

Effects of Radiation and Time Dependent Permeability on Unsteady Two-Dimensional Hydromagnetic Free Convective Flow and Heat Transfer through Porous Medium Bounded between Parallel Plates

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Abstract: Aim of the present paper is to investigate the unsteady, two-dimensional, hydromagnetic, free convective flow with radiative heat transfer through porous medium in the presence of periodic cross-flow velocity and time dependent permeability. The porous medium is confined between two infinite parallel plates. The governing equations of fluid velocity and temperature are solved by perturbation technique and separation of variable technique. The coefficient of skin-friction and Nusselt number at the plates are derived, discussed numerically. Numerical values of skin-friction coefficient for various values of physical parameters are presented through table and the Nusselt number is shown through graphs.

Keywords: Unsteady, Radiative heat flux, MHD, free convection, porous medium, skin-friction, Nusselt number.

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1. Introduction

Free convective flows past different types of bodies are studied because of their wide applications. Unsteady MHD free convection heat transfer with

or without radiation effect have technical importance in the scientific community. Some of the literature surveys and reviews of pertinent work in this field are documented by some mathematicians also.

Free convection flow about a vertical flat plate embedded in a porous medium with application to heat transfer was studied by Cheng and Minkowycz¹. Bharali and Borkakati² investigated the effects of Hall currents on MHD flow and heat transfer between two parallel porous plates. The free-convection in the laminar boundary layer flow of thermomicro-polar fluid past a non-isothermal vertical flat plate with suction/injection was investigated by Jha and Prasad³. A ray tracing method for evaluating the radiative heat transfer in porous medium was analyzed by Argento and Bouvard⁴. The radiative and free convective effects of a MHD flow through a porous medium between infinite parallel plates with time dependent suction was studied by Alagoa et al.⁵. Chamkha et al.⁶ discussed the radiation effects on free convection flow past a semi-infinite vertical plate with mass transfer. Sharma et al.⁷ discussed the unsteady plane poiseuille flow and heat transfer in the presence of oscillatory temperature of the lower plate. Chamkha⁸ studied the unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate with heat absorption. The free convection and radiation characteristics for a vertical plate embedded in a porous medium was investigated by Badruddin et al.⁹. Attia¹⁰ presented the unsteady flow of a dusty conducting fluid between parallel porous plates with temperature dependent viscosity. Sharma and Yadav¹¹ studied the Pulsatile MHD flow and heat transfer through a porous channel with ohmic effect. The unsteady MHD free convective flow of a compressible fluid past a moving vertical plate in the presence of radiative heat transfer was investigated by Mbeledogu et al.. Heat and mass transfer of an unsteady MHD natural convection flow of a rotating fluid past a vertical porous plate in the presence of radiative heat transfer was discussed by Mbeledogu and Ogulu¹². Mahdya et.al.¹³ studied the heat and mass transfer in MHD free convective flow along a vertical wavy plate with variable surface heat and mass flux. The conjugate heat transfer free convection along a vertical plate fin was investigated by Haiso. Murthy and El-amin¹⁴ investigated the thermo diffusion effect on free convection heat and mass transfer in a thermally linearly stratified non Darcy porous media.

The aim of the present paper is to investigate the effects of radiation and time dependent permeability on unsteady two-dimensional free convective flow of a viscous incompressible electrically conducting fluid through porous medium confined between two infinite non-conducting parallel

plates with time dependent suction in the presence of magnetic field applied normal to the plates.

2. Mathematical Formulation

Consider the unsteady two-dimensional viscous incompressible electrically conducting fluid between two infinite parallel non-conducting plates in the presence of transverse magnetic field such that induced magnetic field is neglected. The upper plate is porous and absorbs the fluid with time dependent velocity and moving in its own plane. Also the radiation effects on unsteady free convection flow of the fluid is initiated by a shearing action of the moving wall with resulting suction and pressure gradient along the direction of the motion.

