

Effects of Kappa-Distributed Electrons on Ion-Acoustic Shock Waves in an e-p-i Plasma in Non-Planar Geometry

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Abstract: Ion-acoustic (IA) shock wave in an unmagnetized plasma consisting of electrons, positrons and ions in non-planar geometry has been investigated by employing reductive perturbation technique. In this study, kappa-distributed electrons and Boltzmann distributed positrons are considered, and a modified Burgers' equation is derived. The equation is numerically solved to get the shock wave and it is observed that the basic features of the waves are modified due to the non-planar geometry. It is found that the heights of the developed shock waves are different for different geometries and in both cases the potential of the wave is negative.

1. Introduction

Nonlinear wave propagation in a three-component electron-positron-ion (e-p-i) plasma and electron-ion-dust (e-i-d) plasma¹⁻¹¹ has been a subject of increasing interest in the past decade due to their presence in Van Allen radiation belts and near the polar cap of fast rotation neutron stars¹²⁻¹⁴, active galactic nuclei¹⁵, quasars and pulsar magnetosphere¹⁶⁻¹⁸, semiconductor plasmas¹⁹, intense laser fields²⁰, etc. It is well established that linear and nonlinear waves in plasma having positrons behave differently with the ordinary electron-ion plasmas^{1,21}.

Plasmas containing substantial high energy particles can be effectively modeled by Lorentzian distribution or simply kappa distribution²². In the

planetary magnetospheres, solar wind, whistler emission of Jupiter, and interstellar medium, where plasmas are far from thermal equilibrium, are normally modeled by Lorentzian distribution. The presence of a substantially large number of super-thermal particles, which distinguishes kappa distribution from Maxwellian, can significantly change the rate of resonant energy transfer between particles and plasma waves²³. The kappa distribution with a finite value of the spectral index κ has a power law tail at velocities higher than the thermal velocity and approaches a Maxwellian distribution in the limit as $\kappa \rightarrow \infty$. In space plasmas the spectral index κ varies between 2 and 6²⁴.

Masood *et. al.*²⁵ studied ion-acoustic (IA) shock waves in an unmagnetized plasma consisting of electrons, positrons and adiabatically hot positive ions. In their work, they found that the amplitude of the Kadomstev-Petviashvili (KP) solution decreases with increase of positron concentration. In the limiting case, they found Burger-KP equation and they also discussed the comparison of these two equations.

In their work, Shah *et. al.*²⁶ studied IA shock waves in three-component plasmas consisting of relativistic adiabatic ions, kappa distributed electrons and positrons. The Korteweg-de Vries-Burger equation has been derived and solved numerically. They mentioned that the shock amplitude and steepness flourish as kappa parameter of electrons and positrons are increased.

In this paper, we have considered the non-planar IA shock wave in an unmagnetized plasma consisting of kappa-distributed electrons, ions and Boltzmann distributed positrons. We are particularly interested to observe the shock wave structure in the presence of kappa-distributed electrons in three-component plasma both in cylindrical and spherical geometries.

This paper is organized as follows. The basic governing equations describing the model and Bergers' equation is derived in Section 2. Numerical solution of modified Bergers' equation and its graphical representation is given in Section 3. Finally, the conclusion is stated in Section 4.

2. Basic Equations and Derivation of Burgers' Equation

The normalized fluid equations in an e-p-i plasma system in non-planar geometry are

$$(1) \quad \frac{\partial n_i}{\partial t} + \frac{1}{r^\nu} \frac{\partial(r^\nu n_i u_i)}{\partial r} = 0.$$

$$(2) \quad n_i \frac{\partial u_i}{\partial t} + n_i u_i \frac{\partial u_i}{\partial r} = -n_i \frac{\partial \phi}{\partial r} - \alpha \frac{\partial p_i}{\partial r} + \eta_i n_i \left[\frac{1}{r^\nu} \frac{\partial}{\partial r} \left(r^\nu \frac{\partial u_i}{\partial r} \right) - \frac{\nu u_i}{r^2} \right],$$

$$(3) \quad \frac{\partial p_i}{\partial t} + u_i \frac{\partial p_i}{\partial r} + 3p_i \frac{1}{r^\nu} \frac{\partial(r^\nu u_i)}{\partial r} = 0.$$

