

A Numerical Study on MHD Peristaltic Transport in Rheological Physiology*

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Abstract: A mathematical model is constructed to study the MHD peristaltic flow of a viscoelastic fluid with fractional Maxwell model under physiological flow constraints (creeping flow) viz low Reynolds number and long wavelength approximations. Homotopy Perturbation Method (HPM) is employed to find out the approximate solution of fractional differential equation (FDE). The effects of involved physical parameters, who define the fluid properties, on volumetric flow rate, pressure, and frictional force have been emphasized. Numerical results and simulation have been done by Mathematica Software and discussed with the help of illustrations. On the basis of this study one can make an important conjecture that when the pressure is more for Newtonian fluid in compare to viscoelastic fluid and it is also more for viscoelastic fluid with Maxwell model in compare to viscoelastic fluid with fractional Maxwell model in pumping region, in free pumping region it is same and in co-pumping region reverse trend is noticed. This model is applicable to study the food transportation in digestive system and bio-rheology.

Keywords: Magneto-hydrodynamics; Viscoelastic fluid; Fractional Maxwell model; Homotopy Perturbation Method.

1. Introduction

Magneto-hydrodynamics (MHD) is the study of dynamics of magnetic field in electrically conducting fluids. Examples of such fluids include salty foods, plasma (blood), liquid metals and salt or sea water. The idea of magneto-hydrodynamics is that magnetic fields can induce currents in a moving conducting fluid, which create forces on the fluid, and also change the magnetic field itself. The equations describing magneto-hydrodynamic fluids are a combination of Navier-Stokes equations and Maxwell equations of electromagnetism. First of all, Swedish physicist Alfven¹ initiated the concept of fundamental work and discoveries in magneto-hydrodynamics

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with fruitful applications in different parts of plasma physics, for which he received the Nobel Prize in Physics (1970).

In physiology, biofluid dynamics is the branch of science in which deals the flow/transportation of physiological fluids like blood, food bolus, chime, urine etc. Most of the physiological fluids are being transported from one part to another part by self-generated pumping which is known as peristaltic pumping and transportation is known as peristaltic transport. In view of the huge application of bio-rheology magneto-hydrodynamics and peristaltic transport, Mekheimer² studied the effect of magnetic field on peristaltic flow with couple stress fluid, Ebaid³ reported the MHD effect on peristaltic transport with viscous fluid, Srinivas et al.⁴ reported the effect of heat transfer on MHD peristaltic transport, Hayat and Hina⁵ investigated MHD peristaltic flow of viscoelastic fluids and Tripathi and Beg⁶ explored the MHD peristaltic pumping through coaxial channels. They all considered the steady flow in their studies. In continuation of work on MHD peristaltic transport, some works on unsteady peristaltic flow with the effect of magnetic field have been reported by Pandey and Tripathi⁷⁻⁸ and Tripathi and Beg⁹⁻¹⁰ with Newtonian and non-Newtonian fluids.

Rheology is the study of fluid properties and their physical characterization. In order to describe the viscoelastic properties of materials in various fields, recently the constitutive equations with ordinary and fractional time derivatives have been introduced. Maxwell model is a series combination of purely elastic and viscous components and if fractional element is included; it is called fractional Maxwell model. In view of wide role of fractional viscoelastic fluids in bio-rheology, Tan et al.¹¹ wrote a note on unsteady flow of fractional Maxwell fluid, Nadeem¹² for fractional Oldroyd-B fluid, Vieru et al.¹⁴ discussed flow of a fractional Maxwell fluid between two side walls perpendicular to a plate, Khan et al.¹⁵ studied the decay of potential vortex for a fractional Maxwell fluid, Wang and Xu presented a model for Couette flow of fractional viscoelastic fluids, Khan et al.¹⁶ used fractional Burgers' model in their study, Qi and Xu¹⁷ modelled for unsteady unidirectional flows of Oldroyd-B fluid with fractional derivative, Liu et al.¹⁸ saw the effect of magneto-hydrodynamics on flow of a fractional Oldroyd-B fluid however they have not discussed the peristaltic transport of fractional viscoelastic fluid which is very important in bio-rheology point of view. Incorporating the concept of peristaltic transport in rheology of fractional viscoelastic fluids, Tripathi¹⁹ and Tripathi and Beg²⁰⁻²³ have been added some mathematical models and depicted numerically.