The two-dimensional governing equations of flow [Schlichting and Gersten¹⁵, Bansal¹⁶] of a viscous incompressible electrically conducting fluid through porous medium [Beavers and Joseph¹⁷] confined between two infinite non-conducting parallel plates in the presence of transverse magnetic field [Jefferey¹⁸, Bansal¹⁹] are

Equation of Continuity

$$(1) \quad \frac{\partial v^*}{\partial y^*} = 0,$$

Equations of Motion

$$(2) \quad \frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} = \nu \frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\nu u^*}{K_0 (1 + \epsilon e^{i\omega^* t^*})} - \frac{e B_0^2 u^*}{\rho} + g \beta (T^* - T_\infty),$$

$$(3) \quad \frac{\partial v^*}{\partial t^*} = -\frac{1}{\rho} \frac{\partial p^*}{\partial y^*} - \frac{\nu v^*}{K_0 (1 + \epsilon e^{i\omega^* t^*})},$$

Equation of Energy

$$(4) \quad \frac{\partial T^*}{\partial t^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{\kappa}{\rho C_p} \frac{\partial^2 T^*}{\partial y^{*2}} - \frac{1}{\rho C_p} \frac{\partial q^* R}{\partial y^*},$$

Equation of Radiative Heat Flux

$$(5) \quad \frac{\partial^2 q^*}{\partial y^{*2}} + 3\alpha^2 q^* - 16\sigma \alpha T^{*3} \frac{\partial T^*}{\partial y^*} = 0,$$

where T^* represents temperature of fluid, C_p is the specific heat at constant pressure, κ the thermal conductivity, σ the Stefan Boltzman's constant, α the absorption coefficient, q^* the radiative heat flux parameter, g the acceleration due to gravity, ρ the density of the fluid, T_∞ the temperature of the free stream, ϵ a positive small parameter, B_0 the magnetic field intensity, $\delta\Omega$ the coefficient of volume expansion of the fluid for heat transfer, u^* and v^* are the velocity components along x^* -axis and y^* -axis, respectively, ν^* the kinematic viscosity, ω^* the frequency of vibration of the fluid particles, K_0 the permeability parameter, t^* the time and p^* is the pressure.

Now, applying the condition of 'optically thin' environment such as one would find in the intergalactic layers, where the plasma gas is assumed to be of low density. This assumption suggests that, the absorption coefficient α is much less than unity (i.e. $\alpha \ll 1$). Following Cheng²⁰ the radiative heat flux equation (5) reduces to,

$$(6) \quad \frac{\partial q^*}{\partial y^*} = 4\sigma \alpha (T^{*4} - T_\infty^4).$$

From equations (4) and (6), we have

$$(7) \quad \frac{\partial T^*}{\partial t^*} + v^* \frac{\partial T^*}{\partial y^*} + \frac{\kappa}{\rho C_p} \frac{\partial^2 T^*}{\partial y^{*2}} - \frac{4\sigma \alpha}{\rho C_p} (T^{*4} - T_\infty^4) = 0.$$

The boundary conditions are

$$(8) \quad y^* = 0 : \quad u^* = 0, \quad v^* = 0, \quad T^* = T_w^* \\ y^* = h : u^* = u_0 \left(1 + \epsilon e^{i\omega^* t^*}\right), T^* = T_\infty \left(1 + \epsilon e^{i\omega^* t^*}\right)$$

where h is the distance between the plates, T_w^* is the very high temperature of upper plate such that $T_\infty \ll T_w^*$. Equation (1) gives

$$(9) \quad v^* = v_0 \left(1 + \epsilon A e^{i\omega^* t^*}\right),$$

where v_0 is a non-zero constant suction velocity and A is a real positive constant such that $\epsilon A \ll 1$.

3. Method of Solution

Introducing the following non-dimensional quantities

$$y^* = h\eta, t^* = \left(\frac{4h}{v_0}\right)t, \omega^* = \left(\frac{v_0}{4h}\right)\omega, (u^*, v^*) = (u, v)v_0.$$

$$(10) \quad (T^*, T_w^*) = (\theta, \theta_w)v_0, \text{Re} = \frac{v_0 h}{\nu}, \nu = \frac{\mu}{\rho}, K^2 = \frac{h^2}{K_0}, M^2 = \frac{e B_0^2 h^2}{\rho \nu},$$

