

Unsteady MHD Free Convective Flow and Heat Transfer between Heated Inclined Plates with Magnetic Field in the Presence of Radiation Effects

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Abstract: Aim of the paper is to investigate unsteady MHD natural convective flow of a viscous incompressible electrically conducting fluid and heat transfer between heated inclined non-conducting parallel plates in the presence of uniform magnetic field and radiative effect. Neglecting the induced magnetic field, the velocity and temperature distributions are derived, discussed numerically and their profiles for various values of physical parameters are shown through graphs. The skin-friction coefficient and Nusselt number at the plate are derived, discussed numerically and their numerical values for various values of physical parameters are presented through Tables.

Keywords: Unsteady, MHD, radiation, magnetic field, inclined channel, skin-friction and Nusselt number.

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1. Introduction

The free convection flow of a viscous incompressible electrically conducting fluid through the channel in the presence of a transverse magnetic field has important applications in magnetohydrodynamic generators, pumps, accelerators, cooling systems, centrifugal separation of matter from fluid, petroleum industry, purification of crude oil, electrostatic precipitation, polymer technology and fluid droplets sprays etc. The performance and efficiency of these devices are influenced by the presence of suspended solid particles in the wear activities and/or the combustion processes in MHD generators and plasma MHD accelerators. Many researchers investigated the intricate nature of solution structure from a fundamental point of view in idealizing settings.

Free convection in binary mixtures has been an attractive field of research over the last decades. The natural convection between heated vertical plates with magnetic field was studied by Osterle and Young¹.

Prakash² investigated the liquid flowing down an open inclined channel. The viscous flow down an open inclined channel with naturally permeability bed was presented by Verma and Vyas³. Sanyal and Sanyal⁴ analyzed the hydromagnetic slip flow with heat transfer in an inclined channel. The mixed convection heat transfer in an open ended inclined channels with reversal was studied by Rheault and Bilgen⁵. Sharma and Kumar⁶ investigated the unsteady flow of a non-Newtonian fluid down an open inclined channel. Wang and Robillard⁷ analyzed the mixed convection in an inclined channel with localized heat sources. Chamkha et al.⁸ presented radiation effects on free convection flow past a semi-infinite vertical plate with mass transfer. Malashetty et al.⁹ studied the convective magneto-hydrodynamic two fluid flow and heat transfer in an inclined channel. The unsteady flow and heat transfer of an electrically conducting viscous incompressible fluid between two non-conducting parallel porous plates under uniform transverse magnetic field was analyzed by Sharma and Chaturvedi¹⁰. Chamkha¹¹ discussed the unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate with heat absorption. Combined free and forced convection flow in an inclined channel with discrete heat sources was investigated by Guimaraes and Menon¹². The two fluid flow and heat transfer in an inclined channel containing porous and fluid layer was presented by Malashetty et al.¹³. Said et al.¹⁴ studied the numerical investigation of natural convection inside an inclined parallel walled channel. The pulsatile MHD flow and heat transfer through a porous channel with Ohmic effect was investigated by Sharma and Yadav¹⁵. The Soret effects due to natural convection between heated inclined plates with magnetic field was analyzed by Raju et al.¹⁶. The fully developed mixed convection flow between inclined infinite parallel porous plates filled with porous medium was discussed by Cimpean¹⁷.

Aim of the paper is to investigate the unsteady MHD fully developed natural convection flow of a viscous incompressible electrically conducting fluid and heat transfer between heated inclined infinite non-conducting plates in the presence of magnetic field and radiation effect.

