

MHD Flow of a Dusty Non-Newtonian Fluid through a Rectangular Channel

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Abstract: Here, we have studied the MHD flow of a dusty non-Newtonian fluid through a rectangular channel in presence of a uniform transverse magnetic field and found the relation for mean velocity of flow and magnetic field which includes other parameters like non-Newtonian factor, electrical conductivity of fluid, non dimensional parameter and viscosity of fluid. Similarly the effect of other parameters can be observed. In the end, we have mentioned references.

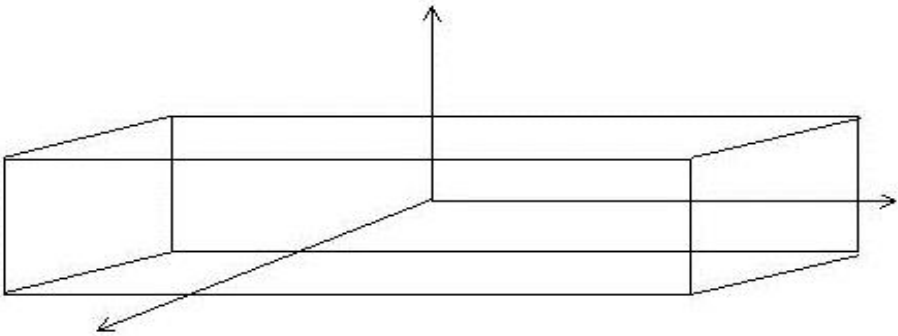
1. Introduction

The applications of dusty fluids through different channels in the presence of a magnetic field have got an important place in the present time e.g. power generation, chemical processing, nuclear reactors, space science and mechanical engineering. Therefore, it has attracted many mathematicians to concentrate on new researches which can be made applicable in this field. First of all, Reddy (1972) discussed the effect of the presence of dust particles with velocity profiles for the unsteady laminar flow of a fluid with uniform distribution of dust through a rectangular channel. Gupta and Gupta (1976) obtained the expressions using operational method for velocities of gas and particles in a dusty gas flow through a rectangular channel. Sastry and Bhadram (1976) found an analytical expression for hydromagnetic slow motion in a curved channel of rectangular cross-section in presence of magnetic field applied at right angle to the horizontal plane. Singh (1976) gave theory about the unsteady flow of a conducting dusty viscous incompressible fluid through a long rectangular channel under the influence of uniform magnetic field and time dependent

pressure gradient and discussed the particular case for periodic function of time, an absolute and exponentially decreasing function of time. Kumar and Singh (1991) discussed the unsteady MHD flow of a dusty viscous liquid through an open rectangular channel. Kundu and Sengupta (2001) investigated the unsteady flow of a visco- elastic Oldroyd fluid with pressure gradient through a rectangular channel. In 2004, I also found the expressions for the flow of a dusty fluid through rectangular channel. Madhuret. al (2009) discussed the motion of a dusty gas through porous media in an open rectangular channel and obtained the expression of skin friction at the boundaries. Gireesha et. al (2010) studied the unsteady flow of a dusty fluid through a rectangular channel under the influence of pulsatile pressure gradient with effect of uniform magnetic field. But as most of the fluids are non -Newtonian and very important in industries and therefore, after inserting necessary factor for it in the previous models, we have considered these fluids flowing through a rectangular channel and using the method variable separable, the flow has been discussed graphically.

2. Formulation and Solution of the Problem

Consider an unsteady flow of an incompressible non-Newtonian electrically conducting viscous fluid with uniform distribution of dust particles through rectangular channel placed in a transversely applied magnetic field. The flow is taking along z-axis assumed horizontally and it is as shown in figure.