$$Gr = \frac{g \beta T_\infty h^2}{v_0 \nu}, \text{Pr} = \frac{\mu C_p}{\kappa}, N^2 = \frac{4 \sigma \alpha h^2 T_\infty^*}{\kappa};$$

into the equations (2) and (7), we get

$$(11) \quad \frac{\text{Re}}{4} \frac{\partial u}{\partial t} + \text{Re}(1 + \epsilon A e^{i\omega t}) \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial \eta^2} - \left(\frac{K^2}{1 + \epsilon A e^{i\omega t}} + M^2 \right) u + Gr(\theta - 1)$$

$$(12) \quad \frac{\text{Re Pr}}{4} \frac{\partial \theta}{\partial t} + \text{Re Pr}(1 + \epsilon A e^{i\omega t}) \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial \eta^2} - N^2(\theta^4 - 1),$$

where Pr is the Prandtl number, N the radiation parameter, M the Hartmann number, Gr the Grashof number, K the permeability parameter and Re is the Reynolds number.

The boundary conditions are reduced to

$$(13) \quad \eta = 0 : \quad u = 0, \quad \theta = \theta_w;$$

$$\eta = 1 : u = (1 + \epsilon e^{i\omega t}), \quad \theta = (1 + \epsilon e^{i\omega t}).$$

Since the equations (11) and (12) are non-linear second order coupled differential equations. Applying the regular perturbation technique, $u(\eta, t)$ and $\theta(\eta, t)$ are expended as given below

$$(14) \quad u(\eta, t) = u_0(\eta) + \epsilon u_1(\eta, t) + \dots, \\ \theta(\eta, t) = \theta_0(\eta) + \epsilon \theta_1(\eta, t) + \dots.$$

Substituting (14) into the equations (11) and (12), and equating the coefficient of like powers of ϵ , we get

Zeroth- order Equations

$$(15) \quad \frac{d^2 u_0}{d\eta^2} - \text{Re} \frac{du_0}{d\eta} - (K^2 + M^2) u_0 = -Gr(\theta_0 - 1),$$

$$(16) \quad \frac{d^2 \theta_0}{d\eta^2} - \text{Pr Re} \frac{d\theta_0}{d\eta} - N^2 (\theta_0^4 - 1) = 0,$$

First- order Equations

$$(17) \quad \frac{\text{Re}}{4} \frac{\partial u_1}{\partial t} + \text{Re} \frac{\partial u_1}{\partial \eta} + \text{Re} e^{i\omega t} \frac{\partial u_0}{\partial t} + \text{Re} A e^{i\omega t} \frac{\partial u_0}{\partial \eta} = e^{i\omega t} \frac{\partial^2 u_0}{\partial \eta^2} + \frac{\partial^2 u_1}{\partial \eta^2} - (K^2 + M^2) u_1 - M^2 e^{i\omega t} u_0 + Gr \theta_1 + Gr e^{i\omega t} (\theta_0 - 1),$$

$$(18) \quad \frac{\partial^2 \theta_1}{\partial \eta^2} - \text{Pr Re} \frac{\partial \theta_1}{\partial \eta} - \frac{\text{Pr Re}}{4} \frac{\partial \theta_1}{\partial t} - 4 N^2 \theta_0^3 \theta_1 = A \text{Pr Re} e^{i\omega t} \frac{\partial \theta_0}{\partial \eta}.$$

The corresponding boundary conditions are reduced to

$$(19) \quad \begin{aligned} \eta = 0 : \quad & u_0 = 0, \quad u_1 = 0, \quad \theta_0 = \theta_w, \quad \theta_1 = 0; \\ \eta = 1 : \quad & u_0 = 1, \quad u_1 = e^{i\omega t}, \quad \theta_0 = 1, \quad \theta_1 = e^{i\omega t}. \end{aligned}$$

Following Bestman²¹ such that the temperature difference between lower plate and its nearest adjacent point is small, therefore θ_0 can be expanded as given below

$$(20) \quad \theta_0 = \theta_w + \psi,$$

where ψ is the small correction factor such that, $O(\theta_w) < \psi < O(1)$.

