

A New Approach to Solve Renee and Peter Extensive form Game

S. C. Sharma and Ganesh Kumar

Department of Mathematics

University of Rajasthan, Jaipur, India

Email: sureshchand26@gmail.com, ganeshmandha1988@gmail.com

(Received September 14, 2015)

Abstract: An extensive form game strongly gives the ability of decision making in the global scenarios. In the present paper a particular extensive form game, namely Renee and Peter game is reduced in normal form. Approximate values and list of strategies are obtained using a new approach. Results are compared with value of game using Snow-Shapely procedure for normal form games.

Keywords: Extensive form game, expectation, optimal strategy and inequality, zero sum game.

2010 Mathematics subject classification: 91A05, 91A18.

1. Introduction

One of the solution concepts in extensive form games is an approximation method. In two players zero sum games, this is equivalent to mini-max strategy. Such strategies were obtained using linear programming by Koller, Megiddo and Stengel¹. The original iterative method to solve a game was described by Robinson². Sergienko, Lebedeva and Roshchin³ obtained approximate solutions of many of the game theoretic problems like discrete optimization problems, combinatorial game problems. The competition between two farmers is described, mathematical models are constructed and using permutation constraints the approximate game value is obtained by Yemets and Olkhovska⁴. Morozov and Shalbuzzov⁵ formulated and solved a discrete model of attack and defense using numerical solution method of special matrix games. In present paper we will find the approximate value of Renee and Peter game introduced⁶. Extensive-form games provide a representation which is more suitable for description of scenarios which evolve in time. This representation allows every player

*Presented at NSRDPAM & 19th PDVML -2015, Department of Mathematics, University of Rajasthan, Jaipur during September 12-13, 2015.

to choose action in each state of the game, using exponentially smaller representations in the form of a game tree instead of the utility matrix. Nodes of this tree represent the states of the game and edges actions available in the corresponding states. Leafs represent the terminal states and has the utility value for all players associated with them, representing their preference of this outcome. We distinguish two types of extensive-form games, perfect-information and imperfect information ones. In perfect-information games every player knows everything about the state of the game, whilst in imperfect-information games some of these information's may be hidden. The first part of the paper gives description of Renee and Peter game. In second part game tree is constructed also Renee and Peter game is reduced in normal form. Third part of the paper contains exact value of the game using shapely-snow procedure. Fourth part of the paper is main section in which we obtained the value of Renee and Peter game using approximation method. The approximate value and strategies are written in form of tables. In the last section of the article we compared the approximate value of game with exact value.

2. Renee v/s Peter Extensive form Game

Renee and peter game is played with three cards King, Ten and Deuce. Renee chooses one and lays it face down on the table. Peter calls high or low. If he is right (King=high, Deuce=law), he wins three dollars from Renee, but if he is wrong he loses two dollars. If the card is the Ten he wins two dollars if he called low, but Renee must choose between the King and Deuce if he called high. This time after she lays down her card face down on the table and he calls high or low, he wins one dollar if he is right, but loses three dollars if he is wrong.

3. Game Tree and Normal form of Game

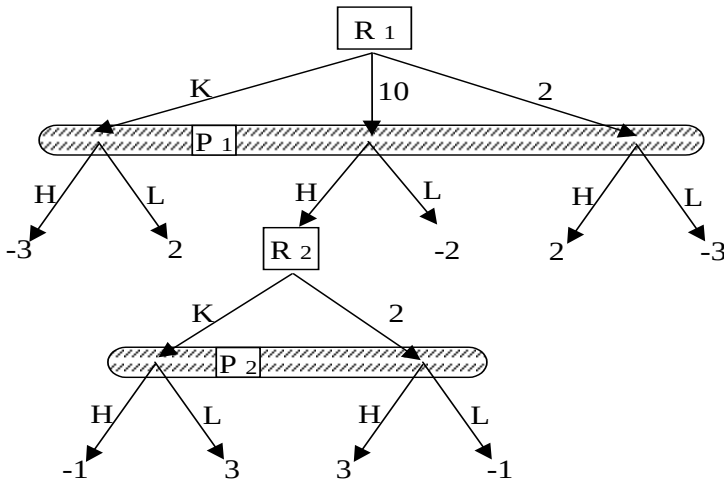


Table 1: Pure strategies for Renee v/s Peter extensive form game

Pure strategies of Renee	Pure strategies of Peter
1. Choose K	1. choose hi, if again choose hi
2. choose Ten, if again choose K	2. choose hi, if again choose low
3. choose Ten, if again choose D	3. choose low
4. choose D	