The velocities of both fluid and dust particles are given by

$$u = u(x, y, t), v = v(x, y, t).$$

The equations of motion will be given by

(1.0) For fluid phase : $\nabla \cdot u = 0$ and

$$(2.0) \quad \frac{\partial u}{\partial t} + (u \cdot \nabla) u = -\frac{1}{\rho} \nabla p + \nu \nabla^2 u + \frac{f}{\tau} (v - u) - \frac{\sigma B_0^2 u}{\rho(1+c^2)}.$$

(1.1) For dust phase: $\nabla \cdot v = 0$ and $\frac{\partial v}{\partial t} + (v \cdot \nabla) v = \frac{1}{\tau} (u - v)$

where u is the velocity of fluid, v is the velocity of particle, m is mass of dust particle, ρ is the density of fluid, p is the static pressure of fluid, N_0 is the number of dust particles per unit volume, K is the Stoke's resistance coefficient, B_0 is the external magnetic field, σ is the electrical conductivity of fluid, ν is the kinetic viscosity of fluid, f is the $\frac{mN_0}{\rho}$, i.e. mass concentration ratio, $\tau = \frac{m}{K}$ is time relaxation parameter and c is the non-Newtonian factor.

The equations (1.0) and (1.1) can be written as:

For fluid phase

$$(1.2) \quad \frac{\partial u}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \frac{f}{\tau} (v - u) - \frac{\sigma B_0^2}{\rho(1+c^2)} u.$$

For dust phase

$$(1.3) \quad \frac{\partial v}{\partial t} = \frac{1}{\tau} (u - v).$$

The boundary conditions are

$$(1.4) \quad \begin{aligned} u &= 0, & v &= 0 & \text{at } y &= 0, \\ u &= 0, & v &= 0 & \text{at } y &= h, \\ u &= 0, & v &= 0 & \text{at } x &= -d \\ u &= 0, & v &= 0 & \text{at } x &= +d \end{aligned}$$

where h is the distance between the plates.

We assume that the pressure gradient is of the form

$$(1.5) \quad -\frac{1}{\rho} \frac{\partial p}{\partial z} = A' (1 + \varepsilon e^{i\omega t})$$

where ε is small quantity and A' and ω are constants.

To solve the above system of equations, we introduce the following non dimensional variables.

$$\bar{u} = \frac{u\omega}{A}, \quad \bar{v} = \frac{v\omega}{A}, \quad \bar{t} = t\omega, \quad \xi = \frac{x}{h}, \quad \eta = \frac{y}{h}, \quad \varphi = \frac{z}{h}$$

$$\bar{p} = \frac{p}{A\rho h} \text{ and } \text{Re} = \frac{\omega h^2}{\nu}$$

(Re: Reynolds number of fluid)

Using above non dimensional variables in equations (1.4) and (1.5), we get

$$\frac{\partial \bar{u}}{\partial \bar{t}} = -\frac{\partial \bar{p}}{\partial \varphi} + \frac{\nu}{\omega h^2} \left(\frac{\partial^2 \bar{u}}{\partial \xi^2} + \frac{\partial^2 \bar{u}}{\partial \eta^2} \right) + \frac{f}{\tau \omega} (\bar{v} - \bar{u}) - \frac{\sigma B_0^2}{\rho(1+c^2)\omega} \bar{u} \text{ and}$$

$$\frac{\partial \bar{v}}{\partial \bar{t}} = \frac{1}{\tau \omega} (\bar{u} - \bar{v}),$$

Now removing bars, the above equations become

$$(1.6) \quad \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial \varphi} + \frac{1}{\text{Re}} \left(\frac{\partial^2 u}{\partial \xi^2} + \frac{\partial^2 u}{\partial \eta^2} \right) + f\beta (v - u) - \alpha u$$

$$(1.7) \quad \frac{\partial v}{\partial t} = \beta (u - v) \text{ where } \beta = \frac{1}{\tau \omega}, \quad \alpha = \frac{\sigma B_0^2}{\rho(1+c^2)\omega}$$

Then the boundary conditions are modified as

$$\begin{aligned} u &= 0, & v &= 0 & \text{at } \eta &= 0, \\ u &= 0, & v &= 0 & \text{at } \eta &= 1, \\ u &= 0, & v &= 0 & \text{at } \xi &= -r \\ u &= 0, & v &= 0 & \text{at } \xi &= r \end{aligned}$$