Substituting (20) into the equation (16) and neglecting the terms containing squares and product of correction quantity ψ , we have

$$(21) \quad \frac{d^2 \psi}{d\eta^2} - \text{Pr Re} \frac{d\psi}{d\eta} - 4 N^2 \theta_w^3 \psi = N^2 (\theta_w^4 - 1).$$

The equation (21) is an ordinary second order linear differential equation in ψ with constant coefficients and solved under the boundary conditions (19). Through straight forward algebra the expression of $\psi(\eta)$ is known but not given for the sake of brevity.

Using the value of ψ into the equation (20) and (15), $\theta_0(\eta)$ and $u_0(\eta)$ are known but not given for the sake of brevity.

In order to solve the first-order equations, assuming

$$(22) \quad u_1(\eta, t) = u_{10}(\eta) e^{i\omega t} \text{ and}$$

$$\theta_1(\eta, t) = H(\eta) e^{i\omega t}$$

and substituting into the equations (17) and (18), we have

$$(23) \quad \frac{\text{Re } i\omega}{4} u_{10} + \text{Re} \frac{\partial u_{10}}{\partial \eta} + \text{Re} \frac{\partial u_0}{\partial t} + \text{Re} A \frac{\partial u_0}{\partial \eta} = \frac{\partial^2 u_0}{\partial \eta^2} + \frac{\partial^2 u_{10}}{\partial \eta^2} - (K^2 + M^2) u_{10} - M^2 u_0 + Gr H + Gr e^{i\omega t} (\theta_0 - 1),$$

$$(24) \quad \frac{\partial^2 H}{\partial \eta^2} - \text{Pr Re} \frac{\partial H}{\partial \eta} - \frac{\text{Pr Re } i\omega t}{4} H - 4 N^2 \theta_0^3 H = A \text{Pr Re} \frac{\partial \theta_0}{\partial \eta}.$$

The corresponding boundary conditions are reduced to

$$(25) \quad \begin{aligned} \eta = 0 : \quad & u_{10} = 0, \quad H = 0; \\ \eta = 1 : \quad & u_{10} = 1, \quad H = 1. \end{aligned}$$

Now, the equations (23) and (24) are ordinary linear differential equations with constant coefficients. Through straight forward algebra, the expression of $u_{10}(\eta)$ and $H(\eta)$ are known but not given here for the sake of brevity.

4. Skin- Friction Coefficient

Coefficient of the skin-friction at the lower plate is given by

$$(26) \quad C_{f_0} = \frac{1}{\text{Re}} \left(\frac{\partial u}{\partial \eta} \right)_{\eta=0} = \frac{1}{\text{Re}} \left\{ \frac{\partial u_0}{\partial \eta} + \epsilon \frac{\partial u_1}{\partial \eta} \right\}_{\eta=0}.$$

Coefficient of the skin-friction at the upper plate is given by

$$(27) \quad C_{f_1} = \frac{1}{\text{Re}} \left(\frac{\partial u}{\partial \eta} \right)_{\eta=1} = \frac{1}{\text{Re}} \left\{ \frac{\partial u_0}{\partial \eta} + \epsilon \frac{\partial u_1}{\partial \eta} \right\}_{\eta=1}.$$

5. Nusselt Number

The rate of heat transfer in terms of Nusselt number at the lower plate is given by

$$(28) \quad Nu_0 = -\left(\frac{\partial \theta}{\partial \eta}\right)_{\eta=0} = -\left\{\frac{\partial \theta_0}{\partial \eta} + \epsilon \frac{\partial \theta_1}{\partial \eta}\right\}_{\eta=0}.$$

The rate of heat-transfer in terms of Nusselt number at the upper plate is given by

$$(29) \quad Nu_1 = -\left(\frac{\partial \theta}{\partial \eta}\right)_{\eta=1} = -\left\{\frac{\partial \theta_0}{\partial \eta} + \epsilon \frac{\partial \theta_1}{\partial \eta}\right\}_{\eta=1}.$$

6. Results and Discussion

It is observed from Fig.1 that fluid velocity increases with an increase of Reynolds number, Hartmann number, permeability parameter, phase angle or Prandtl number; while it decreases due to increase of frequency of vibration of the fluid particles or Grash of number. It is also noticed that due to increase of the value of the radiation parameter, the fluid velocity is minimum in the neighborhood of lower plate, while adverse behavior is observed in the neighborhood of upper plate.