The aim of this paper is to incorporate magneto-hydrodynamics in study of peristaltic transport of viscoelastic fluid with fractional model. HPM is applied to find approximate analytical solution of the problem and

numerical results for different cases are depicted graphically. The influences of fractional parameter, relaxation time, and Hartmann number on the flow pattern are discussed numerically. This model is more applicable in food transportation in digestive system.

2. Mathematical Formulation

The fractional Maxwell model for viscoelastic fluid is given as

$$(2.1) \quad (1 + \tilde{\lambda}_1^\alpha \frac{\partial^\alpha}{\partial \tilde{t}^\alpha}) \tilde{\tau} = \mu \dot{\gamma},$$

where $\tilde{\lambda}_1, \alpha, \tilde{t}, \tilde{\tau}, \mu, \dot{\gamma}$ are the relaxation time, fractional parameter, time, shear stress, viscosity, rate of shear strain.

The governing equations of motion for viscoelastic fluid under the effect of transverse magnetic field are

$$(2.2) \quad \begin{cases} \rho \left(\frac{\partial}{\partial \tilde{t}} + \tilde{u} \frac{\partial}{\partial \tilde{\xi}} + \tilde{v} \frac{\partial}{\partial \tilde{\eta}} \right) \tilde{u} = -\frac{\partial \tilde{p}}{\partial \tilde{\xi}} + \frac{\partial \tilde{\tau}_{\tilde{\xi}\tilde{\xi}}}{\partial \tilde{\xi}} + \frac{\partial \tilde{\tau}_{\tilde{\xi}\tilde{\eta}}}{\partial \tilde{\eta}} - \sigma B_0^2 \tilde{u}, \\ \rho \left(\frac{\partial}{\partial \tilde{t}} + \tilde{u} \frac{\partial}{\partial \tilde{\xi}} + \tilde{v} \frac{\partial}{\partial \tilde{\eta}} \right) \tilde{v} = -\frac{\partial \tilde{p}}{\partial \tilde{\eta}} + \frac{\partial \tilde{\tau}_{\tilde{\eta}\tilde{\xi}}}{\partial \tilde{\xi}} + \frac{\partial \tilde{\tau}_{\tilde{\eta}\tilde{\eta}}}{\partial \tilde{\eta}} - \sigma B_0^2 \tilde{v}. \end{cases}$$

The non-dimensional parameters are

$$(2.3) \quad \begin{cases} \xi = \frac{\tilde{\xi}}{\lambda}, \eta = \frac{\tilde{\eta}}{a}, t = \frac{c\tilde{t}}{\lambda}, \lambda_1 = \frac{c\tilde{\lambda}_1}{\lambda}, u = \frac{\tilde{u}}{c}, v = \frac{\tilde{v}}{c\delta}, \phi = \frac{\tilde{\phi}}{a}, \\ p = \frac{\tilde{p}a^2}{\mu c \lambda}, \tau = \frac{a\tilde{\tau}}{\mu c}, \text{Re} = \frac{\rho c a \delta}{\mu}, \delta = \frac{a}{\lambda}, M = \sqrt{\frac{\sigma}{\mu}} a B_0, \end{cases}$$

where $\rho, \tilde{u}, \tilde{\xi}, \tilde{v}, \tilde{\eta}, \tilde{p}, B_0, \sigma, \delta, \phi, \text{Re}$, and M are the fluid density, axial velocity, axial coordinate, transverse velocity, transverse coordinate, pressure, transverse magnetic field, electrical conductivity, wave number, ratio of half width of channels, Reynolds number, and Hartmann number respectively and applying the long wavelength and low Reynolds number approximations and using Eq. (2.1) in Eq. (2.2), we get

$$(2.4) \quad \begin{cases} (1 + \lambda_1^\alpha \frac{\partial^\alpha}{\partial t^\alpha}) \frac{\partial p}{\partial \xi} = \frac{\partial^2 u}{\partial \eta^2} - M^2 u, \\ \frac{\partial p}{\partial \eta} = 0. \end{cases}$$