Also $r = d/h$

Now the pressure gradient is given by

$$(1.9) \quad -\frac{\partial p}{\partial \varphi} = (1 + \varepsilon e^{it})$$

Now we suppose the solution of the velocities are of the form

$$(2.0) \quad u(\xi, \eta, t) = u_0(\xi, \eta) + \varepsilon u_1(\xi, \eta) e^{it}$$

$$v(\xi, \eta, t) = v_0(\xi, \eta) + \varepsilon v_1(\xi, \eta) e^{it}$$

Putting the value of $\frac{\partial p}{\partial \varphi}$ and u from equations (1.6) and (2.0) in equations (1.6) and (1.7), we get

$$\begin{aligned} \varepsilon i u_1(\xi, \eta) e^{it} = & (1 + \varepsilon e^{it}) + \frac{1}{\text{Re}} \left(\frac{\partial^2 u_0}{\partial \xi^2} + \varepsilon \frac{\partial^2 u_1}{\partial \xi^2} e^{it} + \frac{\partial^2 u_0}{\partial \eta^2} + \varepsilon \frac{\partial^2 u_1}{\partial \eta^2} e^{it} \right) \\ & + f\beta (v_0 - u_0) + f\beta \varepsilon (v_1 - u_1) e^{it} - \alpha (u_0 + \varepsilon u_1 e^{it}) \end{aligned}$$

and

$$\varepsilon i v_1(\xi, \eta) e^{it} = \beta (u_0 - v_0) + \beta \varepsilon (u_1 - v_1) e^{it}.$$

Collecting the coefficient of similar power of ε on both sides, we get the following equations.

Steady part (coefficient of ε^0)

$$(2.1) \quad \frac{1}{\text{Re}} \left(\frac{\partial^2 u_0}{\partial \xi^2} + \frac{\partial^2 u_0}{\partial \eta^2} \right) + f\beta (v_0 - u_0) - \alpha u_0 = -1$$

and

$$(2.2) \quad \beta (u_0 - v_0) = 0.$$

Unsteady part (coefficient of ε)

$$(2.3) \quad \frac{1}{\text{Re}} \left(\frac{\partial^2 u_1}{\partial \xi^2} + \frac{\partial^2 u_1}{\partial \eta^2} \right) + f\beta (v_1 - u_1) - (\alpha + i) u_1 = -1$$

and

$$(2.4) \quad \beta (u_1 - v_1) - i v_1 = 0.$$

Then the corresponding dimensionless boundary conditions become

$$(2.5) \quad \begin{aligned} u_0 = 0, \quad v_0 = 0 \text{ at } \xi = -r, \quad u_0 = 0, \quad v_0 = 0 \text{ at } \xi = r, \\ u_0 = 0, \quad v_0 = 0 \text{ at } \eta = 0 \text{ and } u_0 = 0, \quad v_0 = 0 \text{ at } \eta = 1 \end{aligned}$$

$$(2.6) \quad \begin{aligned} u_1 = 0, \quad v_1 = 0 \text{ at } \xi = -r, \quad u_1 = 0, \quad v_1 = 0 \text{ at } \xi = r, \\ u_1 = 0, \quad v_1 = 0 \text{ at } \eta = 0 \text{ and } u_1 = 0, \quad v_1 = 0 \text{ at } \eta = 1 \end{aligned}$$

By putting equation (6.13) in equation (6.12), we have

$$(2.7) \quad \frac{\partial^2 u_0}{\partial \xi^2} + \frac{\partial^2 u_0}{\partial \eta^2} - \alpha \operatorname{Re} u_0 = -\operatorname{Re}$$

To solve equation (2.7), we assume the solution of the form

$$(2.8) \quad u_0(\xi, \eta) = X(\xi, \eta) + Y(\xi)$$

Then equations become

$$\frac{\partial^2 X}{\partial \xi^2} + \frac{\partial^2 X}{\partial \eta^2} + \frac{\partial^2 Y}{\partial \xi^2} - \alpha \operatorname{Re}(X + Y) = -\operatorname{Re},$$

