

A Deterministic Inventory Model with Weibull Distribution Deteriorating Item with Selling Price and the Time Dependent Demand Rate

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Abstract: This study represents a deterministic inventory model for deteriorating products under the condition of instantaneous replenishment. In the paper, we developed an inventory model for obtaining optimum cycle length by minimizing the total cost per unit time under usual condition. The demand is taken as a function of selling price and decreases exponentially with time and holding cost are linear and shortages are not allowed. The deterioration rate is taken as a special form of two parameter Weibull function considered by Covert and Phillip. The model is solved by minimizing the total cost. The model reduces to well known result by choosing appropriate values of the parameter. The model is illustrated with the help of numerical example. Sensitivity analysis has also been conducted study the effect of the parameter on optimal solution.

Keywords: Inventory, Deteriorating items, Holding Cost & Optimum cost.

1. Introduction

Inventory theory deals with the management of stock levels of goods with aim of ensuring that demand for these goods is met. The demand rate of any product is normally dynamic in nature. This phenomenon is due to price or time or even introducing of new products in the market. Researcher extensively studied in the different aspects of time varying demand patterns considering continuous and discrete. Mostly in continuous time inventory models researcher studied linearly increasing or decreasing demand models. The purpose of inventory theory is to determine rules that management can use to minimize the cost associated with maintaining inventory and meeting customers demand. A good number of authors have taken different rate of deterioration in their analysis such as constant, linear function of time and two parameters weibull function etc. as the utility of goods does not remain constant over time.

Ghare and Schrader¹ developed a model for exponentially decaying inventory considering constant demand. Emmons² also developed a model for exponentially decaying products where the product decayed at one rate into a new product which decayed at a second rate. Cohen³ developed a model for joint pricing and ordering policy for exponentially decaying inventory with known demand and constant decay rate. Mukherjee⁴ extended it by considering time varying demand rate. Kumar and Sharma⁵ had extended Mukherjee⁴ model by considering shortages.

Nahmias⁶ reviewed the ordering policy for perishable inventories, covering both fixed life and random life models. Hollier and Mark⁷ explored the demand rate to be decreasing negative exponentially without shortages. Cohen⁸ considered joint pricing and ordering policy for exponentially decaying inventory model with known demand considering constant deterioration rate. Mukherjee⁴ developed an inventory model for optimum ordering interval for time varying decay rate of inventory. Chung et al⁹ developed a solution procedure to determine joint pricing and replenishment policy for deteriorating inventory systems with declining market.

In this study we consider demand as function of selling price and decrease exponentially with time and linear holding cost and shortage are not allowed. Deterioration rate is taken as a special form of two parameter weibull function. Considered by covert and Philip¹⁰. The model reduces to well-known result by choosing appropriate value of the parameter.

2. Assumptions and Notations

1. $I(t)$ is the on hand inventory at any time t .
2. $Z(t)$ is the stock loss due to decay at time t .
3. Demand rate $D(t, p)$ is known and decrease exponentially i.e. at time t ,

$$t \geq 0, \quad D(t, p) = \frac{e^{-\theta t}}{d(p)} t$$

where θ is constant governing the decreasing rate of demand and p is the selling price per unit and $d(p)$ is the function of p .

4. Lead time is zero and shortages are not allowed.
5. In the present model, the deterioration function $\theta(t, \alpha)$ is assumed in the form $\theta(t, \alpha) = \theta_0(\alpha)t$, $0 < \theta_0(\alpha) < 1, t > 0$ which is a special form of two

parameter weibull function considered by Covet and Philip¹⁰. The function is some function of the random variable α and in which a probability density function $p(\alpha)$ is defined such as $\int p(\alpha)d\alpha=1$.

6. There is no replacement or repair of the decayed units during the period under consideration.
7. The unit purchase cost is c , h is holding cost per unit per unit time and k is the ordering cost per order.
8. The replenishment rate is infinite and T is the cycle time.