The extensive form game is reduced in normal form. Then matrix form of Renee and Peter game is given by

Table 2: normal form of game

Players Strategies	B ₁	B ₂	B ₃
A ₁	-3	-3	2
A ₂	-1	3	-2
A ₃	3	-1	-2
A ₄	2	2	-3

4. Exact Solution of Rectangular Game using Shapely-Snow Procedure

Lemma 3.1: For a rectangular game G whose payoff matrix is A , suppose that $v(G)$, the value of game, is different from zero, and let $X \in T_1(G)$ and $Y \in T_2(G)$. Then a necessary and sufficient condition that $X \in K[T_1(G)]$ and $Y \in K[T_2(G)]$ is that there exist a nonsingular sub matrix B of A such that

$$(4.1) \quad v = \frac{1}{J_r B^{-1} J_r^T},$$

$$(4.2) \quad \dot{X} = \frac{J_r B^{-1}}{J_r B^{-1} J_r^T},$$

$$(4.3) \quad \dot{Y} = \frac{J_r B^{-T}}{J_r B^{-1} J_r^T},$$

where r is order of matrix B , $\dot{X}$ is the vector obtained from X by deleting the elements corresponding to the rows deleted to obtain B from A , and $\dot{Y}$ is the vector obtained from Y by deleting the elements corresponding to the columns deleted to obtain B from A .

Also above results can also be written as

$$(4.4) \quad v = \frac{|B|}{J_r(adj B)J_r^T},$$

$$(4.5) \quad \dot{X} = \frac{J_r(adj B)}{J_r(adj B)J_r^T},$$

$$(4.6) \quad \dot{Y} = \frac{J_r(adj B)^T}{J_r(adj B)J_r^T}.$$

By using above method and of Robinson², we get the value of game $v = -\frac{1}{2}$,

Optimal strategy of Renee: $\left(\frac{1}{2}, 0, 0, \frac{1}{2}\right)$,

Optimal strategy of peter: $\left(\frac{1}{4}, \frac{1}{4}, \frac{1}{2}\right)$.

5. A New Approach to Find the Value of Renee and Peter Extensive form Game

In the approximation method two players play a long sequence of plays in given game. Each of them plays in such a way as to maximize his expectation under the assumption that the future will resemble the past. At each point of the sequence one can calculate upper and lower bounds for the value of the game as well as an approximation to an optimal strategy for each player.

Table: 3

Play	R chooses	P chooses		R expect				P expect		
			A ₁	A ₂	A ₃	A ₄	B ₁	B ₂	B ₃	
1.	A ₁	B ₁	-3	-1	3	2	3	3	-2	
2.	A ₃	B ₁	-6	-2	6	4	0	4	0	
3.	A ₃	B ₂	-9	1	5	6	-3	5	2	
4.	A ₄	B ₂	-12	4	4	8	-5	3	5	
5.	A ₄	B ₃	-10	2	2	5	-7	1	8	
6.	A ₄	B ₃	-8	0	0	2	-9	-1	11	
7.	A ₄	B ₃	-6	-2	-2	-1	-11	-3	14	

