

Some Results on IF Pre m_x Open Sets in IF Topological Space*

Jayanti Tripathi Pandey

Department of Mathematics
Amity University, Noida, India
Email: jtpandey@lko.amity.edu

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Abstract: The purpose of this paper is to introduce the concept of IF pre m_x open sets in IF topological space. Some of its properties are discussed with existing IF open m_x sets and IF open sets. The topological structure formed by these sets and related continuities are also discussed in this paper.

Keywords: IF topology, IF generalized minimal closed sets, IF strongly generalized closed sets.

1. Introduction

The concept of fuzzy sets was introduced by Zadeh¹ in 1965 and later Atanassov² generalized this idea to intuitionistic fuzzy sets. But a contradiction arose with the concept of Intuitionistic Logic. The concept of Intuitionistic Logic is not similar with Intuitionistic fuzzy logic and hence to avoid this contradiction various researchers suggested various nomenclatures. Following some of their suggestion we are using the nomenclature as IF Set in place of Intuitionistic fuzzy set.

D.Coker³ introduced the notions of IF topological space and other related concepts. The concept of m_x structure was introduced by H. Maki in ordinary topological space which was extended in fuzzy m_x structure by M. Alimohammady and M. Roohi⁴. Later on many other researchers such as F. Nakaoka and N. Oda have shown the applications of this set. A lot of research work is done in both the areas in ordinary as well as fuzzy topology.

In section 2, the basic preliminary has been given. In section 3, the concept of IF pre m_x -open set has been introduced and some properties of this set is studied and also connection of this set with other IF sets are discussed in this section of the paper.

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2. Preliminary

Definition 2.1²: Let X is a nonempty set and I the unit interval $[0, 1]$. An IF set (IFS for short) U is an object having the form $U = \{ \langle x, \mu_u(x), \gamma_u(x) \rangle : x \in X \}$, where the functions $\mu_u : X \rightarrow I$ and $\gamma_u : X \rightarrow I$ denote respectively the degree of membership and degree of non-membership of each element $x \in X$ to the set U and $0 \leq \mu_u(x) + \gamma_u(x) \leq 1$ for each $x \in X$.

For the sake of simplicity we shall use the symbol $U = \langle x, \mu_u, \gamma_u \rangle$ for the IF set $U = \{ \langle x, \mu_u(x), \gamma_u(x) \rangle : x \in X \}$.

Definition 2.2⁵: Let X be a nonempty set and let IF sets U and V be in the form $U = \{ \langle x, \mu_u(x), \gamma_u(x) \rangle : x \in X \}$ and $V = \{ \langle x, \mu_v(x), \gamma_v(x) \rangle : x \in X \}$. Then

- (1) $U^c = \{ \langle x, \gamma_u(x), \mu_u(x) \rangle : x \in X \}.$
- (2) $U \cap V = \{ \langle x, \min(\mu_u(x), \mu_v(x)), \max(\gamma_u(x), \gamma_v(x)) \rangle : x \in X \}.$
- (3) $U \cup V = \{ \langle x, \max(\mu_u(x), \mu_v(x)), \min(\gamma_u(x), \gamma_v(x)) \rangle : x \in X \}.$
- (4) $(U^c)^c = U, 0_-^c = 1_-, 1_-^c = 0_-.$
- (5) $U \subseteq V$ iff $(\forall x \in X)(\mu_u(x) \leq \mu_v(x)), (\gamma_u(x) \geq \gamma_v(x)).$
- (6) $U = V$ iff $U \subseteq V$ and $V \subseteq U.$

Definition 2.3³: Let $\{A_i : i \in I\}$ be an arbitrary family of IF sets in X , where $A_i = (\mu_{A_i}, \gamma_{A_i})$ for each $i \in I$. Then

- (1) $\cap A_i = (\wedge \mu_{A_i}, \vee \gamma_{A_i}).$
- (2) $\cup A_i = (\vee \mu_{A_i}, \wedge \gamma_{A_i}).$