The boundary conditions are taken as

$$(2.5) \quad \frac{\partial u(\xi, 0, t)}{\partial \eta} = 0, \quad u(\xi, h, t) = 0, \quad \left. \frac{\partial p}{\partial \xi} \right|_{\xi=0} = 0.$$

Imposing the above boundary conditions on the governing Eqs.(2.4), the axial velocity yields

$$(2.6) \quad u = \frac{1}{M^2} \left(\frac{\cosh(M\eta)}{\cosh(Mh)} - 1 \right) \left(1 + \lambda_1^\alpha \frac{\partial^\alpha}{\partial t^\alpha} \right) \frac{\partial p}{\partial \xi},$$

The volumetric flow rate is defined as $Q = \int_0^h u \, d\eta$, which, by virtue of Eq.(2.6), reduces to

$$(2.7) \quad Q = \frac{1}{M^3} (\tanh(Mh) - Mh) \left(1 + \lambda_1^\alpha \frac{\partial^\alpha}{\partial t^\alpha} \right) \frac{\partial p}{\partial \xi}.$$

The transformations between the wave and the laboratory frames, in the dimensionless form, are given by

$$(2.8) \quad x = \xi - t, \quad y = \eta, \quad U = u - 1, \quad V = v, \quad \hat{q} = Q - h,$$

where the left side parameters are in the wave frame and the right side parameters are in the laboratory frame.

The geometry of peristaltic transport is mathematically formulated in wave frame (see Figure1) as:

$$(2.9) \quad h = 1 - \phi \cos^2(\pi x).$$

The averaged flow rate is given in form of the flow rate in both frames:

$$(2.10) \quad \bar{Q} = \hat{q} + 1 - \frac{\phi}{2} = Q - h + 1 - \frac{\phi}{2}.$$

Eq.(2.7), in view of Eq.(2.10) gives

$$(2.11) \quad \frac{\partial^\alpha}{\partial t^\alpha} \left(\frac{\partial p}{\partial x} \right) + \frac{1}{\lambda_1^\alpha} \frac{\partial p}{\partial x} = \frac{1}{\lambda_1^\alpha} \frac{M^3 \left(\bar{Q} + h - 1 + \frac{\phi}{2} \right)}{\tanh(Mh) - Mh}.$$

3. HPM Solution

Eq.(2.11) can be simplified as

$$(3.1) \quad D_t^\alpha f + \frac{1}{\lambda_1^\alpha} f = -\frac{A}{\lambda_1^\alpha},$$

where $f(\xi, t) = \frac{\partial p}{\partial \xi}$ and

$$(3.2) \quad A = -\frac{M^3 \left(\bar{Q} + h - 1 + \frac{\phi}{2} \right)}{\tanh(Mh) - Mh},$$

with the initial condition

$$(3.3) \quad f(\xi, 0) = 0.$$

According to the homotopy perturbation method given by Hi²⁴, we construct the following homotopy

$$(3.4) \quad D_t^\alpha f = -q \left[\frac{1}{\lambda_1^\alpha} f + \frac{A}{\lambda_1^\alpha} \right].$$

In view of homotopy perturbation method, we use the homotopy parameter 'q' to expand the solution

$$(3.5) \quad f = f_0 + q f_1 + q^2 f_2 + q^3 f_3 + q^4 f_4 + \dots.$$

When $q \rightarrow 1$, Eq.(3.4) corresponds Eq.(3.1) and Eq.(3.5) becomes the approximate solution of Eq.(3.1). Substituting Eq.(3.5) in Eq.(3.4) and comparing the like powers of q , we obtain the following set of linear differential equations

$$(3.6) \quad q^0 : D_t^\alpha f_0 = 0,$$

$$(3.7) \quad q^1 : D_t^\alpha f_1 = -\frac{1}{\lambda_1^\alpha} f_0 - \frac{A}{\lambda_1^\alpha},$$

$$(3.8) \quad q^2 : D_t^\alpha f_2 = -\frac{1}{\lambda_1^\alpha} f_1,$$

$$(3.9) \quad q^3 : D_t^\alpha f_3 = -\frac{1}{\lambda_1^\alpha} f_2,$$

$$(3.10) \quad q^4 : D_t^\alpha f_4 = -\frac{1}{\lambda_1^\alpha} f_3,$$

and so on.