8.	A ₄	B ₃	-4	-4	-4	-4	-13	-5	17
9.	A ₁	B ₃	-2	-6	-6	-7	-10	-2	15
10.	A ₁	B ₃	0	-8	-8	-10	-7	1	13
11.	A ₁	B ₃	2	-10	-10	-13	-4	4	11
12.	A ₁	B ₃	4	-12	-12	-16	-1	7	9
13.	A ₁	B ₃	6	-14	-14	-19	2	10	7
14.	A ₁	B ₂	3	-11	-15	-17	5	13	5
15.	A ₁	B ₂	0	-8	-16	-15	8	16	2
16.	A ₁	B ₂	-3	-5	-17	-13	11	19	0
17.	A ₁	B ₂	-6	-2	-18	-11	14	22	-2
18.	A ₂	B ₂	-9	1	-19	-9	15	19	0
19.	A ₂	B ₂	-12	4	-20	-7	16	16	2
20.	A ₂	B ₁	-15	3	-17	-5	17	13	4

$\bar{v}_i \rightarrow$ Maximum of numbers in the row of the table under the heading R expect.

$\underline{v}_i \rightarrow$ The negative of maximum of numbers in the row of the table under the heading P expect. And using the inequality $\frac{\underline{v}_i}{i} \leq v \leq \frac{\bar{v}_i}{i}$

Table: 4

Play	$\underline{v}_i$	$\bar{v}_i$	Interval of approximate value $\frac{\underline{v}_i}{i} \leq v \leq \frac{\bar{v}_i}{i}$
1.	-3	3	[-3, 3]
2.	-4	6	[-1.5, 3]
3.	-5	6	[-1.6667, 2]
4.	-5	8	[-1.25, 2]
5.	-8	5	[-1.6, 1]
6.	-11	2	[-1.8333, 0.3333]
7.	-14	-1	[-2, -0.1429]
8.	-17	-4	[-2.125, -0.5]
9.	-15	-2	[-1.6667, -0.4444]
10.	-13	0	[-1.3, 0]
11.	-11	2	[-1, 0.1818]
12.	-9	4	[-0.75, 0.3333]
13.	-10	6	[-0.7692, 0.4615]
14.	-13	3	[-0.9286, 0.2143]
15.	-16	0	[-1.0667, 0]
16.	-19	-3	[-1.1875, -0.1875]
17.	-22	-2	[-1.2941, -0.1176]
18.	-19	1	[-1.0556, 0.0556]
19.	-16	4	[-0.8421, 0.2105]
20.	-17	3	[-0.85, 0.15]