$$(4) \quad n_e - p n_p - (1-p)n_i = 0.$$

where $n_e = [1 - \phi / (\kappa - 3/2)]^{-\kappa+1/2}$, $n_p = \exp(-\sigma\phi)$, $\nu = 1$ for cylindrical geometry and $\nu = 2$ for spherical geometry. In the above equation, n_i is the ion number density normalized by its equilibrium value n_{i0} , u_i is the ion fluid speed normalized by $C_i = (k_B T_e / m_i)^{1/2}$, ϕ is the wave potential normalized by $k_B T_e / e$, p_i is the ion-thermal pressure normalized by $n_{i0} k_B T_i$, $\alpha = T_i / T_e$, $p = n_{p0} / n_{e0}$. The time and space variables are normalized by reciprocal plasma frequency $\omega_{pi}^{-1} = (m_i / 4\pi n_{i0} e^2)^{1/2}$ and the Debye length $\lambda_D = (k_B T_e / 4\pi n_{i0} e^2)^{1/2}$, respectively and η_i is the viscosity coefficient of ion fluid normalized by $\omega_{pi} \lambda_D^2$. We have assumed the quasi-neutrality condition in equation (4).

We can expand the variables n_i, u_i, p_i and ϕ in a power series of ε as

$$(5) \quad n_i = 1 + \varepsilon n_i^{(1)} + \varepsilon^2 n_i^{(2)} + \dots$$

$$(6) \quad u_i = \varepsilon n_i^{(1)} + \varepsilon^2 n_i^{(2)} + \dots$$

$$(7) \quad p_i = 1 + \varepsilon p_i^{(1)} + \varepsilon^2 p_i^{(2)} + \dots$$

$$(8) \quad \phi = \varepsilon \phi^{(1)} + \varepsilon^2 \phi^{(2)} + \dots$$

Now, to derive the cylindrical and spherical Burgers' equation we used the stretched coordinates $\xi = \varepsilon^{1/2}(x - V_0 t)$ and $\tau = \varepsilon^{3/2}t$, where ε is a smallness parameter measuring the weakness of the nonlinearity and V_0 is the phase speed of the IA wave normalized by c_i , we now express (1) to (4) in terms of ξ and τ as

$$(9) \quad \varepsilon^{3/2} \frac{\partial n_i}{\partial \tau} - V_0 \varepsilon^{1/2} \frac{\partial n_i}{\partial \xi} + \varepsilon^{1/2} \frac{\partial (n_i u_i)}{\partial \xi} + \frac{v \varepsilon^{3/2}}{V_0 \tau (1 + \frac{\varepsilon \xi}{V_0 \tau})} (n_i u_i) = 0.$$

$$(10) \quad \varepsilon^{3/2} n_i \frac{\partial u_i}{\partial \tau} - V_0 \varepsilon^{1/2} n_i \frac{\partial u_i}{\partial \xi} + \varepsilon^{1/2} n_i u_i \frac{\partial u_i}{\partial \xi} = -\varepsilon^{1/2} n_i \frac{\partial \phi}{\partial \xi} - \varepsilon^{1/2} \alpha \frac{\partial p_i}{\partial \xi} +$$

$$\varepsilon^{1/2} \eta_0 n_i \left[\varepsilon \frac{\partial^2 u_i}{\partial \xi^2} + \frac{v \varepsilon^2}{V_0 \tau (1 + \frac{\varepsilon \xi}{V_0 \tau})} \frac{\partial u_i}{\partial \xi} - \frac{v \varepsilon^3}{V_0^2 \tau^2 (1 + \frac{\varepsilon \xi}{V_0 \tau})^2} u_i \right]$$

$$(11) \quad \varepsilon^{3/2} \frac{\partial p_i}{\partial \tau} - V_0 \varepsilon^{1/2} \frac{\partial p_i}{\partial \xi} + \varepsilon^{1/2} u_i \frac{\partial p_i}{\partial \xi} + 3 p_i \left[\varepsilon^{1/2} \frac{\partial u_i}{\partial \xi} + \frac{v \varepsilon^{3/2}}{V_0 \tau (1 + \frac{\varepsilon \xi}{V_0 \tau})} u_i \right] = 0.$$