3. Mathematical Modelling and Analysis

The differential equation governing the system is given by

$$(1) \quad \frac{dI(t)}{dt} = -\theta_0(t, \alpha)I(t) - d(p)$$

Solution of (1) is given by after adjusting constant of integration at $t=0$, $\beta=I(0)$

$$(2) \quad I(t) = -\frac{1}{d(p)} \exp\left(-\theta_0(\alpha)\frac{t^2}{2}\right) \left[t - \frac{\theta t^2}{2} + \theta_0(\alpha)\frac{t^3}{6} \right] + I(0) \exp\left(-\theta_0(\alpha)\frac{t^2}{2}\right)$$

Inventory without decay at time t , the differential equation of the system will be

$$(3) \quad \frac{d}{dt}I_w(t) = \frac{e^{-\theta t}}{d(p)} \Rightarrow I_w(t) = \frac{e^{-\theta t}}{\theta d(p)} + \beta_2$$

The solution of which gives

$$(4) \quad I_w(t) = I(0) + \frac{e^{-\theta t} - 1}{\theta d(p)} .$$

The stock loss $Z(t)$ due to decay in $[0, T]$ is given by

$$(5) \quad Z(t) = I_w(t) - I(t) = I(0) + \frac{e^{-\theta t} - 1}{\theta d(p)} - I(t) .$$

Using (2), equation (5) reduces to

$$(6) \quad Z(t) = I(t) \frac{\theta_0(\alpha) t^2}{2} + \frac{1}{d(p)} \left[t - \frac{\theta t^2}{2} + \frac{\theta_0(\alpha) t^3}{6} + \frac{e^{-\theta t}}{\theta} - \frac{1}{\theta} \right].$$

If T is the cycle length (ordering interval), the order quantity Q_T required to satisfy the demand during a cycle of length T is equal to

$$(7) \quad Q_T = Z(t) + \frac{e^{-\theta T}}{\theta d(p)}.$$

In case of instantaneous replenishment $I(0) = Q_T$ and $I(T) = 0$. From equation (6), we have

$$(8) \quad Q(T) = I(t) \theta_0(\alpha) \frac{T^2}{2} + \frac{1}{d(p)} \left[T - \frac{\theta T^2}{2} + \frac{\theta_0(\alpha) T^3}{6} + \frac{e^{-\theta T}}{\theta} - \frac{1}{\theta} \right] + \frac{e^{-\theta T}}{\theta d(p)}.$$

$$(9) \quad Q(T) = \frac{1}{d(p)} \left[T - \theta \frac{T^2}{2} + \frac{\theta_0(\alpha) T^3}{6} \right] - \frac{1}{Q d(p)}.$$

From equation (2), we have

$$(10) \quad I(t) = -\frac{1}{d(p)} e^{-\theta_0(\alpha) \frac{t^2}{2}} \left[t - \theta \frac{t^2}{2} + \frac{\theta_0(\alpha) t^3}{6} \right] + \left\{ \frac{1}{d(p)} \left[T^2 - \theta \frac{T^2}{2} + \frac{\theta_0(\alpha) T^3}{6} \right] - \frac{1}{Q d(p)} \right\} \\ \times \exp \left(-\theta_0(\alpha) \frac{t^2}{2} \right)$$

Cost per unit time $C(T, P, \alpha)$ is given by

$$C(T, P, \alpha) = \frac{K}{T} + \frac{C Q_T}{T} + \frac{h}{T} \int_0^T I(t) dt \\ = \frac{K}{T} + \frac{C}{T d(p)} \left[T - \theta \frac{T^2}{2} + \frac{\theta_0(\alpha) T^3}{6} - \frac{1}{\theta} \right] \\ + \frac{h}{T} \int_0^T \frac{1}{d(p)} \left[t - \theta \frac{t^2}{2} + \frac{\theta_0(\alpha) t^3}{6} - T^2 + \theta \frac{T^2}{2} - \frac{\theta_0(\alpha) T^3}{6} + \frac{1}{\theta} \right] \exp(-\theta_0(\alpha) \frac{t^2}{2}) dt$$

$$\begin{aligned}
(11) \quad C(T, P, \alpha) = & \frac{K}{T} + \frac{C}{Td(p)} \left[T - \theta \frac{T^2}{2} + \frac{\theta_0(\alpha) T^3}{6} - \frac{1}{\theta} \right] + \frac{h}{Td(p)} + \frac{T^2}{2} \\
& - \frac{2}{3} \theta \frac{T^3}{6} + \frac{\theta_0(\alpha) T}{12} - \frac{\theta \theta_0(\alpha) T^5}{30} + \frac{\theta_0(\alpha) T^3}{30} + \frac{\theta T^3}{6} - T^3
\end{aligned}$$