So that
$$\frac{\partial^2 Y}{\partial \xi^2} - \alpha \operatorname{Re} Y + \operatorname{Re} = 0$$

$$(2.9) \quad \frac{d^2 Y}{d\xi^2} - \alpha \operatorname{Re} Y + \operatorname{Re} = 0$$

and

$$(3.0) \quad \frac{\partial^2 X}{\partial \xi^2} + \frac{\partial^2 X}{\partial \eta^2} - \alpha \operatorname{Re} X = 0.$$

The corresponding boundary conditions change as

$$(3.1) \quad \begin{aligned} u_0(-r, \eta) &= X(-r, \eta) + Y(-r) = 0, \\ u_0(r, \eta) &= X(r, \eta) + Y(r) = 0 \\ u_0(\xi, 0) &= X(\xi, 0) + Y(\xi) = 0, \\ u_0(\xi, 1) &= X(\xi, 1) + Y(\xi) = 0 \end{aligned}$$

and

$$(3.2) \quad \begin{aligned} u_1(-r, \eta) &= X_1(-r, \eta) + Y_1(-r) = 0, \\ u_1(r, \eta) &= X_1(r, \eta) + Y_1(r) = 0 \\ u_1(\xi, 0) &= X_1(\xi, 0) + Y_1(\xi) = 0, \\ u_1(\xi, 1) &= X_1(\xi, 1) + Y_1(\xi) = 0. \end{aligned}$$

The solution of equation (2.9) is given as $Y(\xi) = C.F + P.I$

$$(3.3) \quad Y(\xi) = B_1 e^{\sqrt{\alpha \text{Re}} \xi} + B_2 e^{-\sqrt{\alpha \text{Re}} \xi} + \frac{1}{\alpha},$$

where B_1 and B_2 are constants and they may be determined by applying conditions (3.1) as

$$B_1 = -\frac{1}{\alpha} \left[\frac{1}{2 \cosh(\sqrt{\alpha \text{Re}} r)} \right], B_2 = -\frac{1}{\alpha} \left[\frac{1}{2 \cosh(\sqrt{\alpha \text{Re}} r)} \right]$$

Putting the value of B_1 and B_2 in equation (3.3), we have

$$(3.4) \quad Y(\xi) = \frac{1}{\alpha} \left[1 - \frac{\cosh \sqrt{\alpha \text{Re}} \xi}{\cosh \sqrt{\alpha \text{Re}} r} \right].$$

Now solution for equation (3.0) is obtained by the method of separation of variables.

Let

$$(3.5) \quad X(\xi, \eta) = Z(\xi) N(\eta).$$

Then the equation (3.0) becomes

$$(3.6) \quad \frac{1}{Z} \frac{d^2 Z}{d\xi^2} = -\frac{1}{N} \frac{d^2 N}{d\eta^2} + \alpha \text{Re} = k \text{ (say } -p^2 \text{)}.$$

Therefore, solution of equation (3.5) is given by

$$(3.7) \quad X(\xi, \eta) = (C_1 \cosh p \xi + C_2 \sinh p \xi) \left((C_3 + C_4) \cosh \sqrt{\alpha \text{Re} + p^2} \eta + (C_3 - C_4) \sinh \sqrt{\alpha \text{Re} + p^2} \eta \right),$$

$$(3.8) \quad X(\xi, \eta) = \frac{2}{\alpha} \sum_{n=1}^{\infty} \sin \left(\frac{n\pi}{r} \xi \right) \left(\frac{\sinh A(\eta-1) - \sinh A\eta}{\sinh A} \right) \times \left(\frac{1}{n\pi} - \frac{\alpha \text{Re}}{A^2 n\pi} (-1)^n - \frac{n\pi}{r^2 A^2 \cosh \sqrt{\alpha \text{Re}} r} \right).$$

