Effect of Migration on the Population Growth Rate:
Models Approach

S. R. J. Singh
J. N. Krishi Vishwavidyalaya, College of Agriculture, Rewa, India

(Received February 25, 2007)

Abstract: The size of the population and its growth depends on many characteristics of the population as well as on the manner of reduction in the schedules of fertility and/or mortality. In this paper, some migration policies are worked out to compare the reduction in the rate of increase in population growth in rural areas due to rural-urban migration of males with that of females migration at different age levels. Models are illustrated with the two regions (rural and urban systems) for India. For simplicity, expressions deal with female sex only. This study emphasizes the need for the use of contraceptive devices at two or more age levels.

Keywords: Population projection, Population growth rate, Migration stream, Stationary population, Stable Population.

1. Introduction

The inter-relation between migration and reproduction and the implications of such relations for demographic policy and socioeconomic planning provides unique opportunity for the integrated research, especially when pursued within the broader context of urbanization and modernization. For most part of the world, the fertility of rural to urban migrants falls between rural and urban rates. It may be said that after the occurrence of movement, migrants may quickly adopt new surrounding, attitudes and behavior, including the lower fertility that are like those of other urbanities and that suit city life. A number of countries are using transmigration (a term used to denote government sponsored population movement programme) to control their populations or to have a proper population distribution to each region depending on the available resources or to make a development. For example, every year some inhabitants of Java, Bali and Lombik move to Sumatra, Kalimantan, Sulawesi and East Timer in Indonesia under her transmigration programme (Tirtosudarmo, 1987).

In India like many other developing countries, there is no such type of transmigration programme, but the spontaneous rural-urban migration has played its significant role in reducing the pressure of population growth in rural areas where village resources mostly depending on land or more or less stagnant over a period of time.

Though most of the migration in India is circular in nature, it occurs mostly among males who migrate to urban areas for their livelihood leaving their wives and children in
villages, such separation of wife-husband certainly reduce their fertility and hence overall growth rate of rural areas. The effect of various policies adopted so far in this direction can be observed in Widjojo (1970) where a number of realistic population projections are made under alternative assumptions regarding population movements. For India, Davis¹ has also estimated that hundred of millions Indians would have to migrate over a twenty-year period in order to compensate for the population growth. A rigorous theoretical analysis of the effects of migration on the population growth has been given by Keyfitz² ³ ⁴, Singh⁵, Singh and Yadava⁶, Sivamurthi⁷ and Yadava⁸.

In this paper, to find out the effects of migration on the population growth rate, some migration policies are worked out to compare the reduction in the rate of increase in population growth in rural areas due to rural-urban migration of males with that of females migration at different age levels, on the lines demarcated and perused by Keyfitz² ³ ⁴ and Singh et al.⁶ Models are illustrated with the two regions (rural and urban system) for India. For simplicity, expressions deal with female sex only.

2. Proportion of Migrants for Desired Reduction in the Rate of Increase

Keyfitz⁴ exploits the integral equation of Lotka model of the population dynamics to assess quantitatively the effect of age-specific migration over a limited period of time and of migration streams that are continued indefinitely into the future. It is known that migration over a limited period of time has no effect on the ultimate rate of increase, while on the other hand an indefinitely continued stream of emigration has effect on it. A continued certain stream of out migration at a given age or at some ages, can be thought of as either reducing the age specific fertility or creating discrete drop in the survivor function. The intrinsic rate of increase, say 'r' is determined by fertility and mortality schedules alone and it is the only real root of Lotka's integral equation.

\[ \beta \int_a^\infty e^{-ra} p(a) m(a) \ da = 1, \]

where \( p(a) \) is the fraction of female population that survives to age \( a \), \( m(a) \) da is the probability, that a female who is of age \( a \) will bear a female child in next \( da \) period of her life and \( a \) and \( \beta \) are respectively the lower and upper limits of the reproductive period.