Definition 2.4³: An intuitionistic fuzzy topology (IFT for short) on a nonempty set X is a family τ of IF sets in X satisfying

- (i) $0_-, 1_- \in \tau.$
- (ii) $G_1 \cap G_2 \in \tau.$
- (iii) $\cup G_i \in \tau$ for any family $\{G_i : i \in I\} \subset \tau.$

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IF topological space for short) and each IF set in τ is known as an

intuitionistic fuzzy open set (IFOS for short). The compliment U^c of an intuitionistic fuzzy open set (IFOS, for short) U in an intuitionistic fuzzy topological space (IF topological space for short) (X, τ) is called an intuitionistic fuzzy closed set (IFCS, for short) in X .

Definition 2.5³: Let (X, τ) is an intuitionistic fuzzy topological space and A an intuitionistic fuzzy set in X . Then

- (a) IF closure of A is defined by $Cl(A) = \cap \{F : F \text{ is an IFCS in } X \text{ and } A \subseteq F\}$.
- (b) IF interior of A is defined by $Int(A) = \cup \{G : G \text{ is an IFOS in } X \text{ and } A \supseteq G\}$.

It can also be shown that $Cl(A)$ is an Intuitionistic fuzzy closed set and $Int(A)$ is an IFOS in X , and

- (a) A is an intuitionistic fuzzy closed set in X iff $Cl(A) = A$;
- (b) A is an intuitionistic fuzzy open set in X iff $Int(A) = A$.
- (c) $Int(1_-) = 1_-$
- (d) $Cl(0_-) = 0_-$

Definition 2.6⁶: A subset A of IF topological space (X, τ) is called an intuitionistic fuzzy semi open set if $A \subseteq Cl(Int(A))$ and is said to be an IF semi closed set if $Int(Cl(A)) \subseteq A$.

Definition 2.7⁶: Let A is an IFS of X . Then A is said to be intuitionistic fuzzy $-\alpha$ open if $A \subseteq Int(Cl(IntA))$ and is said to be intuitionistic fuzzy $-\alpha$ closed if $Cl(Int(ClA)) \subseteq A$.

Definition 2.8⁶: A subset A of IF topological space (X, τ) is called an intuitionistic fuzzy pre open set if $A \subseteq Int(Cl(A))$ and is said to be an intuitionistic fuzzy pre closed set if $Cl(Int(A)) \subseteq A$.

Definition 2.9⁷: A family m_x of intuitionistic fuzzy sets is called an intuitionistic fuzzy m_x -structure if 0_- and 1_- are the members of m_x . (X, m_x) denote a nonempty subset X with a minimal structure m_x on X . Each member of m_x is said to be an IF m_x open and the compliment of an m_x open set is an IF m_x closed.

Definition 2.10⁸: Let (X, m_x) be an IF m_x structure on (X, τ) , where (X, τ) is an IF topological space generated by IF m_x structure. An IF Set A is said to be an IF regular closed m_x set iff $Cl(m_x - Int(A)) = A$ and an IF regular open m_x set iff $Int(m_x - Cl(A)) = A$.

Definition 2.11⁷: Let (X, τ) be an intuitionistic fuzzy topological space and m_x an intuitionistic fuzzy m_x structure on X . For a subset A of (X, τ)

- (1) IF $m_x - Cl(A) = \{F : A \subset F, F^C \in m_x\}$,
- (2) IF $m_x - Int(A) = \cup\{G : G \subset A, G \in m_x\}$.

