, V) = -u(X) - u\left(\left(\tilde{\nabla}_\xi \phi\right)X\right) - X\lambda.$$

Now if M is totally umbilical, then $A_{\hat{N}} = \zeta I$, ζ is Kahlerian metric and (2.18) reduces into $\sigma(X, Y) = g(A_{\hat{N}}X, Y) = g(\zeta X, Y) = \zeta g(X, Y)$, therefore $\sigma(X, Y) = \zeta g(X, Y)$, and equation (3.24) gives

$$(3.25) \quad u(X) + u(\tilde{\nabla}_{\xi}\phi)X + X\lambda + \zeta v(X) = 0.$$

If M is totally geodesic i.e. $\zeta = 0$, then (3.25) gives

$$u(X) + u(\tilde{\nabla}_{\xi}\phi)X + X\lambda = 0.$$

Now, if nearly Sasakian manifold is equipped with contact structure then (3.21) takes the form $u(X) + X\lambda = 0$. This concludes the required condition (3.22).

Theorem 3.6: *Let M be a noninvariant hypersurface with (f, g, u, v, λ) -structure of a nearly Sasakian manifold. If f is parallel, then we have*

$$(3.26) \quad \sigma(X, Y) = \frac{1}{3(1-\lambda^2)}(u(X)v(Y)) + \frac{\mu}{3(1-\lambda^2)^2}u(X)u(Y),$$

where $\mu = \sigma(U, U) = g(A_{\hat{N}}U, U)$. Also, M is totally geodesic if and only if

$$(3.27) \quad \frac{4}{3}u(X) + u(\tilde{\nabla}_{\xi}\phi)X + X\lambda = 0$$

Proof: Since f is parallel then equation (3.19) reduces into

$$\begin{aligned} 2\sigma(X, Y)U &= v(Y)X + v(X)Y - 2g(X, Y)V \\ &\quad - u(Y)A_{\hat{N}}X - u(X)A_{\hat{N}}Y + v(X)fY. \end{aligned}$$

Applying u both sides, we obtain

$$(3.28) \quad \begin{aligned} 2(1-\lambda^2)\sigma(X, Y) &= u(Y)v(X) - u(Y)u(A_{\hat{N}}X) \\ &\quad - u(X) - u(X)u(A_{\hat{N}}Y). \end{aligned}$$

In view of (3.28), we have

$$3(1-\lambda^2)\sigma(X, U) = (1-\lambda^2)u(X)u(A_{\hat{N}}U)$$

or

$$(3.29) \quad u(A_{\hat{N}}X) = \frac{1}{3}v(X) - \frac{\mu}{3(1-\lambda^3)}u(X),$$

where $\mu = \sigma(U, U)$.

Following the similar way, we can get

$$(3.30) \quad u(A_{\hat{N}}Y) = \frac{1}{2}v(Y) - \frac{\mu}{3(1-\lambda^3)}u(Y),$$

In the light of equations (3.28), (3.29) and (3.30), we have

$$\begin{aligned} 2(1-\lambda^2)\sigma(X, Y) &= \frac{2}{3}u(Y)v(X) + \frac{2}{3}u(X)v(Y) \\ &\quad + \frac{\mu}{3(1-\lambda^2)}u(X)u(Y). \end{aligned}$$

Next, from (3.24) and (3.26), we have

$$\frac{4}{3}u(X) + u(\tilde{\nabla}_{\xi}\phi)X + X\lambda = 0.$$

Further, if nearly Sasakian manifolds posses contact structure, then

$$\frac{4}{3}u(X) + X\lambda = 0 \Rightarrow \frac{4}{\lambda}u + 3d(\log \lambda) = 0.$$

Theorem 3.7: *Let M be a noninvariant hypersurface with (f, g, u, v, λ) -structure of a nearly Sasakian manifold. If U is parallel, then we have*

$$(3.31) \quad \lambda X + f(A_{\hat{N}}X) + (\nabla_{\hat{N}}f)X = 0.$$

Proof: Consider

$$\begin{aligned} (\tilde{\nabla}_X\phi)\hat{N} &= \tilde{\nabla}_X\phi\hat{N} - \phi\tilde{\nabla}_X\hat{N} \\ &= -\tilde{\nabla}_XU - \phi(-A_{\hat{N}}X) \\ &= -\tilde{\nabla}_XU - \phi(X, U)\hat{N} + f(A_{\hat{N}}X) + u(A_{\hat{N}}X)\hat{N}, \end{aligned}$$

$$(3.32) \quad (\tilde{\nabla}_X\phi)\hat{N} = -\nabla_XU + f(A_{\hat{N}}X).$$

From equation (2.4), we have

$$(3.33) \quad (\tilde{\nabla}_X \phi)Y + (\tilde{\nabla}_Y \phi)X = 2g(X, Y)V - \eta(Y)X - \eta(X)Y \\ - \eta(Y)fX + \eta(X)fY + 2g\lambda(X, Y).$$

On putting $Y = \hat{N}$, we have

$$(3.34) \quad (\tilde{\nabla}_X \phi)\hat{N} + (\tilde{\nabla}_{\hat{N}} \phi)X = -\lambda X - \nu(X)\hat{N}.$$

Also

$$(3.35) \quad (\tilde{\nabla}_{\hat{N}} \phi)X = (\tilde{\nabla}_{\hat{N}} f)X - u(X)A_{\hat{N}}\hat{N} + \{(\nabla_{\hat{N}} u)X + \phi(\hat{N}, fX)\}\hat{N},$$

Using (3.32), (3.34) and (3.35), we get

$$-\nabla_X U + f(A_{\hat{N}}X) + (\tilde{\nabla}_{\hat{N}} \phi)X = -\lambda X - \nu(X)\hat{N}$$

Now equating tangential part, we have

$$(3.36) \quad \nabla_X U = \lambda X + f(A_{\hat{N}}X) + (\nabla_{\hat{N}} f)X.$$

If U is parallel, then (3.36) implies

$$\lambda X + f(A_{\hat{N}}X) + (\nabla_{\hat{N}} f)X = 0.$$

This completes the proof.

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