$$(12) \quad \left[1 - \frac{\frac{1}{2} - \kappa_e}{\kappa_e - \frac{3}{2}} \phi - \frac{(\frac{1}{2} + \kappa_e)(\frac{1}{2} - \kappa_e)}{2(\kappa_e - \frac{3}{2})^2} \phi^2 + \dots \right] - p \left[1 - \sigma \phi + \frac{\sigma^2 \phi^2}{2} + \dots \right] \\ = (1 - p) n_i$$

where $\eta_i = \varepsilon^{1/2} \eta_0$ is considered.

Now substituting (5) – (8) into (9) – (12), and equating the lowest order of ε , we get

$$(13) \quad n_i^{(1)} = \mu \phi^{(1)},$$

$$(14) \quad u_i^{(1)} = \mu V_0 \phi^{(1)},$$

$$(15) \quad p_i^{(1)} = 3\mu \phi^{(1)},$$

$$(16) \quad V_0 = \sqrt{3\alpha + \frac{1}{\mu}},$$

$$(17) \quad \mu = \frac{p}{1-p} \sigma - \left(\frac{1}{1-p} \right) \left(\frac{\frac{1}{2} - \kappa_e}{\kappa_e - \frac{3}{2}} \right)$$

To next higher order in ϵ , we obtain the following set of equations

$$(18) \quad \frac{\partial n_i^{(1)}}{\partial \tau} - V_0 \frac{\partial n_i^{(2)}}{\partial \xi} + \frac{\partial u_i^{(2)}}{\partial \xi} + \frac{\partial (n_i^{(1)} u_i^{(1)})}{\partial \xi} + \frac{\nu u_i^{(1)}}{V_0 \tau} = 0.$$

$$(19) \quad \begin{aligned} \frac{\partial u_i^{(1)}}{\partial \tau} - V_0 \frac{\partial u_i^{(2)}}{\partial \xi} + u_i^{(1)} \frac{\partial u_i^{(1)}}{\partial \xi} - V_0 n_i^{(1)} \frac{\partial u_i^{(1)}}{\partial \xi} + \alpha \frac{\partial p_i^{(2)}}{\partial \xi} \\ = -\frac{\partial \phi^{(2)}}{\partial \xi} - n_i^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} + \eta_0 \frac{\partial^2 u_i^{(1)}}{\partial \xi^2} \end{aligned}$$

$$(20) \quad \frac{\partial p_i^{(1)}}{\partial \tau} - V_0 \frac{\partial p_i^{(2)}}{\partial \xi} + u_i^{(1)} \frac{\partial p_i^{(1)}}{\partial \xi} + 3 \frac{\partial u_i^{(2)}}{\partial \xi} + 3 p_i^{(1)} \frac{\partial u_i^{(2)}}{\partial \xi} + \frac{3 \nu u_i^{(1)}}{V_0 \tau} = 0.$$

$$(21) \quad n_i^{(2)} = \mu \phi^{(2)} - \mu_1 \{\phi^{(1)}\}^2,$$

where

$$(22) \quad \mu_1 = \frac{p \sigma^2}{2(1-p)} - \frac{(\frac{1}{2} + \kappa_e)(\frac{1}{2} - \kappa_e)}{2(1-p)(\kappa_e - \frac{3}{2})^2}.$$

Combining equation (18) – (21), we get a modified Burgers' equation

$$(23) \quad \frac{\partial \phi^{(1)}}{\partial \tau} + \frac{\nu}{2\tau} \phi^{(1)} + A \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} = C \frac{\partial^2 \phi^{(1)}}{\partial \xi^2},$$

where

$$(24) \quad A = \frac{3\mu^2 + 12\mu^3\alpha + 2\mu_1}{2\mu^2 \sqrt{3\alpha + \frac{1}{\mu}}},$$

$$(25) \quad C = \frac{\eta_0}{2}.$$

3. Numerical Solution of Burgers' Equation and Graphical Representation

Analytic solution of equation (23) is not available. Hence, Burgers' equation is numerically investigated for cylindrical and spherical geometry for describing the nonlinear propagation of the shock waves in an unmagnetized plasma. For planar geometry ($\nu = 0$), a stationary solution of the equation (23) has the following form:

$$(26) \quad \phi^{(1)} = \frac{V}{A} \left[1 - \tanh \left\{ \frac{V(\xi - V\tau)}{2C} \right\} \right],$$

where V is the shock wave speed normalized by C_i .