2. Formulation of the Problem

Consider unsteady natural convective flow of a viscous incompressible electrically conducting binary mixture fluid between two inclined parallel non-conducting plates in the presence of magnetic field applied normal to the plates with the co-ordinates system employed and depicted in the figure-1. The radiation effects are also taken into account and plates are inclined with an angle ψ with horizon as shown in figure-1:

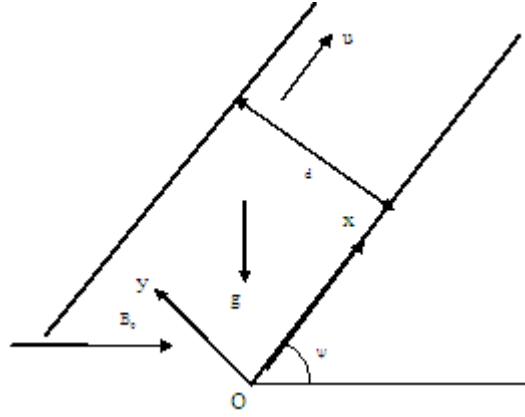


Fig.1. Physical Configuration and Coordinate Systems

The binary mixture of fluid of thermally and electrically conducting viscous incompressible fluids is sheared between two infinitely wide inclined plates separated by distance d . The density and viscosity of the mixture is independent of the distribution of the component. The flow problem of the binary mixture is identical to that of a single fluid but the velocity is to be understood as the mass average velocity $V = (\rho_1 V_1 + \rho_2 V_2) / \rho$ and the density $\rho = \rho_1 + \rho_2$, where the subscripts 1 and 2 denote the rare and the more abundant components, respectively.

The flow of the fluid due to buoyancy force is in the direction parallel to the plates and is of magnitude ' u '. The inclined magnetic field is of the order of the product of magnetic Reynolds number and imposed magnetic field. Since fully developed natural convection flow of a fluid with very small electrical conductivity is considered here, therefore it is the case of low magnetic Reynolds number and hence the induced magnetic field due to the weak applied magnetic field is neglected.

Within the frame of these assumptions, the governing equations for unsteady MHD free convection flow of a binary mixture of viscous incompressible thermally and electrically conducting viscous fluids sheared between two electrically non-conducting inclined parallel flat plates with the radiation effect in the presence of uniform magnetic field under the usual Bossinesq approximation are:

Equation of Motion

$$(1) \quad \frac{\partial u^*}{\partial t^*} = \nu \frac{\partial^2 u^*}{\partial y^{*2}} + g\beta(T^* - T_s)\sin\psi - \left(\frac{\sigma_e B_0^2}{\rho} \right) u^* .$$

Equation of Energy

$$(2) \quad \rho C_p \frac{\partial T^*}{\partial t^*} = \kappa \frac{\partial^2 T^*}{\partial y^{*2}} + \mu \left(\frac{\partial u^*}{\partial y^*} \right)^2 - \sigma_e B_0^2 u^{*2} - \frac{\partial q^*}{\partial y^*},$$

Heat Flux Equation

$$(3) \quad \frac{\partial^2 q^*}{\partial y^{*2}} - 3 \alpha^2 q^* - 16 \sigma^* \alpha T_\infty^3 \frac{\partial T^*}{\partial y^*} = 0,$$

where u^* is the velocity component in x^* - direction, t^* the time, ν the Kinematic viscosity, g the acceleration due to gravity, β the thermal expansion coefficient, T^* the temperature of the fluid, T_s the static temperature of the fluid, ψ the inclination angle of the plates with the horizontal axis, σ the electrical conductivity of the fluid, B_0 the intensity of applied magnetic field, ρ the fluid density, κ the thermal conductivity of the fluid, C_p the specific heat at constant pressure, q^* the radiation heat flux, α the absorption coefficient and σ^* is the Stefan Boltzman constant.

Since the medium is optically thin with relatively low density and $\alpha \ll 1$, the radiative heat flux given by equation (3), in the spirit of Cogley et al. (1968) gives

$$(4) \quad \frac{\partial q^*}{\partial y^*} = 4 \alpha^2 (T^* - T_\infty), \quad \alpha^2 = \int_0^\infty \delta \lambda \frac{\partial B}{\partial T^*}, \backslash$$

where B is Plank's function.

Grief et al.¹⁸ showed that for an optically thin limit, the fluid does not absorb its own emitted radiation hence there is no self absorption, but fluid absorbs radiation emitted by the boundaries.

