

Study of Reflection and Transmission of Thermoelastic Plane Waves at Liquid- Solid Interface

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Abstract: The present paper is concerned with the study of reflection and transmission characteristics of plane waves at liquid-solid interface. The liquid is chosen to be inviscid and the solid half-space is homogeneous isotropic, thermally conducting elastic. Both classical (coupled) and non-classical (generalized) theories of linear thermoelasticity have been employed to investigate the characteristics of reflected and transmitted waves. The amplitude ratios of reflected waves to that of incident one have been obtained for quasi-longitudinal (qP) wave incident. The analytical calculation is carried out by using Gauss Elimination Method. The numerical computations are carried out for Water-Aluminium-epoxy structure by using MATLAB software and the results have been presented graphically.

Keywords: Reflection, Transmission, Elastic Solid, Inviscid fluid, Critical angle.

1. Introduction

The problems of reflection and transmission of waves at an interface between liquid and solid media has many applications in under water acoustics and seismology. Deresiewicz² studied the reflection of plane waves from a plane stress-free boundary in the coupled theory of thermoelasticity. Gutenberg⁴ and Jeffreys⁵ considered the reflection of elastic plane waves at a solid half space. Knott⁷ derived the general equations for reflection and refraction of waves at plane boundaries. Sharma⁹, et al. studied the reflection of piezothermoelastic waves from the charge free and stress free boundary of transversely isotropic half space. Sharma¹⁰, et al. studied the generalized Rayleigh waves in thermoelastic solids under viscous fluid loading.

In this paper, we discuss the reflection and transmission of thermoelastic plane waves at the interface between inviscid liquid half-space and thermo-viscoelastic solid half-space. The effects of incident angles and fluid loading

on reflection and transmission coefficients are considered. The analytical results so obtained have been verified numerically and are illustrated graphically.

2. Formulation of the Problem

We consider a homogeneous isotropic, thermally conducting, elastic solid in the undeformed state initially at uniform temperature T_0 , underlying an inviscid liquid half space. We take the origin of the co-ordinate system (x, y, z) at any point on the plane surface (interface) and the z -axis pointing vertically downward into the solid half space which is thus represented by $z \geq 0$. We choose the x -axis along the direction of wave propagation in such a way that all the particles on the line parallel to the y -axis are equally displaced. Therefore, all the field quantities are independent of y -co-ordinate. Further, the disturbances are assumed to be confined to the neighborhood of the interface $z = 0$ and hence vanish as $z \rightarrow \infty$. In the linear theory of homogeneous isotropic, the basic governing field equations of motion and heat conduction for solid and liquid (inviscid) medium, in the absence of heat sources and body forces, are given by

$$(2.1) \quad \mu \nabla^2 \vec{u} + (\lambda + \mu) \nabla \nabla \cdot \vec{u} - \beta \nabla (T + t_1 \delta_{2k} \dot{T}) = \rho \ddot{\vec{u}},$$

$$(2.2) \quad K \nabla^2 T - \rho C_e (\dot{T} + t_0 \ddot{T}) = \beta T_0 \nabla \cdot (\vec{u} + t_0 \delta_{1k} \ddot{\vec{u}}),$$

where $\beta = (3\lambda + 2\mu)\alpha_t$, α_t being the coefficient of linear thermal expansion. Here λ, μ are Lamé's parameters; ρ is the density of the solid; $T(x, y, t)$ is the temperature change and $\vec{u}(x, z, t) = (u, 0, w)$ is the displacement vector; δ_{jk} is the Kronecker's delta with $k = 1$ for LS theory and $k = 2$ for GL theory; k is the thermal conductivity; t_0 and t_1 are thermal relaxation times; C_e is the specific heat at constant strain of the solid and dot notation is used for time differentiation. The basic governing equations for inviscid fluid (liquid) medium are given by

$$(2.3) \quad \lambda_L \nabla \nabla \cdot \vec{u}_L - \beta_L^* \nabla T_L = \rho_L \ddot{\vec{u}}_L,$$

$$(2.4) \quad T_L = -\frac{\beta_L^* T_0^*}{\rho_L C_V^*} \nabla \cdot \vec{u}_L,$$

where $\beta_L^* = 3\lambda_L\alpha^*$, λ_L is the bulk modulus, ρ_L and α^* are the density and coefficient of volume thermal expansion, $\vec{u}_L$ is the velocity vector and T_L is the temperature deviation in the liquid temperature from ambient temperature T_0^* .