Keyfitz³ has shown that if a proportion \( f \) of population migrates when reaching age \( x \), \( x < \beta \) and \( 0 \leq f \leq 1 \), then the new rate of increase \( r^* \), say satisfies

\[ \beta \int_a^\infty e^{-r^*a} p(a) m(a) \ da = 1, \]

with

\[ m(a) = \begin{cases} m(a) & \text{if } a < x \\ (1-f) m(a) & \text{if } a \geq x. \end{cases} \]

Solving (2.2) for the proportion \( f \) of emigrants at age \( x \), he found
\[
\begin{align*}
\beta 
\int_0^\alpha e^{-r \alpha} p(a) m(a) \, da - 1 \\
\int_0^\alpha e^{-r \alpha} p(a) m(a) \, da
\end{align*}
\]

(2.3)

\[
\frac{\beta}{\alpha} \int_0^\alpha e^{-r \alpha} p(a) m(a) \, da - 1 \\
\int_0^\alpha e^{-r \alpha} p(a) m(a) \, da
\]

Thus the equation (2.3) can be used to find the amount of out-migration at age \( x \) that would be required to achieve the desired rate of increase \( r^* \). In order to get a stationary population, the population that is governed by a regime of unchanging fertility and mortality schedules for a long time, age structure remains fixed and size changes with a constant rate of increase in known as stable population and the population, where rate of increase in population is zero and its size become fixed is called stationary population, i.e., for \( r^* = 0 \), equation (2.3) reduces to

\[
f = \frac{R_0 - 1}{\int_0^\alpha p(a) m(a) \, da}
\]

(2.4)

where \( R_0 = \int_0^\alpha p(a) m(a) \, da \), is the net reproduction rate, Also for \( x < \alpha \),

\[
f = \frac{R_0 - 1}{R_0}
\]

(2.5)

The equation (2.5) gives the proportion of migrants before reaching the age of reproduction to bring down the rate of increase to zero.

In the case of rural-urban migration, it is seen that a certain proportion of migrants return to their native place. If these return migrations are at the ages before the upper limit of the reproductive period, they must affect the rate of increase. Suppose a certain proportion \( f (0 \leq f \leq 1) \) of the population migrates when reaching age \( x \) and a certain proportion \( \sigma (\sigma < f) \) returns when reaching age \( y \) for \( x < y < \beta \) to their place of origin, then the new rate of increase, \( r^* \), say satisfied

\[
\beta \int_0^\alpha e^{-r \alpha} p(a) m^*(a) \, da = 1
\]

(2.6)

with

\[
m^*(a) = \begin{cases} 
m(a) & \text{if } a < x \\
(1 - f) m(a) & \text{if } x \leq a < y \\
(1 - f + \sigma) m(a) & \text{if } a \geq y.
\end{cases}
\]

(2.7)

Putting the value of \( m^*(a) \) and solving for \( f \), we have
\[
\beta \int_{a}^{\infty} e^{-r \cdot a} p(a) m(a) \, da + \sigma \int_{y}^{\infty} e^{-r \cdot a} p(a) m(a) \, da - 1
\]

Thus for the known value of \( \sigma \), the value of \( f \) can be computed for the desired reduction in the rate of increase. The expression (2.8) may also be treated as a temporary contraceptive type modal.

If a continued stream of out-migration is made at two ages, \( x \) and \( y \) say, then the new rate of increase \( \bar{r} \) say, satisfies

\[
\int_{a}^{\infty} e^{-\bar{r} \cdot a} p(a) \bar{m}(a) \, da = 1,
\]

with

\[
\bar{m}(a) = \begin{cases} 
  m(a) & \text{if } a < x \\
  (1 - f_1)m(a) & \text{if } x \leq a < y \\
  (1 - f_2)(1 - f_1)m(a) & \text{if } a \geq y 
\end{cases}
\]

where \( f_1 \) and \( f_2 \) are respectively the proportions of migrants at age \( x \) and \( y \). From (2.9) and (2.10), we get

\[
f_1 = \frac{\int_{a}^{x} e^{-\bar{r} \cdot a} p(a) m(a) \, da - f_2 \int_{y}^{\infty} e^{-\bar{r} \cdot a} p(a) m(a) \, da - 1}{\int_{a}^{x} e^{-\bar{r} \cdot a} p(a) m(a) \, da - f_2 \int_{y}^{\infty} e^{-\bar{r} \cdot a} p(a) m(a) \, da}
\]

For given \( \bar{r} \) and \( f_2 \), \( f_1 \) can be estimated with this equation. A similar expression for \( f_2 \) in terms of \( f_1 \) and \( \bar{r} \) can be worked out.