The method is based on applying the operator J_t^α (the inverse operator of Caputo derivative D_t^α) on both sides of the Eqs.(3.6) – (3.10), we obtain

$$(3.11) \quad f_0 = 0,$$

$$(3.12) \quad f_1 = -\frac{A}{\lambda_1^\alpha} \frac{t^\alpha}{\Gamma(\alpha+1)},$$

$$(3.13) \quad f_2 = \frac{A}{\lambda_1^{2\alpha}} \frac{t^{2\alpha}}{\Gamma(2\alpha+1)},$$

$$(3.14) \quad f_3 = -\frac{A}{\lambda_1^{3\alpha}} \frac{t^{3\alpha}}{\Gamma(3\alpha+1)},$$

$$(3.15) \quad f_4 = \frac{A}{\lambda_1^{4\alpha}} \frac{t^{4\alpha}}{\Gamma(4\alpha+1)}.$$

Thus the approximate solution may be obtained as

$$(3.16) \quad f(\xi, t) = \sum_{r=0}^{\infty} f_r$$

$$= \sum_{r=0}^{\infty} [\psi(\alpha)]^r \frac{t^{r\alpha}}{\Gamma(r\alpha+1)}, \text{ where } [\psi(\alpha)]^r = (-1)^r \frac{A}{\lambda_1^{r\alpha}}, \quad r \geq 1$$

$$= 0, \quad r = 0$$

$$= E_\alpha(\psi(\alpha)t^\alpha).$$

where $E_\alpha(t) = \sum_{r=0}^{\infty} \frac{t^r}{\Gamma(r\alpha+1)}$, ($\alpha > 0$) is the Mittag-Leffler function in one parameter.

The pressure difference across one wavelength (Δp) and friction force across one wavelength (F) are defined as

$$(3.17) \quad \Delta p = \int_0^1 \frac{\partial p}{\partial x} dx ,$$

$$(3.18) \quad F = \int_0^1 h \left(-\frac{\partial p}{\partial x} \right) dx .$$

4. Numerical Results and Discussion

In this section, the numerical and computational results are discussed through the graphical illustrations. Figure 1 shows the geometry of peristaltic transport. The salient feature of MHD peristaltic transport of viscoelastic fluids with fractional Maxwell model are presented in Figures 2-9. Figures 2-5 illustrate the variation of the volumetric flow rate of peristaltic waves with pressure gradient for different values of the relaxation time (λ_1), fractional parameter (α), Hartmann number (H), and amplitude ratio (ϕ). It is observed that different regions on the basis of the values of pressure gradient examined in the present study, the region for $\Delta p > 0$ is entire pumping region, the region for $\Delta p = 0$ is free pumping region and the region for $\Delta p < 0$ is co-pumping region.

Figure 2 shows that in the entire pumping region the volumetric flow rate as well as pressure decreases with the increase in magnitude of relaxation time, it is same in free pumping region whereas in the co-pumping region, it increases with the increase in magnitude of relaxation time. From the same Figure, we further derive the information that for a Newtonian fluid ($\lambda_1 \rightarrow 0$), the magnitude of the pressure is more than that for the viscoelastic fluid in pumping region and it is less in co-pumping region. Figure 3 observes the volumetric flow rate as well as pressure enhances in the pumping region and same in free pumping region and it reduces in co-pumping region by increasing the magnitude of fractional parameter. It is further observed that for a viscoelastic fluid with ordinary Maxwell model ($\alpha = 1$), the pressure is more than that for the viscoelastic fluids with fractional Maxwell model in pumping region and it is opposite in co-pumping region. It is found from Figure 4 that with the rise in the magnitude of Hartmann number, pressure enhances in pumping region at a certain value of volumetric flow rate after then it diminishes in pumping,

free pumping and co-pumping regions. Figure 5 depicts the pressure increases in the pumping region, free pumping region and co-pumping region when the magnitude of amplitude ratio increases but in co-pumping region after a certain value of volumetric flow rate a reverse trend is noticed.