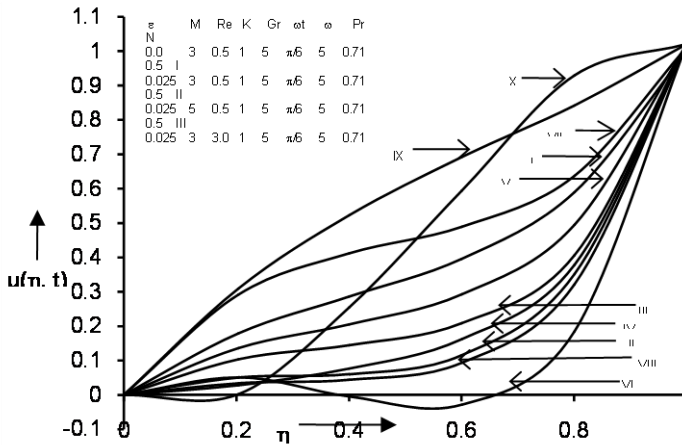
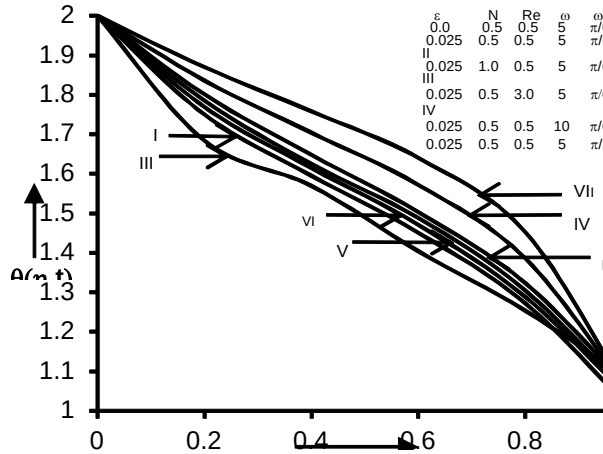


Fig.1. Velocity distribution versus η when $A = 1$.

It is noted from Fig.2 that fluid temperature increases due to increase of Reynolds number or Prandtl number; while it decreases due to increase of frequency of vibration of the fluid particles or phase angle.



It is observed from the Fig.3 that the Nusselt number at the lower plate increases with the increase of phase angle or radiation parameter, while it decreases due to increase of frequency, Reynolds number or the Prandtl number.

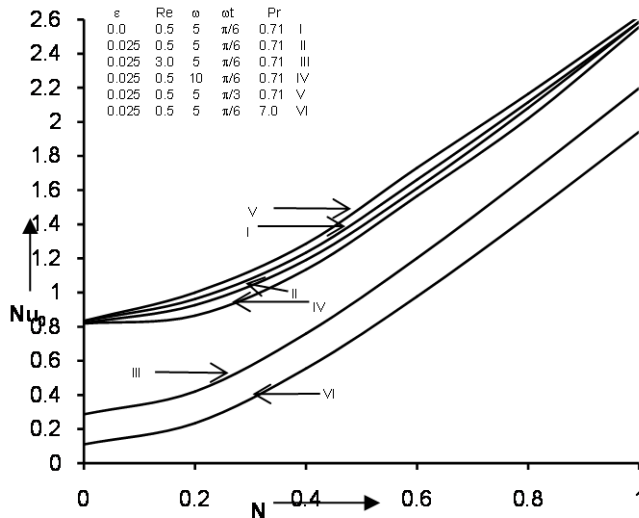
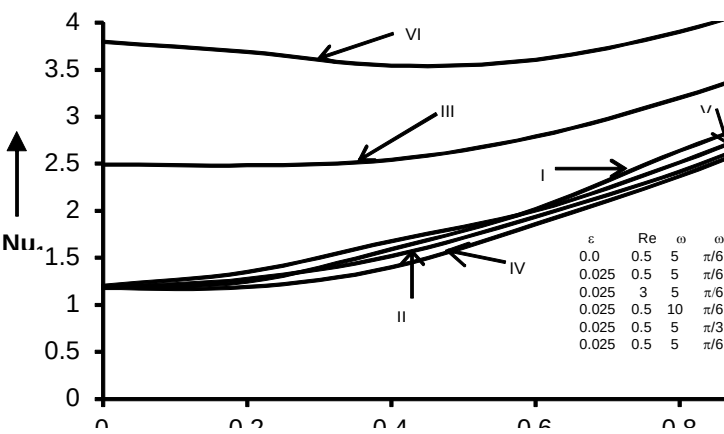