In those situations where survivor function \( p(a) \) is not available or not reliable, an approximate formula for the proportion of migrants at age \( x \) \((x < \alpha)\) to achieve the desired rate of increase is worked out in this section. An approximate formula for the rate of increase is given by Coale\(^6\) as

\[
r \approx \frac{\log(GRR) + \log p(\bar{m})}{\bar{m}},
\]

where \( GRR = \int_{a}^{\infty} m(a) \, da \) is the gross reproduction rate and \( p(\bar{m}) \) is the fraction of female population surviving to the mean age \( \bar{m} \) of the child bearing;
\[
\frac{\beta}{\int a m(a) \, da} = \frac{\alpha}{\int \frac{m(a)}{m(\alpha)} \, da}.
\]

If a constant proportion \( f \) of the population migrates before reaching the age of child bearing, the new rate of increase \( \bar{r} \) say, satisfies

\[
(2.13) \quad \bar{r} \approx \frac{\log(1 - f)GRR + \log p(\bar{m})}{\bar{m}}.
\]

From (2.12) and (2.13), we have

\[
(r - \bar{r})\bar{m} = -\log(1 - f),
\]

i.e.,

\[
1 - f = e^{-(r-\bar{r})\bar{m}},
\]

i.e.

\[
(2.14) \quad f = 1 - e^{-(r-\bar{r})\bar{m}}.
\]

In case \( \bar{r} = 0 \)

\[
(2.15) \quad f = 1 - e^{r\bar{m}}
\]

All expressions given in 2 deals with the female population. The male population can be obtained from

\[
\text{Male population} = \text{Female population} \times \text{Sex ratio at birth} \times \frac{\text{life expectancy of males}}{\text{life expectancy of females}}.
\]

3. Numerical Illustrations

The different migration policies aimed at finding out the proportions of migrants at one or more age levels for the desired reduction in the rate of increase as proposed in this paper are examined with the help of the data obtained during Rural Development and Population Growth-Survey-1987 (RD PG - Survey). The details of the survey are given by Singh\(^8\). The reduction in \( m(a) \) due to rural-urban migration of males and hence in the rate of increase is compared with the proportion of female migrants at one or more ages in order to have the same reduction in the rate of increase. For this purpose we need the values of net maternity function \( p(a)m(a) \). The value of \( m(a) \) for the migrated population (male migrated and female living at home permanently), non-migrated population and total population combining both migrated and non-migrated are computed with the help of the data collected in the RDPG Survey-1987. The computational procedures are given in Appendix 1 to this paper.

The value of \( p(a) \) are taken from the Regional Model Life Table of Coale and Demeny\(^9\). This level has been chosen due to the similarity in mortality experience of the
region under study. Table 1 presents the values of net maternity function for migrated, non-migrated and total population.

The values of integrals in the 5 yearly age intervals are approximated by Coale\(^9\) to be

\[
\int_{z}^{z+4} e^{-r\alpha} p(\alpha) m(\alpha) \, d\alpha = e^{-r(z+4)} \int_{z}^{z+4} F_z \, d\alpha,
\]

where \( \int_{z}^{z+4} p(\alpha) \, d\alpha \) and \( F_z \) is the age specific fertility rate for women aged \( z \) to \( z+4 \) at last birth day.

The rate of increase \( r \) for all three groups of the population are computed by using the standard formulae of stable population\(^9\) as:

\[
r = \frac{\log R_0}{T},
\]

where \( T \) is the mean length of generation in the stable population and \( T = \mu \), where \( \mu \) is the mean generation time\(^9\). The value of \( r \) for migrated, non-migrated and total populations respectively are 0.019, 0.027 and 0.026. This shows that the rate of increase for non-migrated population of the study area is 0.027 and rural-urban migration of males lowers this rate of increase by 0.001.

Table 1: Values of net maternity function \( p(\alpha) \, m(\alpha) \) alongwith \( R_0, r \) and \( \mu \) for Migrated, Non-migrated and total Couples.