Frictional force (F) in the case of the peristaltic flow of a viscoelastic fluid is calculated over one wave period in the term of averaged volumetric flow rate. Figures 6-9 show the variation of frictional force with averaged flow rate for different values pertinent parameters. Figure 6 illustrates that the frictional force enhances with increasing the relaxation time at a certain value of volumetric flow rate then it has opposite behavior. From Figure 7, it is observed that the impact of fractional parameter on frictional force is opposite to that for pressure. The quantum of influence of the Hartmann number on frictional force is shown in Figure 8. This Figure indicates that the magnitude of frictional force reduces in very small interval of volumetric flow rate then it increases with increasing the magnitude of Hartmann number. The effect of amplitude ratio on frictional force is shown in Figure 9 and it is found that the frictional force reduces at a certain value of volumetric flow rate then increase with increasing the magnitude of amplitude ratio.

4. Conclusion

The influence of viscoelastic behavior, fractional nature of fluid model and transverse magnetic field on peristaltic flow pattern have been discussed numerically in the above section. A very significant conclusion that can be made on the basis of this study is that the pressure as well as the peristaltic pumping performance reduces by increasing the magnitude of relaxation time in pumping region, it is constant in free pumping region and it enhances in co-pumping region. The effect of fractional parameter on pressure is opposite to that for relaxation time in all regions. It is also revealed that pressure increases in pumping region at a certain value of volumetric flow rate then it reduces in all regions by increasing the Hartmann number. The effect of amplitude ratio on pressure is found that pressure increases in all three regions but after a certain value of volumetric flow rate in co-pumping region it starts opposite trend. The study further reveals the important fact that the effects of all pertinent parameters on frictional forces are opposite to that of pressure.

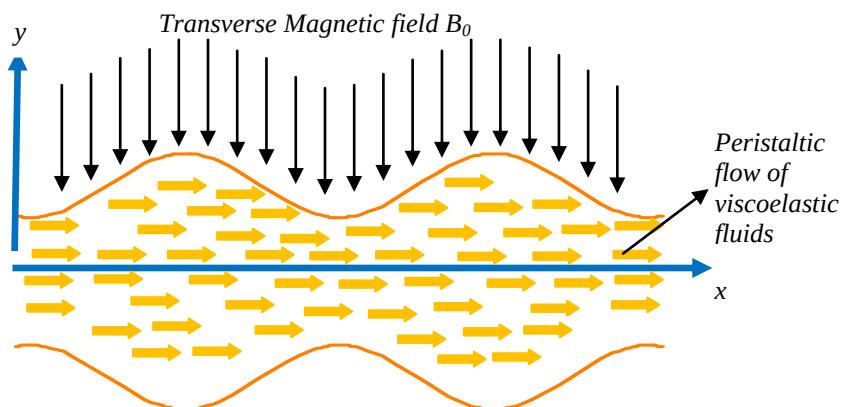


Figure1: Geometry of the Peristaltic Transport under the effect of transverse magnetic field.

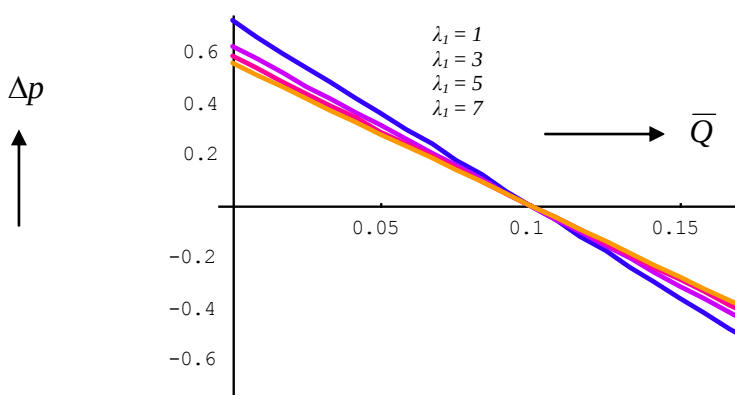


Figure 2: Pressure vs. averaged flow rate for various values of λ_1 at $t = 1, \alpha = 1/4$, $M = 1.0, \phi = 0.5$