Fig.3. Nusselt number at the lower plate versus N when $A = 1$

It is noted from the Fig.4 that the Nusselt number at the upper plate increases with the increase of phase angle, Reynolds number, Prandtl

number or radiation parameter, while it decreases due to increase frequency of vibrations of the fluid particles.



It is inferred from Table-1 that the skin-friction coefficient at the lower plate increases due to increase in the Prandtl number, the Reynolds number, frequency of vibration of the fluid particles, phase angle, radiation parameter, permeability parameter or the Hartmann number, while it decreases due to increase in the Grash of number. Skin-friction coefficient at the upper plate increases due to increase in permeability parameter, phase angle, the Prandtl number or the Reynolds number, while it decreases due to increase in the Grash of number, frequency, radiation parameter or the Hartmann number.

Table- 1. Numerical values of coefficient of skin-friction at lower and upper plates for various values of physical parameters

	ε	K	Gr	ωt	ω	Pr	N	Re	M	C_{f_0}	C_{f_1}
I	0.025	1	5	$\pi/6$	5	0.71	0.5	0.5	2	-11.7434	-3.72153
II	0.0	1	5	$\pi/6$	5	0.71	0.5	0.5	2	-2.40119	-1.05574
III	0.025	3	5	$\pi/6$	5	0.71	0.5	0.5	2	16.5494	7.9059
IV	0.025	1	10	$\pi/6$	5	0.71	0.5	0.5	2	-24.2406	-12.5962
V	0.025	1	5	$\pi/3$	5	0.71	0.5	0.5	2	-1.380415	2.15521
VI	0.025	1	5	$\pi/6$	10	0.71	0.5	0.5	2	-8.600273	-4.7734
VII	0.025	1	5	$\pi/6$	5	7.0	0.5	0.5	2	6.50405	1.13604
VIII	0.025	1	5	$\pi/6$	5	0.71	1.0	0.5	2	-9.6218	-4.25199
IX	0.025	1	5	$\pi/6$	5	0.71	0.5	3.0	2	8.62229	10.30957

X	0.025	1	5	$\pi/6$	5	0.71	0.5	0.5	1	-2.375398	-3.31999
XI	0.025	1	5	$\pi/6$	5	0.71	0.5	0.5	0.5	-2.109535	2.56455

7. Conclusions

A study of velocity and temperature distributions of a electrically conducting viscous compressible fluid through porous medium bounded by two infinite horizontal thin porous plates when the conservation of energy involves the free-convection radiative heat transfer have been carried out. The non-dimensional coefficient of shearing stress and the rate of heat transfer in terms of Nusselt number at the plates are obtained.

The following observations are made as given below

- (i) The magnitude of fluid velocity for water is more in comparison to air.
- (ii) The fluid velocity decreases as viscosity of fluid increases as seen from increase of Reynolds number.
- (iii) The magnitude of fluid velocity decreases as the cooling of plate increases.
- (iv) The velocity of fluid flow decreases with the increase of radiative heat transfer in the lower half zone, while reverse behavior is observed in the upper half zone between the plates.
- (v) Fluid temperature decreases as the radiative heat transfer increases.
- (vi) The magnitude of fluid temperature for water is more in comparison to air.
- (vii) The fluid temperature increases as the Reynolds number increases.
- (viii) The Nusselt number is more for air in comparison to water, while it decreases with the increase of frequency of fluid particle.
- (ix) The coefficient of skin-friction is more for water in comparison to air at both plates. At lower plate, it becomes maximum with the increase of permeability parameter, while it goes to minimum for highly cooled case. The coefficient of skin-friction is maximum for greater values of Reynolds number, while it is least for highly cooled plate.