Table: 5

Play	Strategies of player R $X^{(i)} = [x_1^{(i)} x_2^{(i)} x_3^{(i)} x_4^{(i)}]$	Strategies of player P $Y^{(i)} = [y_1^{(i)} y_2^{(i)} y_3^{(i)}]$
1	$[1\ 0\ 0\ 0]$	$[1\ 0\ 0]$
2	$[\frac{1}{2}\ 0\ \frac{1}{2}\ 0]$	$[1\ 0\ 0]$
3	$[\frac{1}{3}\ 0\ \frac{2}{3}\ 0]$	$[\frac{2}{3}\ 0]$
4	$[\frac{1}{4}\ 0\ \frac{1}{2}\ \frac{1}{4}]$	$[\frac{1}{2}\ 0]$
5	$[\frac{1}{5}\ 0\ \frac{2}{5}\ \frac{2}{5}]$	$[\frac{2}{5}\ \frac{2}{5}\ \frac{1}{5}]$
6	$[\frac{1}{6}\ 0\ \frac{1}{3}\ \frac{1}{2}]$	$[\frac{1}{3}\ \frac{1}{3}\ \frac{1}{3}]$
7	$[\frac{1}{7}\ 0\ \frac{2}{7}\ \frac{4}{7}]$	$[\frac{2}{7}\ \frac{2}{7}\ \frac{3}{7}]$
8	$[\frac{1}{8}\ 0\ \frac{1}{4}\ \frac{5}{8}]$	$[\frac{1}{4}\ \frac{1}{4}\ \frac{1}{2}]$
9	$[\frac{2}{9}\ 0\ \frac{2}{9}\ \frac{5}{9}]$	$[\frac{2}{9}\ \frac{2}{9}\ \frac{5}{9}]$
10	$[\frac{3}{10}\ 0\ \frac{1}{5}\ \frac{1}{2}]$	$[\frac{1}{5}\ \frac{1}{5}\ \frac{3}{5}]$
11	$[\frac{4}{11}\ 0\ \frac{2}{11}\ \frac{5}{11}]$	$[\frac{2}{11}\ \frac{2}{11}\ \frac{7}{11}]$
12	$[\frac{5}{12}\ 0\ \frac{1}{6}\ \frac{5}{12}]$	$[\frac{1}{6}\ \frac{1}{6}\ \frac{2}{3}]$
13	$[\frac{6}{13}\ 0\ \frac{2}{13}\ \frac{5}{13}]$	$[\frac{2}{13}\ \frac{2}{13}\ \frac{9}{13}]$
14	$[\frac{1}{2}\ 0\ \frac{1}{7}\ \frac{5}{14}]$	$[\frac{1}{7}\ \frac{3}{14}\ \frac{9}{14}]$
15	$[\frac{8}{15}\ 0\ \frac{2}{15}\ \frac{1}{3}]$	$[\frac{2}{15}\ \frac{4}{15}\ \frac{3}{5}]$
16	$[\frac{9}{16}\ 0\ \frac{1}{8}\ \frac{5}{16}]$	$[\frac{1}{8}\ \frac{5}{16}\ \frac{9}{16}]$
17	$[\frac{10}{17}\ 0\ \frac{2}{17}\ \frac{5}{17}]$	$[\frac{2}{17}\ \frac{6}{17}\ \frac{9}{17}]$
18	$[\frac{5}{9}\ \frac{1}{18}\ \frac{1}{9}\ \frac{5}{18}]$	$[\frac{1}{9}\ \frac{7}{18}\ \frac{1}{2}]$
19	$[\frac{10}{19}\ \frac{2}{19}\ \frac{2}{19}\ \frac{5}{19}]$	$[\frac{2}{19}\ \frac{8}{19}\ \frac{9}{19}]$
20	$[\frac{1}{2}\ \frac{3}{20}\ \frac{1}{10}\ \frac{1}{4}]$	$[\frac{3}{20}\ \frac{2}{5}\ \frac{9}{20}]$

6. Conclusions

In the third section of the paper value of Renee and Peter game using Shapely-Snow procedure is -0.5, in approximation method the best inequality we obtained is

$$v \leq \frac{\bar{v}_8}{8} = -0.5 \text{ and } v \geq \frac{v_{12}}{12} = -0.75.$$

This is a good approximation value of Renee and Peter game. Nowadays every firm uses techniques of operations research to get better payoff to continue their existence and complete their daily operations. In this paper, we have proposed a new approach to handle such problems an approximation solution procedure is proposed to solve extensive form game.

References

1. D. Koller, N. Megiddo and B. Von Stengel, Fast algorithms for finding randomized strategies in game trees, *In annual ACM symposium on theory of computing, STOC's* (1994) 750 – 759.
2. J. Robinson, An iterative method for solving a game, *Annals of mathematics*, **54** (1951) 296-301.
3. V. Sergienko, O. O. Lebedeva and V. A. Roshchin, *Approximate Methods to solve Discrete Optimization Problems*, Russian, Naukova Dumka, Kyiv, 1980.
4. O. O. Yemets and O. V. Ol'khovs'ka, Solution of combinatorial game problems with permutation constraints in both players Iterative method, *Sist. Doslidzh. Inf. Tekhnol*, **4** (2012) 80–93.
5. V. V. Morozov and K. D. Shalbuzov, on a solution of the discrete resource allocation game, *Moscow Univ. Computer Math. Cybern*, **38** (2014) 37–44.
6. A. J. Jones, *Mathematical models of conflict*, Game Theory, Ellis Horwood series, 2000.
7. O. A. Iemets and L. N. Kolehkina, *Combinatorial Optimization Problems with Linear Fractional Objective Functions*, Russian, Naukova Dumka, Kyiv, 2005.