Then, we have

$$(3.9) \quad u_0(\xi, \eta) = \frac{1}{\alpha} \left(1 - \frac{\cosh \sqrt{\alpha \text{Re}} \xi}{\cosh \sqrt{\alpha \text{Re}} r} \right) + \frac{2}{\alpha} \sum_{n=1}^{\infty} \sin \left(\frac{n\pi}{r} \xi \right) \left(\frac{\sinh A(\eta-1) - \sinh A\eta}{\sinh A} \right) \times$$

$$\left(\frac{1}{n\pi} - \frac{\alpha \text{Re}}{A^2 n\pi} (-1)^n - \frac{n\pi}{r^2 A^2 \cosh \sqrt{\alpha \text{Re} r}} \right).$$

Also the equation (2.3) can be written as

$$(4.0) \quad \frac{\partial^2 u_1}{\partial \xi^2} + \frac{\partial^2 u_1}{\partial \eta^2} - \text{Re} \left(\frac{f\beta i}{\beta + i} + (\alpha + i) \right) u_1 = -\text{Re},$$

Let

$$(4.1) \quad u_1(\xi, \eta) = X_1(\xi, \eta) + Y_1(\xi)$$

equation (4.1) can be written as

$$(4.2) \quad \frac{\partial^2 Y_1}{\partial \xi^2} - \text{Re} \left(\frac{f\beta i}{\beta + i} + (\alpha + i) \right) Y_1 + \text{Re} = 0$$

or

$$(4.3) \quad \frac{d^2 Y_1}{d\xi^2} - \text{Re} \left(\frac{f\beta i}{\beta + i} + (\alpha + i) \right) Y_1 + \text{Re} = 0$$

Modified form of equation (4.2) is

$$\frac{d^2 Y_1}{d\xi^2} - Q^2 Y_1 + \text{Re} = 0$$

where

$$Q^2 = \text{Re} \left(\frac{f\beta i}{\beta + i} + (\alpha + i) \right),$$

whose solution is

$$(4.4) \quad Y_1 = B_3 e^{Q\xi} + B_4 e^{-Q\xi} + \frac{\text{Re}}{Q^2},$$

where B_3 and B_4 are constants and can be determined by using boundary conditions, we find that

$$B_3 = -\frac{\text{Re}}{Q^2 2 \cosh(Qr)}, \quad B_4 = -\frac{\text{Re}}{Q^2 2 \cosh(Qr)}.$$

Equation (4.4) becomes

$$(4.5) \quad Y_1(\xi) = \frac{\text{Re}}{Q^2} \left(1 - \frac{\cosh(Q\xi)}{\cosh(Qr)} \right).$$

Now we find the solution for equation (4.3) as

$$(4.6) \quad \frac{\partial^2 X_1}{\partial \xi^2} + \frac{\partial^2 X_1}{\partial \eta^2} - Q^2 X_1 = 0,$$

Let

$$(4.7) \quad X_1 = Z_1(\xi) N_1(\eta)$$

then above equation becomes

$$(4.8) \quad \frac{1}{Z_1} \frac{d^2 Z_1}{d\xi^2} = -\frac{1}{N_1} \frac{d^2 N_1}{d\eta^2} + Q^2 = -p^2 \text{ (Say)}$$

$$(4.9) \quad \frac{d^2 N_1}{d\eta^2} - (Q^2 + p^2) N_1 = 0$$

Solution of equation (4.8) is given as

$$(5.0) \quad Z_1 = C.F. + P.I. = (C_5 \cos p\xi + C_6 \sin p\xi)$$

and solution of equation (4.9) is

$$(5.1) \quad N_1 = \left((C_7 + C_8) \cosh \sqrt{p^2 + Q^2} \eta + (C_7 - C_8) \sinh \sqrt{p^2 + Q^2} \eta \right),$$

$$(5.2) \quad X_1(\xi, \eta) = (C_5 \cos p\xi + C_6 \sin p\xi) X$$

$$\left((C_7 + C_8) \cosh \sqrt{p^2 + Q^2} \eta + (C_7 - C_8) \sinh \sqrt{p^2 + Q^2} \eta \right).$$