To facilitate the solution we define the following dimensionless quantities

$$(2.5) \quad \begin{aligned} x' &= \frac{\omega^* x}{c_1}, z' = \frac{\omega^* z}{c_1}, t' = \omega^* t, T' = \frac{T}{T_0}, \sigma'_{ij} = \frac{\sigma_{ij}}{\beta_e T_0}, T'_L = \frac{T_L}{T_0} \\ u' &= \frac{\rho \omega^* c_1 u}{\beta_e T_0}, w' = \frac{\rho \omega^* c_1 w}{\beta_e T_0}, u'_L = \frac{\rho \omega^* c_1 u_L}{\beta_e T_0}, w'_L = \frac{\rho \omega^* c_1 w_L}{\beta_e T_0} \\ t'_1 &= \omega^* t, t'_0 = \omega^* t_0, \bar{\beta} = \frac{\beta_L^*}{\beta_e}, \end{aligned}$$

where

$$\begin{aligned} c_3^2 &= \frac{\mu}{\rho}, c_1^2 = \frac{\lambda + 2\mu}{\rho}, \delta^2 = \frac{c_3^2}{c_1^2}, c_L^2 = \frac{\lambda_L}{\rho_L}, \delta_L^2 = \frac{c_L^2}{c_1^2} \\ \omega^* &= \frac{C_e(\lambda + 2\mu)}{K}, \varepsilon = \frac{\beta_e^2 T_0}{\rho C_e(\lambda + \mu)}, \varepsilon_L = \frac{\beta_L^{*2} T_0^*}{\rho_L c_V^* \lambda_L} \end{aligned}$$

Here ω^* is the characteristic frequency of the solid plate; ε is the thermomechanical coupling constant and c_1, c_2 are respectively, the longitudinal and shear wave velocities in the thermoelastic solid half-space; ε_L is the thermomechanical coupling and c_L is the velocity of sound in the fluid.

Upon using quantities (5) alongwith the relations

$$(2.6) \quad u = \varphi_{,x} + \psi_{,z}, \quad w = \varphi_{,z} - \psi_{,x}$$

$$(2.7) \quad u_L = \varphi_{L,x}, \quad w_L = \varphi_{L,z}$$

in equations (2.1)-(2.4), we get

$$(2.8) \quad \nabla^2 \psi - \frac{1}{\delta^2} \ddot{\psi} = 0,$$

$$(2.9) \quad \nabla^2 \varphi - \ddot{\varphi} = T + t_1 \delta_{2k} \dot{T},$$

$$(2.10) \quad \nabla^2 T - (\dot{T} + t_0 \ddot{T}) = \varepsilon \nabla^2 (\dot{\phi} + t_0 \delta_{1k} \ddot{\phi}),$$

$$(2.11) \quad \nabla^2 \varphi_L - \frac{\ddot{\varphi}_L}{\delta_L^2 (1 + \varepsilon_L)} = 0,$$

$$(2.12) \quad T_L = -\frac{\varepsilon_L \rho_L}{\beta \rho (1 + \varepsilon_L)} \ddot{\varphi}_L.$$

3. Boundary Conditions

The boundary conditions at $z = 0$ can be expressed as

$$(3.1) \quad \tau_{zz} = -p, \tau_{xz} = 0, u = u_L, w = w_L, T_{,z} + H(T - T_L) = 0,$$

where H is the Biot's heat transfer constant.