<table>
<thead>
<tr>
<th>Age</th>
<th>( p(\alpha) )</th>
<th>( p(\alpha) , m(\alpha) )</th>
<th>( p(\alpha) , m(\alpha) )</th>
<th>( p(\alpha) , m(\alpha) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Migrated</td>
<td>Non-migrated</td>
<td>Total</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>0.75517</td>
<td>0.132999</td>
<td>0.251369</td>
<td>0.244719</td>
</tr>
<tr>
<td>20</td>
<td>0.74104</td>
<td>0.403261 *</td>
<td>0.550369</td>
<td>0.536520</td>
</tr>
<tr>
<td>25</td>
<td>0.72334</td>
<td>0.412441</td>
<td>0.510562</td>
<td>0.500584</td>
</tr>
<tr>
<td>30</td>
<td>0.70420</td>
<td>0.416575</td>
<td>0.425007</td>
<td>0.423321</td>
</tr>
<tr>
<td>35</td>
<td>0.68450</td>
<td>0.199115</td>
<td>0.259715</td>
<td>0.255387</td>
</tr>
<tr>
<td>40</td>
<td>0.66350</td>
<td>0.105650</td>
<td>0.131720</td>
<td>0.130348</td>
</tr>
<tr>
<td>45</td>
<td>0.64092</td>
<td>0.071931</td>
<td>0.046241</td>
<td>0.047526</td>
</tr>
<tr>
<td>( R_0 )</td>
<td>1.741369</td>
<td>2.17498</td>
<td>2.138404</td>
<td></td>
</tr>
<tr>
<td>( \mu )</td>
<td>29.9615</td>
<td>28.8801</td>
<td>28.9429</td>
<td></td>
</tr>
<tr>
<td>( r )</td>
<td>0.0185</td>
<td>0.0269</td>
<td>0.263</td>
<td></td>
</tr>
</tbody>
</table>

Example: The estimated proportion of females emigrating at 20 years of age to have the new rate of increase \( r^* = 0.026 \), is

\[
\frac{1.04950 - 1}{0.89004} = 0.05562.
\]
By using the formula (2.3) and Table 2, it is found that at age 20 about 5 percent emigration of female population is required to bring down the rate of increase from 0.027 to 0.026. Similarly at age 25 about 8 percent and at age 30 about 14 percent female population are required to have the same reduction in the rate of increase. Also about 54 percent of the females should be emigrated before reaching the age of reproduction 20 to bring down the rate of increase to zero (using formula 2.5).

If we take \( x = 25, y = 35 \) and \( \sigma = 0.5 \), then from formula (2.8), it is found that about 21 percent of the females at age 25 years should be emigrated in order to have the rate of increase from 0.027 to 0.026. Other expressions can also be illustrated with the help of the data given in Table 2.

Table 2: Calculation of \( e^{-r\sigma} p(a) m(a) \) for Estimating the Fraction of Emigrants to have the Rate of Increase to \( r' = 0.026 \) from \( r = 0.027 \)

<table>
<thead>
<tr>
<th>Age (z)</th>
<th>( e^{-0.026(z+2.5)} )</th>
<th>( e^{-0.026(z+2.5)} )</th>
<th>( \int e^{-r\sigma} p(a) m(a) , da )</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>0.6344</td>
<td>0.15946</td>
<td>1.04950</td>
</tr>
<tr>
<td>20</td>
<td>0.5571</td>
<td>0.30661</td>
<td>0.89004</td>
</tr>
<tr>
<td>25</td>
<td>0.4892</td>
<td>0.24976</td>
<td>0.58343</td>
</tr>
<tr>
<td>30</td>
<td>0.4296</td>
<td>0.18258</td>
<td>0.33367</td>
</tr>
<tr>
<td>35</td>
<td>0.3620</td>
<td>0.09402</td>
<td>0.15109</td>
</tr>
<tr>
<td>40</td>
<td>0.312</td>
<td>0.04363</td>
<td>0.05707</td>
</tr>
<tr>
<td>45</td>
<td>0.2908</td>
<td>0.01345</td>
<td>0.01345</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>1.04950</td>
</tr>
</tbody>
</table>

4. Conclusions

Several results based on different assumptions have been derived under the stability conditions. A number of expressions for the proportion of female migrants at one or more age levels are worked out for ultimate reduction in the rate of increase. It is worth while to mention that the birth and death rates existing in the study area (Varanasi rural) have remained more or less constant for several years, hence as an approximation, the population may be assumed to be stable and results of this paper may be reasonable approximation. Although it is not feasible in India to have migration of females at one or more age levels from population control, some of the proposed migration policies may be useful as temporary contraceptive methods or sterilization. Also it is not feasible to apply any contraceptive device at any single age level and hence provision of the same at two or more age levels may be useful and effective.