References

1. P. Cheng and W. J. Minkowycz, Free convection flow about a vertical flat plate embedded in a porous medium with application to heat transfer, *J. Geophys. Res.*, **82** (1977) 2040-2044.
2. A. Bharali and A. K. Borkakati, The effects of hall currents MHD flow and heat transfer between two parallel porous plates, *Applied Scientific Research*, **39** (1982) 155-165.

3. B. K. Jha and R. Prasad, MHD free- convection and mass transfer flow through a porous medium with heat source, *Astrophys and Space Science*, **181**(1991) 117-123.
4. C. Argento and D. Bouvard, A ray tracing method for evaluating the radiative heat transfer in porous medium, *Int. J. Heat transfer*, **39**(1996) 3175-3180.
5. K. D. Alagoa, G. Tay, and T. M. Abbey, Radiative and free convective effects of a MHD flow through a porous medium between infinite parallel plates with time dependent suction, *Astrophysics and Space science*, **260**(1999) 455-468.
6. A. J. Chamkha, H. S. Takhar, and V. M. Soundalgekar, Radiation effects on free convection flow past a semi-infinite vertical plate with mass transfer, *Chemical Engg. Journal*, **84**(2001) 335-342.
7. P. R. Sharma, S. Chawla, and R. Singh, Unsteady plane Poiseuille flow and heat transfer in the presence of oscillatory temperature of the lower plate, *Indian J. Theo. Phys., India*, **52**(3) (2004) 215-226.
8. A. J. Chamkha, Unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate with heat absorption, *Int. J. of Engg. Science*, **42**(2004) 217-230.
9. I. Z. Badruddin, Z. A. Zainal, P. A. Aswatha Narayana, K. M. Seetharamu and W. S. Lam, Free convection and radiation characteristics for a vertical plate embedded in a porous medium, *Int. J. Numerical Methods Engg.*, **65**(2005) 2265-2278.
10. A. H. Attia, Unsteady flow of a dusty conducting fluid between parallel porous plates with temperature dependent viscosity, *Turk. J. Phys.*, **29**(2005) 257-267.
11. P. R. Sharma and G. R. Yadav, Pulsatile MHD flow and heat transfer through a porous channel with Ohmic effect, *Bull. Pure Appl. Sciences*, **75E** (2006) 499-509.
12. I. U. Mbeledogu and A. Ogulu, Heat and Mass Transfer of an unsteady MHD natural convection flow of a rotating fluid past a vertical porous plate in the presence of radiative heat transfer, *Int. J. Heat Mass Transfer*, **50** (2007) 1902-1908.
13. A. Mahdya, R.A. Mohamed and F. M. Hady, Heat and mass transfer in MHD free convective flow along a vertical wavy plate with variable surface heat and mass flux, *Latin American Applied Reserch*, **39**(2009) 337-344.
14. P.V.S.N. Murthy, M. F. Al-Emin, The thermo diffusion effect on free convection heat and mass transfer in a thermally linearly stratified non Darcy porous media, *The open Transport Penomena Journal*, **3**(2011) 49-55.
15. H. Schlichting and K. Gersten, *Boundary Layer Theory*, Springer, New York, 1999.
16. J. L. Bansal, *Viscous Fluid Dynamics*, Oxford and IBH. Pub. Com., New Delhi, 1977.
17. G. S. Beavers and D. D. Joseph, Boundary condition at a naturally permeable wall, *J. Fluid Mech.*, Vol. 30 (1967) 197-207.
18. A. Jefferey, *Magneto-Hydrodynamics*, Oliver and Boyd., N.Y., 1966.
19. J. L. Bansal, *Megnetofluid Dynamics of Viscous Fluids*, Jaipur Pub. House, Jaipur, India, 1994.
20. P. Cheng, Two-dimensional radiating gas flow by a moment method, *AIAA Journal*, **2**(1964) 1662-1664.
21. A. R. Bestman and S. K. Adjepong, Unsteady hydromagnetic free-convection flow with heat transfer in a rotating fluid, *Astrophys. Space Sci.*, **143** (1988) 73-80.