Applying boundary condition (6.23) in above equation, we have

$$X_1(\xi, \eta) = C_6 \sin \left(\frac{n\pi}{r} \xi \right) \left((C_7 + C_8) \cosh B \eta + (C_7 - C_8) \sinh B \eta \right)$$

$$\text{where } B = \sqrt{\frac{n^2 \pi^2}{r^2} + Q^2}$$

By similar method, we get

$$(5.3) \quad X_1(\xi, \eta) = \frac{2\text{Re}}{Q^2} \sum_{n=1}^{\infty} \sin\left(\frac{n\pi}{r}\xi\right) \left(\frac{\sinh B(\eta-1) - \sinh B\eta}{\sinh B} \right) \times \\ \left(\frac{1}{n\pi} - \frac{Q^2}{n\pi B^2} (-1)^n - \frac{n\pi}{r^2 B^2 \cosh(Qr)} \right)$$

The equation (4.1) becomes

$$(5.4) \quad u_1(\xi, \eta) = \frac{\text{Re}}{Q^2} \left(1 - \frac{\cosh(Q\xi)}{\cosh(Qr)} \right) + \frac{2\text{Re}}{Q^2} \sum_{n=1}^{\infty} \sin\left(\frac{n\pi}{r}\xi\right) \left(\frac{\sinh B(\eta-1) - \sinh B\eta}{\sinh B} \right) \times \\ \left(\frac{1}{n\pi} - \frac{Q^2}{n\pi B^2} (-1)^n - \frac{n\pi}{r^2 B^2 \cosh(Qr)} \right) \\ (5.5) \quad v_1(\xi, \eta) = \frac{\beta}{\beta + i} u_1(\xi, \eta).$$

Fluid velocity and particle velocity are given by

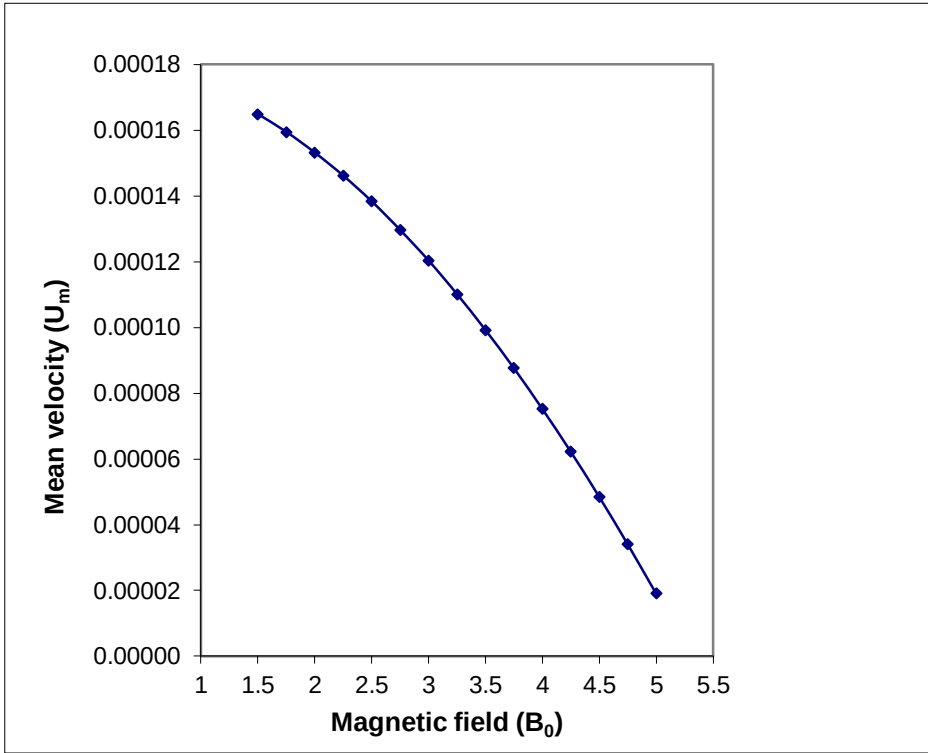
$$(5.6) \quad u(\xi, \eta, t) = u_0(\xi, \eta) + \varepsilon u_1(\xi, \eta) e^{it} \\ = \frac{1}{\alpha} \left(1 - \frac{\cosh \sqrt{\alpha \text{Re}} \xi}{\cosh \sqrt{\alpha \text{Re}} r} \right) + \frac{2}{\alpha} \sum_{n=1}^{\infty} \sin\left(\frac{n\pi}{r}\xi\right) \left(\frac{\sinh A(\eta-1) - \sinh A\eta}{\sinh A} \right) \times \\ \left(\frac{1}{n\pi} - \frac{\alpha \text{Re}}{A^2 n\pi} (-1)^n - \frac{n\pi}{r^2 A^2 \cosh \sqrt{\alpha \text{Re}} r} \right) + \varepsilon \frac{\text{Re}}{Q^2} \left(1 - \frac{\cosh(Q\xi)}{\cosh(Qr)} \right) \\ + \frac{2\text{Re}}{Q^2} \sum_{n=1}^{\infty} \sin\left(\frac{n\pi}{r}\xi\right) \left(\frac{\sinh B(\eta-1) - \sinh B\eta}{\sinh B} \right) \\ \times \left(\frac{1}{n\pi} - \frac{Q^2}{n\pi B^2} (-1)^n - \frac{n\pi}{r^2 B^2 \cosh(Qr)} \right),$$