4. Solution of the Problem

We assume wave solutions of the form

$$(4.1) \quad \{\varphi, \psi, T, \varphi_L\} = \{A, B, C, D\} \exp\{ik(x \sin \theta - z \cos \theta - ct)\},$$

where $c = \frac{\omega}{k}$ is the non dimensional phase velocity, ω is the frequency and k is the wave number. Upon using solution (4.1) in equations (2.8)-(2.12), we obtain a system of algebraic equations in unknowns A, B, C and D . The condition for the existence of non-trivial solution of this system of equations upon solving provide us

$$(4.2) \quad k_j^2 = a_j^2 \omega^2 \quad (j=1, 2, 3, 4),$$

where

$$(4.3) \quad a_1^2 + a_2^2 = 1 + \tau_0 - i\omega \varepsilon \tau'_0 \tau_1, \quad a_1^2 a_2^2 = \tau_0, \quad a_3^2 = \frac{1}{\delta^2},$$

$$a_4^2 = \frac{1}{\delta_L^2 (1 + \varepsilon_L)}, \quad \tau_0 = \tau_1 = \tau'_0 = i\omega^{-1}.$$

In the absence of viscous effects ($\alpha_0 = 0 = \alpha_1$) and thermal field ($T = 0 = \varepsilon, T_L = 0 = \varepsilon_L$), we have

$$(4.4) \quad a_1^2 = 1, \quad a_2^2 = \tau_0, \quad a_3^2 = \frac{1}{\delta^2}, \quad a_4^2 = \frac{1}{\delta_L^2}.$$

qP -Wave Incidence upon a Plane Surface

Let the suffix i and r represent incident and reflected waves, respectively. Omitting the term $\exp(-i\omega t)$, we can write

$$(4.5) \quad \varphi = \varphi_i = \sum_{j=1}^2 A_{rj} \exp\{ik_j(x \sin \theta_j + z \cos \theta_j)\},$$

$$(4.6) \quad \psi = \psi_i = A_{r3} \exp\{ik_3(x \sin \theta_3 + z \cos \theta_3)\},$$

$$(4.7) \quad T = T_i = \sum_{j=1}^2 S_j A_{rj} \exp\{ik_j(x \sin \theta_j + z \cos \theta_j)\},$$

$$(4.8) \quad \varphi_L = \varphi_{Li} + \varphi_{Lr} = A_{i4} \exp\{ik_4(x \sin \theta + z \cos \theta)\} \\ + A_4 \exp\{ik_4(x \sin \theta_4 - z \cos \theta_4)\},$$

$$(4.9) \quad T_L = T_{Li} + T_{Lr} = A_{i4} \exp\{ik_4(x \sin \theta + z \cos \theta)\} \\ + S_L A_4 \exp\{ik_4(x \sin \theta_4 - z \cos \theta_4)\},$$

where

$$(4.10) \quad S_j = i\omega\tau_1^{-1}(1 - a_j^2), (j=1,2) \quad S_L = \frac{\varepsilon_L \rho_L}{\bar{\beta}\rho(1 + \varepsilon_L)} \omega^2.$$

Upon using equations (4.5)-(4.9) in the boundary conditions (3.1) alongwith the fact that all the waves, incident, reflected and transmitted must be in phase at the interface $z = 0$ for all values of x and t , we get

$$(4.11) \quad k_1 \sin \theta_1 = k_2 \sin \theta_2 = k_3 \sin \theta_3 = k_4 \sin \theta_4.$$

This with the help of equation (4.2) implies that

$$(4.12) \quad a_1 \sin \theta_1 = a_2 \sin \theta_2 = a_3 \sin \theta_3 = a_4 \sin \theta_4.$$

The equation (4.12) is modified Snell's law in this situation. The analytical expression of reflection and transmission coefficients $R_k^{qP} = A_{rk} / A_{i1}$ ($k=1,2,3$) and $T_1^{qP} = A_4 / A_{i1}$ in the presence of thermal field for incident qP wave are obtained as

$$(4.13) \quad R_1^{qP} = \frac{\Delta_1}{\Delta}, \quad R_2^{qP} = \frac{\Delta_2}{\Delta}, \quad R_3^{qP} = \frac{\Delta_3}{\Delta}, \quad T_1^{qP} = \frac{\Delta_4}{\Delta},$$

where

$$(4.14) \quad \Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ S_1 a_{41} & S_2 a_{42} & 0 & a_{44} \end{vmatrix}$$

and $\Delta_1, \Delta_2, \Delta_3, \Delta_4$ can be obtained from Δ by replacing first, second, third and fourth column by $[-a_{14} \ a_{24} \ a_{34} \ 0]^T$ respectively.