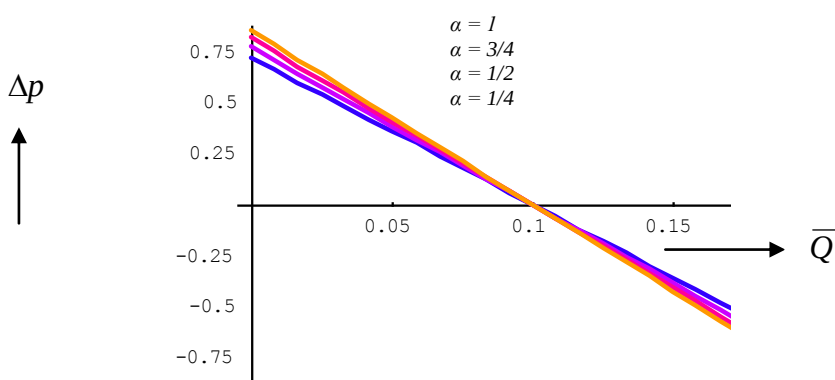


Figure 3: Pressure vs. averaged flow rate for various values of α at $t = 1, \lambda_1 = 1, M = 1.0, \phi = 0.5$

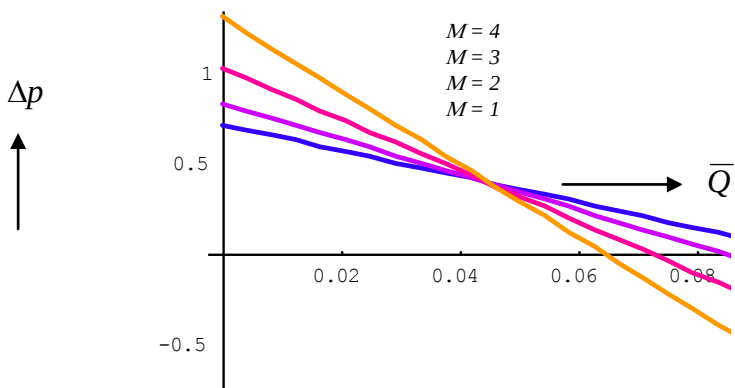


Figure 4: Pressure vs. averaged flow rate for various values of M at $t=1, \alpha=1/4$, $\lambda_1=1, \phi=0.5$

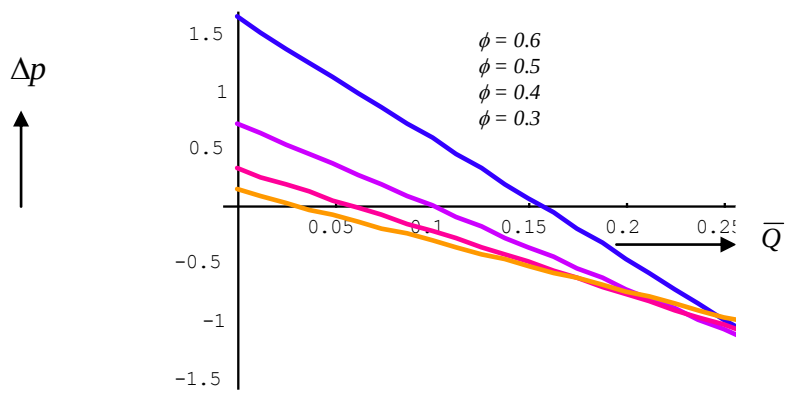


Figure 5: Pressure vs. averaged flow rate for various values of ϕ at $t=1, \alpha=1/4$, $\lambda_1=1, M=1.0$