Lemma 2.12⁷: Let (X, τ) be an IF topological space and m_x an IF m_x structure on X . For a subset A of $((X, \tau))$

1. $m_x - Cl(A^C) = (m_x - Int(A))^C$ and $m_x - Int(A^C) = (m_x - Cl(A))^C$.
2. If $A^C \in m_x$, then $m_x - Cl(A) = A$ and if $A \in m_x$ then $m_x - Int(A) = A$.
3. $m_x - Int(0_-) = 0_-$ and $m_x - Int(1_-) = 1_-$.
4. $m_x - Cl(0_-) = 0_-$ $m_x - Cl(1_-) = 1_-$.
5. If $A \subseteq B$ then $m_x - Cl(A) \subseteq m_x - ClB$ and $m_x - IntA \subseteq m_x - IntB$.
6. $A \subseteq m_x - ClA$ and $m_x - IntA \subseteq A$.
7. $m_x - Cl(m_x - ClA) = m_x - ClA$ and $m_x - Int(m_x - IntA) = m_x - IntA$.

Throughout the paper, an ordinary topological space is denoted as τ and the complement of a set A is denoted as A^C respectively.

3. On IF Pre m_x Open Set

In this section the concept of IF pre m_x open set is introduced and some theorems related to this newly constructed set are studied and related properties are discussed.

Definition 3.1: Let (X, τ) is an IF topological space and (X, m_x) an IF m_x structure on X . An IF set A is said to be IF pre m_x open set (IFP m_x OS for short) if $A \subseteq Int(m_x - ClA)$ and IF pre m_x closed set (IFP m_x CS for short) if $A \subseteq Cl(m_x - IntA)$.

Example 3.2: $A = \{ \langle x, 0.3, 0.5 \rangle, x \in X \}$ and $B = \{ \langle x, 0.4, 0.6 \rangle, x \in X \}$. Let the IF m_x structure be $m_x = \{0_1 A, B\}$ and $\tau = \{0_1 A, B, A \cup B, A \cap B\}$. Consider an IF subset C of X such that $C = \{ \langle x, 0.2, 0.7 \rangle, x \in X \}$, then $m_x - ClC = \{ \langle x, 0.5, 0.4 \rangle, x \in X \}$ and $Int(m_x - ClC) = \{ \langle x, 0.4, 0.5 \rangle, x \in X \}$. Clearly $C \subseteq Int(m_x - ClC)$. Hence C is an IF pre m_x open set.

Theorem 3.3: In an IF topological space generated by an IF m_x structure, IF pre m_x open set and IF pre open set are similar concept.

Proof: Let A be an IF pre m_x open set in an IF m_x structure. Let (X, τ) is an IF topological space generated by the IF m_x structure (X, m_x) . So every, IF closed m_x set will be IF closed set. Thus IF pre m_x open set implies IF pre open set.

Conversely, Let A is an IF pre open set in the said topological space.

$$\Rightarrow A \subseteq Int(ClA) \text{ (as } A \text{ is IF pre open set)}$$

$$\Rightarrow ClA \subseteq m_x - ClA$$

$$\text{As } ClA \subseteq m_x - ClA \Rightarrow Int(ClA) \subseteq Int(m_x - ClA)$$

$$\Rightarrow A \subseteq Int(m_x - ClA)$$

Hence A is an IF pre m_x open set.

Remark 3.4: In an arbitrary IF topological space IF pre m_x open sets and IF pre open sets are independent concepts, which can be proved from the following example:

Let $A = \{ \langle x, 0.3, 0.5 \rangle, x \in X \}$ and $B = \{ \langle x, 0.4, 0.3 \rangle, x \in X \}$. Let the IF m_x structure be $m_x = \{0_1 A\}$ and the corresponding topology be $\tau = \{0_1 A, B\}$. Consider an IF subset C of X such that $C = \{ \langle x, 0.2, 0.4 \rangle, x \in X \}$, then $m_x - ClC = \{ \langle x, 0.5, 0.3 \rangle, x \in X \}$ and $Int(m_x - ClC) = \{ \langle x, 0.4, 0.3 \rangle, x \in X \}$.

Clearly, $C \subseteq Int(m_x - ClC)$. Hence C is an IF pre m_x open set. Now $ClC = \{ \langle x, 0.3, 0.4 \rangle, x \in X \}$, $Int(ClC) = \{ \langle x, 0.3, 0.5 \rangle, x \in X \}$. Here $C \not\subseteq Int(ClC)$. Hence C is not an IF pre open set.