Here

$$(4.15) \quad \begin{aligned} a_{11} &= \cos 2\theta_3, & a_{12} &= \cos 2\theta_3, & a_{13} &= -\sin 2\theta_3, & a_{14} &= -\delta^2 \omega_L, \\ a_{21} &= \delta^2 a_1^2 \sin 2\theta_1, & a_{22} &= \delta^2 a_2^2 \sin 2\theta_2, & a_{23} &= \cos 2\theta_3, & a_{31} &= a_1 \cos \theta_1, \\ a_{32} &= a_2 \cos \theta_2, & a_{33} &= -a_3 \sin \theta_3, & a_{34} &= a_4 \cos \theta_4, & a_{41} &= i\omega a_1 \cos \theta_1 + H, \\ a_{42} &= i\omega a_2 \cos \theta_2 + H, & a_{43} &= 0, & a_{44} &= -HS_L, & \omega_L &= \frac{\rho_L}{\rho \delta^2}. \end{aligned}$$

5. Numerical Results and Discussion

In this section the reflection and transmission coefficients for qP wave incidence at an interface between thermo-viscoelastic solid and inviscid fluid have been computed numerically. The material chosen for this purpose is Aluminium-epoxy composite, the physical data for which is given by Sharma¹⁰, et al.

$$\begin{aligned} \varepsilon &= 0.073, \lambda = 7.59 \times 10^{10} \text{ Nm}^{-2}, \mu = 1.89 \times 10^{10} \text{ Nm}^{-2}, T_0 = 23^\circ \text{C}, \\ \rho &= 2.19 \times 10^3 \text{ kg m}^{-3}, K = 250.8 \text{ W m}^{-1} \text{ deg}^{-1}, C_e = 961.4 \text{ J kg}^{-1} \text{ deg}^{-1}. \end{aligned}$$

The liquid chosen for the purpose of numerical calculations is water, the velocity of sound in which is given by $c_L = 1.5 \times 10^3 \text{ m/s}$ and density is $\rho_L = 1000 \text{ kg m}^{-3}$. $T_0^* = 298 \text{ K}$.

Fig. 1 yields the behaviour of reflection / transmission coefficients for the angles of incidence of longitudinal wave propagates from fluid into solid. It is observed that for longitudinal wave incidence, the reflected longitudinal wave passes through a minimum at critical angle $\theta = 55^\circ$ for both thermoelastic as well as elastic case which is known as Rayleigh-wave angle and at this angle a wave with large surface components is generated.

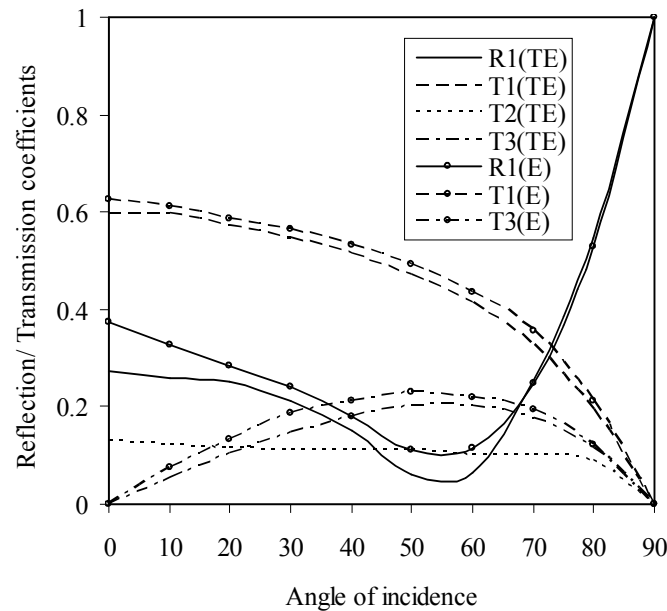


Fig. 1: qP -wave incidence at the interface (TE/E)

6. Conclusions

The reflection and transmission of plane waves at inviscid liquid-thermoelastic solid interface has been studied theoretically. The significant effect of incident angle, thermal and presence of liquid on the amplitude ratios of reflected and transmitted waves have been observed. Rayleigh angle phenomenon is explained. It is shown that reflected surface wave exist for incidence angles greater than Rayleigh wave angle.

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