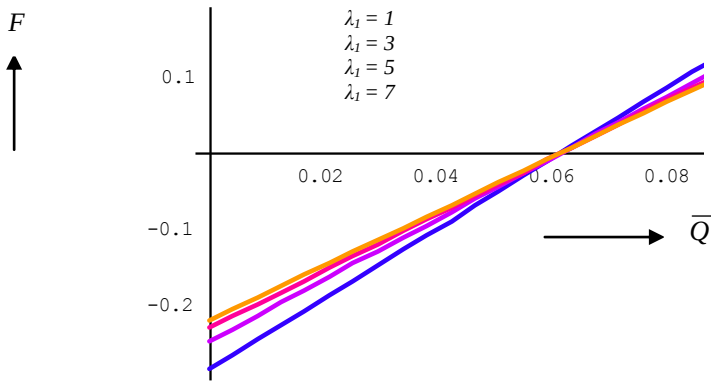


Figure 6: Frictional Force vs. averaged flow rate for various values of λ_1 at $t=1, \alpha=1/4$, $M=1.0, \phi=0.5$

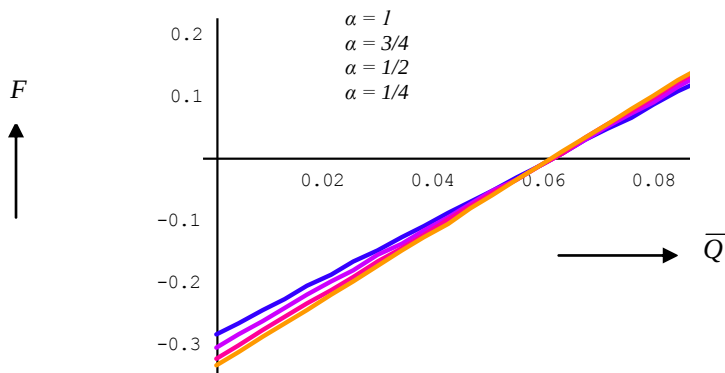


Figure 7: Frictional Force vs. averaged flow rate for various values of α at $t = 1$, $\lambda_1 = 1$, $M = 1.0$, $\phi = 0.5$

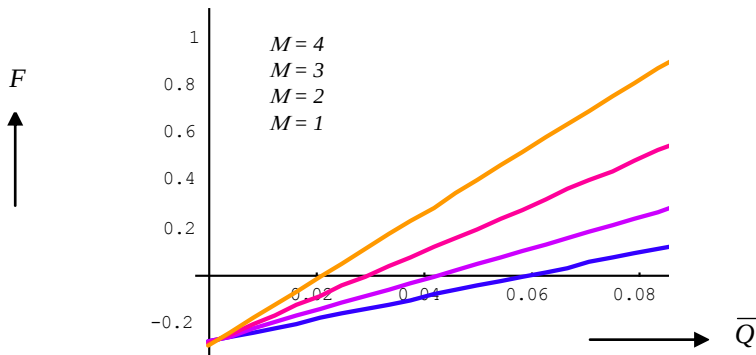


Figure 8: Frictional Force vs. averaged flow rate for various values of M at $t = 1$, $\alpha = 1/4$, $\lambda_1 = 1$, $\phi = 0.5$

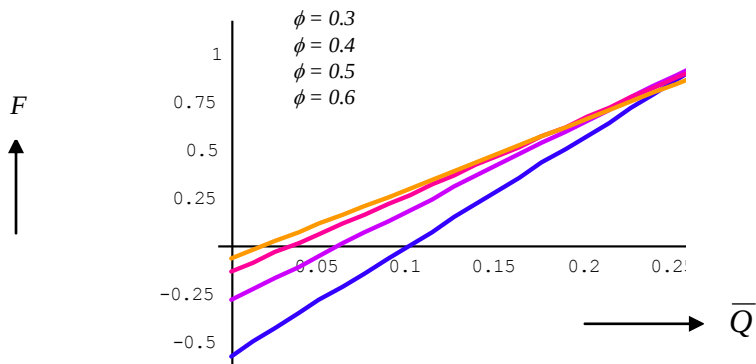


Figure 9: Frictional Force vs. averaged flow rate for various values of ϕ at $t = 1$, $\alpha = 1/4$, $\lambda_1 = 1$, $M = 1.0$

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