We numerically solve (23) by considering the following plasma parameters:

$$r_d = 5\mu m, \quad p = n_{p0}/n_{e0}, \quad \alpha = T_t/T_e, \quad \sigma = T_e/T_p, \quad n_{p0} = 1 \times 10^{-4}, n_{e0} = 3 \times 10^{-5}, \\ n_{i0} = 2 \times 10^{-4}, z_d = 10^3, T_t = 10^3 K, T_e = 10^4 K, T_p = 10^3 K, \kappa = 2.0, \eta_0 = 0.1, \\ m_i = 39m_p.$$

Results are displayed in figure 1 and 2.

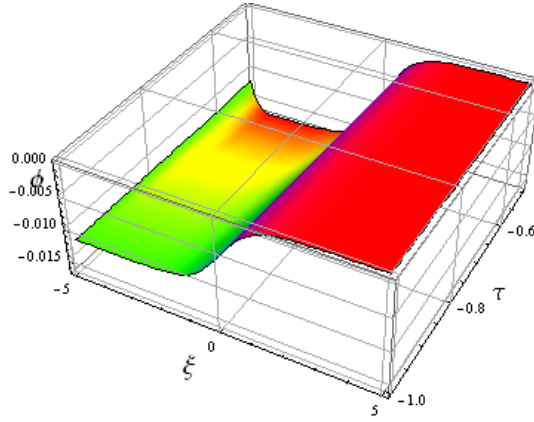


Figure1. The cylindrical shock wave potential ϕ versus spatial coordinate ξ and time τ for $\kappa = 2.0$

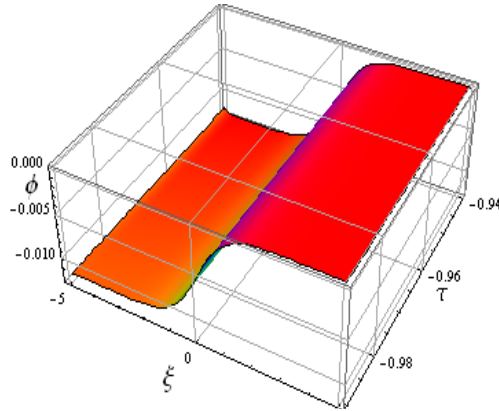


Figure2. The spherical shock wave potential ϕ versus spatial coordinate ξ and time τ for $\kappa = 2.0$.

Figure 1 shows the effect of the cylindrical geometry ($\nu=1$) of IA shock wave whereas figure 2 shows the effect of spherical geometry ($\nu=2$). For a large value of τ , the numerical solution of (23) shows that cylindrical, spherical and one dimensional planar ($\nu=0$) shock waves are similar. But for a small value of τ , the term $\left(\frac{\nu}{2\tau}\right)\phi^{(1)}$ in (23), plays a significant role in the formation of the shock structure. It is shown that the developed shock heights are different from each other in different geometry. From figures we

found that in spherical geometry the shock wave has higher height than that of the cylindrical geometry.

4. Conclusion

We have studied the nonlinear propagation of ion acoustic shock wave in an unmagnetized plasma consisting of kappa-distributed electrons, ions and Boltzmann distributed positrons in a non-planar geometry. The Burgers' equation is derived by using the standard perturbation method. The propagation of the nonlinear shock wave in the e-p-i plasma is considered by analyzing the solution of the Burgers' equation. A detailed numerical analysis of the amplitude of the IA shock wave is performed. We have shown here how the basic features of the nonlinear IA shock waves are modified for kappa-distributed electrons for the cylindrical and spherical geometry. It is shown that the developed shock heights (amplitudes) are different for different geometry and in the case of spherical geometry, the shock wave has much higher amplitude than that of the cylindrical geometry.

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