Theorem 3.5: 0_1 and 1_1 are IF pre open m_x sets.

Proof: Directly from the definition.

Remark 3.6: Collection of all IF pre m_x open sets form an IF m_x structure.

Theorem 3.7: Every IF open m_x set is IF pre m_x open set.

Proof: Let A is an IF open m_x set $\Rightarrow m_x - IntA = A$.

Now $A \subseteq m_x - ClA \Rightarrow m_x - IntA = A \subseteq m_x - Int(m_x - ClA) \subseteq Int(m_x - ClA)$.

$\Rightarrow A \subseteq Int(m_x - ClA)$.

Hence A is an IF pre m_x open set.

Remark 3.8: The converse of above may not be true. This can be proved with the help of example 3.2. Here $m_x - IntC = 0 \neq 0$. Hence C is not an IF open m_x set.

Theorem 3.9: Every IF open set is an IF pre m_x open set.

Proof: Let A is an IF open set $\Rightarrow IntA = A$.

Now $A \subseteq m_x - ClA \Rightarrow IntA \subseteq Int(m_x - ClA) \Rightarrow A \subseteq Int(m_x - ClA)$

$\Rightarrow A$ is an IF pre m_x open set.

Remark 3.10: The converse of above need not be true. This can be seen from following example:

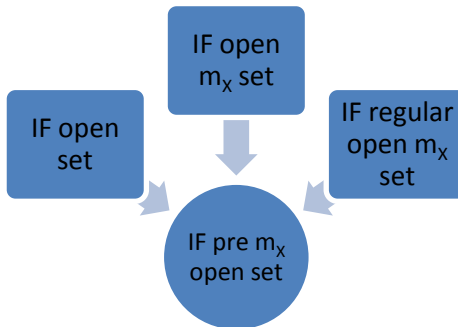
$A = \{ \langle x, 0.2, 0.5 \rangle, x \in X \}$ and $B = \{ \langle x, 0.4, 0.6 \rangle, x \in X \}$. Let the IF m_x structure be $m_x = \{ 0, 1, A \}$ and $\tau = \{ 0, 1, A, B, A \cup B, A \cap B \}$.

Consider an IF subset C of X such that $C = \{ \langle x, 0.3, 0.6 \rangle, x \in X \}$, then

$m_x - ClC = \{ \langle x, 0.5, 0.4 \rangle, x \in X \}$ and $Int(m_x - ClC) = \{ \langle x, 0.4, 0.5 \rangle, x \in X \}$

Clearly $C \subseteq Int(m_x - ClC)$. Hence C is an IF pre m_x open set. But is C not an IF open set as $C \notin \tau$.

So from the above discussion we get the following structural relation



Theorem 3.11: An IF regular open m_x set is IF pre m_x open set.

Proof: Let U be an IF regular open m_x set.

$$U = \text{Int}(m_x - ClU) \Rightarrow U \subseteq \text{Int}(m_x - ClU) \Rightarrow U \text{ open set.}$$

Remark 3.12: Converse of above is not true as can be seen from following example:

$A = \{ \langle x, 0.3, 0.5 \rangle, x \in X \}$ and $B = \{ \langle x, 0.4, 0.6 \rangle, x \in X \}$. Let the IF m_x structure be $m_x = \{0_1_A, B\}$ and $\tau = \{0_1_A, B, A \cup B, A \cap B\}$.

Consider an IF subset C of X such that $C = \{ \langle x, 0.2, 0.7 \rangle, x \in X \}$, then $m_x - ClC = \{ \langle x, 0.5, 0.4 \rangle, x \in X \}$, and $\text{Int}(m_x - ClC) = \{ \langle x, 0.4, 0.5 \rangle, x \in X \}$. Clearly $C \subseteq \text{Int}(m_x - ClC)$. Hence C is an IF pre m_x open set. But C is not an IF regular open m_x set as $C \neq \text{Int}(m_x - ClC)$.

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