Neglecting higher power of $\theta_0(\alpha)$. Hence mean cost $\langle C(T, P, \alpha) \rangle$ is obtained by the integral

$$\langle C(T, P) \rangle = \int_T C(T, P, \alpha) p(\alpha) d\alpha$$

We use $\int_T p(\alpha) d\alpha = 1$ and $\int_T \theta_0(\alpha) p(\alpha) d\alpha = A$

$$\begin{aligned}
(12) \quad \langle C(T, P) \rangle = & \frac{K}{T} + \frac{C}{d(p)} \left[1 - \frac{\theta T}{2} + \frac{AT^2}{2} - \frac{1}{T\theta} \right] \\
& + \frac{h}{d(p)} \left[\frac{T}{2} - \frac{T^2}{9} + \frac{AT^3}{12} - \frac{\theta AT^4}{30} - \frac{AT^2}{30} + \frac{\theta T^2}{6} - T^2 \right]
\end{aligned}$$

The minimum mean cost $\langle C(T, P) \rangle$ for fixed prime p is minimized i.e.

$$\frac{\partial \langle C(T, P) \rangle}{\partial T} = 0$$

$$\begin{aligned}
(13) \quad & -\frac{K}{T^2} + \frac{C}{d(p)} \left[-\frac{\theta}{2} + TA - \frac{1}{T^2\theta} \right] \\
& + \frac{h}{d(p)} \left[\frac{1}{2} - \frac{2T}{9} + \frac{AT^2}{14} - \frac{4\theta AT^3}{3} - \frac{2TA}{3\theta} - \frac{T\theta}{3} + 2T \right] = 0
\end{aligned}$$

Equation (13) can be solved numerically for T_p for given values of $K, C, h, d(p)$ etc. Also

$$\frac{\partial^2 \langle C(T, P) \rangle}{\partial T^2} = \frac{2K}{T^3} + \frac{CA}{d(p)} + \frac{C}{\theta d(p) T^4} + \frac{hTA}{2d(p)} + \frac{2h\theta}{d(p)} > 0$$

We maximize profit rate $F(T, P)$ for optimum price decision for a fixed period length. Where $F(T, P) = \frac{P}{d(p)} - \langle C(T, P) \rangle$ by holding T Fixed, find

$$\begin{aligned} \frac{\partial F(T, P)}{\partial T} &= 0 \\ \frac{\partial \langle C(T, P) \rangle}{\partial p} &= -\frac{C d'(p)}{d(p)^2} \left[1 - \frac{\theta T}{2} + \frac{AT^2}{6} - \frac{1}{T\theta} \right] \\ &\quad - \frac{h d'(p)}{d(p)^2} \left[\frac{T}{2} - \frac{T^2}{9} + \frac{AT^3}{12} - \frac{\theta AT^4}{3} - \frac{AT^2}{3\theta} + \frac{\theta T^2}{6} - T^2 \right] \\ \frac{\partial f}{\partial p} &= \frac{p d'(p) - d(p)}{d(p)^2} + \frac{C d'(p)}{d(p)^2} \left[1 - \frac{\theta T}{2} + \frac{AT^2}{6} - \frac{1}{T\theta} \right] \\ &\quad + \frac{h d'(p)}{d(p)^2} \left[\frac{T}{2} - \frac{T^2}{9} + \frac{AT^3}{12} - \frac{\theta AT^4}{3} - \frac{AT^2}{3\theta} - \frac{\theta T^2}{6} + T^2 \right] = 0. \end{aligned}$$

Also $\frac{\partial^2 F}{\partial p^2} > 0$ since $d'(p)$ and $d''(p) < 0$.

4. Model Analysis Special Case

If $\theta_0(\alpha) = 0$ and $d(p) = R$ i.e. when there is no decay and demand rate is constant, then

$$\langle C(T) \rangle = \frac{K}{T} + CR + \frac{hRT}{2}$$

which is standard result for non-decay inventory.

5. Conclusion

In this paper, we developed a deterministic inventory model for deteriorating items when demand is a function of time and selling price. The result of the model is important for formulating the decisions when the inventory decay with constant rate and demand is a function of time and price both. Shortages are not allowed and constant inflation is also considered. The model is solved by cost minimizing criterion. This model

reduces to standard result for non-decaying inventory. Special case is also provided for further study to incorporate any factual relation that may exists between time and price both in the demand rate function and taken suitable deterioration rate. The result of the model is important for formulating the decisions when the inventory deteriorates with time together with another parameter such as temperature humidity etc.

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