Appendix

Computation of age-specific fertility rates:

Age-specific fertility rates, defined as the average number of children born in a year to a female of a given age, have been computed by utilizing data on number of children ever born to the females belonging in different age-groups.
Firstly, females are classified according to their present age, into five-years age-groups. The average number of children ever born to those women in the successive age groups is computed and plotted on a graph against the age (see figure 1). A smooth curve is drawn to approximate these points. The number of children ever born at the ends of the age-intervals are read from graph, and by successive differences, the average number of children born during the intervals, a division by five gave the estimates of the age-specific fertility rate (ASFR) given table 3.

![Graph](image)

Table 3 presents the age-specific fertility rates for 'migrated' and 'non-migrated' couples. The ASFR for 'migrated couples' are consistently lower than for 'non-migrated' couples and the gap becomes narrower at higher ages. The narrow gap at higher age is mainly due to the fact that the observance of abstinence may be more prevalent at higher age for 'non-migrated' couples causing relatively larger reduction in fertility at higher ages, than in 'migrated' couples.
Table 3: Values of age-specific fertility rates for migrated, non-migrated and total couples

<table>
<thead>
<tr>
<th>Age Groups</th>
<th>Migrated</th>
<th>Non-migrated</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-19</td>
<td>0.100</td>
<td>0.189</td>
<td>0.184</td>
</tr>
<tr>
<td>20-24</td>
<td>0.233</td>
<td>0.318</td>
<td>0.310</td>
</tr>
<tr>
<td>25-29</td>
<td>0.248</td>
<td>0.307</td>
<td>0.301</td>
</tr>
<tr>
<td>30-34</td>
<td>0.247</td>
<td>0.252</td>
<td>0.251</td>
</tr>
<tr>
<td>35-39</td>
<td>0.138</td>
<td>0.180</td>
<td>0.177</td>
</tr>
<tr>
<td>40-44</td>
<td>0.077</td>
<td>0.096</td>
<td>0.095</td>
</tr>
<tr>
<td>45-49</td>
<td>0.056*</td>
<td>0.036</td>
<td>0.037</td>
</tr>
<tr>
<td>Total</td>
<td>1.099</td>
<td>1.378</td>
<td>1.355</td>
</tr>
<tr>
<td>T.M.F.R.</td>
<td>6.890</td>
<td>5.495</td>
<td>6.775</td>
</tr>
</tbody>
</table>

* Based on less than 20 observations.

The Computation of \( m(a) \):

Since the computation of ASFR includes all the females, this does not take into account whether the female is eligible or not. Thus, multiplying these ASFR by the proportion of eligible couples of the particular age-group, we can find ASFR for the eligible couples. Further, multiplying by sex-ratio and 5 (due to 5 yearly age group), we can get values of \( m(a) \) i.e.

\[
m(a) = \text{ASFR} \times S(a) \times Q \times 5.
\]

where \( Q \) is the sex ratio at birth and \( S(a) \) is the proportion of eligible couples at age \( a \). The value of \( Q \) in the RDPG – Survey 1987, is 0.487 female birth / per birth and the values of \( S(a) \) are presented in Table 4.

Table 4: Values of \( S(a) \) for each age-group

<table>
<thead>
<tr>
<th>Age-groups</th>
<th>( S(a) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-19</td>
<td>0.72328</td>
</tr>
<tr>
<td>20-24</td>
<td>0.95915</td>
</tr>
<tr>
<td>25-29</td>
<td>0.94421</td>
</tr>
<tr>
<td>30-34</td>
<td>0.98356</td>
</tr>
<tr>
<td>35-39</td>
<td>0.86567</td>
</tr>
<tr>
<td>40-44</td>
<td>0.84926</td>
</tr>
<tr>
<td>45-49</td>
<td>0.82305</td>
</tr>
</tbody>
</table>
References