$$\begin{aligned}
 (5.7) \quad v(\xi, \eta, t) = & \frac{1}{\alpha} \left(1 - \frac{\cosh \sqrt{\alpha \operatorname{Re} \xi}}{\cosh \sqrt{\alpha \operatorname{Re} r}} \right) \\
 & + \frac{2}{\alpha} \sum_{n=1}^{\infty} \sin \left(\frac{n\pi}{r} \xi \right) \left(\frac{\sinh A(\eta-1) - \sinh A\eta}{\sinh A} \right) \times \\
 & \left(\frac{1}{n\pi} - \frac{\alpha \operatorname{Re}}{A^2 n\pi} (-1)^n - \frac{n\pi}{r^2 A^2 \cosh \sqrt{\alpha \operatorname{Re} r}} \right) \\
 & + \varepsilon \left(\frac{\beta}{\beta + i} \right) \frac{\operatorname{Re}}{Q^2} \left(1 - \frac{\cosh(Q\xi)}{\cosh(Qr)} \right) \\
 & + \frac{2\operatorname{Re}}{Q^2} \sum_{n=1}^{\infty} \sin \left(\frac{n\pi}{r} \xi \right) \left(\frac{\sinh B(\eta-1) - \sinh B\eta}{\sinh B} \right) \\
 & \times \left(\frac{1}{n\pi} - \frac{Q^2}{n\pi B^2} (-1)^n - \frac{n\pi}{r^2 B^2 \cosh(Qr)} \right) e^{it}
 \end{aligned}$$

and the mean velocity is given by

$$\begin{aligned}
 (5.8) \quad U_m = & \frac{1}{\alpha} \left(1 - \frac{\cosh \sqrt{\alpha \operatorname{Re} \xi}}{\cosh \sqrt{\alpha \operatorname{Re} r}} \right) + \frac{2}{\alpha} \sum_{n=1}^{\infty} \sin \left(\frac{n\pi}{r} \xi \right) \left(\frac{\sinh A(\eta-1) - \sinh A\eta}{\sinh A} \right) \times \\
 & \left(\frac{1}{n\pi} - \frac{\alpha \operatorname{Re}}{A^2 n\pi} (-1)^n - \frac{n\pi}{r^2 A^2 \cosh \sqrt{\alpha \operatorname{Re} r}} \right).
 \end{aligned}$$

Thus we have derived relations for the velocities of both fluid and particle phases and now we assume suitable values of various parameters to observe the variations of mean velocity with respect to the magnetic field.



Graph between Magnetic field and Mean velocity (1.0)

3. Result and Discussion

This graph shows the variation of mean velocity with respect to magnetic field keeping all other parameters constant. It is very clear that mean velocity decreases with respect to the magnetic field. This can be utilized for practical purpose. Hence the magnetic field has been found very effective in controlling the velocity of the system along with other parameters.

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