The boundary conditions for velocity and temperature fields are

$$(5) \quad y^* = 0 : \quad \frac{\partial u^*}{\partial y^*} = 0, \quad \frac{\partial T^*}{\partial y^*} = 0;$$

$$y^* = d : u^* = U_d^* (1 + \epsilon e^{i\omega^* t^*}), \quad T^* = T_d^* (1 + \epsilon e^{i\omega^* t^*}),$$

where U_d^* is the characteristic velocity of the fluid, T_d^* the characteristic temperature of the fluid and ω^* is the frequency of vibration of fluid particles.

3. Method of Solution

Introducing the following dimensionless quantities

(6)

$$\begin{aligned}
y &= \frac{y^*}{d}, t = \frac{\nu t^*}{d^2}, \omega = \frac{d^2 \omega^*}{\nu}, u = \frac{\nu u^*}{g \beta d^2 (T_d^* - T_s)}, \\
\theta &= \frac{(T^* - T_s)}{(T_d^* - T_s)}, M = \frac{\sigma_e B_0^2 d^2}{\rho \nu}, N = \frac{4 \alpha^2 d^2}{\kappa}, \\
\text{Pr} &= \frac{\rho C_p \nu}{\kappa}, Gr = \frac{\rho g^2 \beta d^4 (T_d^* - T_s)}{\kappa \nu}, U_d = \frac{\nu U_d^*}{g \beta d^2 (T_d^* - T_s)}, \\
T_d &= \frac{T_d^*}{(T_d^* - T_s)},
\end{aligned}$$

into the equations (1) and (2), we get

$$(7) \quad \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + \theta \sin \psi - M u,$$

$$(8) \quad \text{Pr} \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + \text{Gr} \left(\frac{\partial u}{\partial y} \right)^2 + M \text{Gr} u^2 - N \theta,$$

where M is the Hartmann number, Pr the Prandtl number, Gr the buoyancy force parameter, i.e. the Grashof number for heat transfer, and N the radiation parameter. The equations (7) and (8) are second order coupled partial differential equations.

The boundary conditions in dimensionless form are given by

$$\begin{aligned}
(9) \quad y=0: \quad & \frac{\partial u}{\partial y} = 0, \frac{\partial \theta}{\partial y} = 0; \\
y=1: \quad & u = U_d (1 + \epsilon e^{i\omega t}), \theta = (1 + T_d \epsilon e^{i\omega t}),
\end{aligned}$$

where U_d is the dimensionless characteristic velocity and T_d is the dimensionless characteristic temperature of the fluid.

In view of the boundary conditions (9), the velocity and temperature distributions are separated into steady and unsteady parts by taking

$$\begin{aligned}
(10) \quad u(y, t) &= u_0(y) + \epsilon u_1(y) e^{i\omega t}, \\
\theta(y, t) &= \theta_0(y) + \epsilon \theta_1(y) e^{i\omega t},
\end{aligned}$$

Substituting (10) into the equations (7) and (8) and equating the harmonic and non-harmonic terms, we obtain

$$(11) \quad u_0'' - M u_0 = -\theta_0 \sin \psi,$$

$$(12) \quad u_1'' - (M + i\omega)u_1 = -\theta_1 \sin \psi,$$

$$(13) \quad \theta_0'' - N\theta_0 = -M \text{Gr} u_0^2 - \text{Gr} u_0'^2,$$

$$(14) \quad \theta_1'' - (N + i\omega \text{Pr})\theta_1 = -2M \text{Gr} u_0 u_1 - 2\text{Gr} u_0' u_1',$$

where prime denotes differentiation with respect to y .

The corresponding boundary conditions are reduced to

$$(15) \quad y=0: u_0' = 0, u_1' = 0, \theta_0' = 0, \theta_1' = 0;$$

$$y=1: u_0 = U_d, u_1 = U_d, \theta_0 = 1, \theta_1 = T_d.$$

The equations (11) to (14) are still coupled second order differential equations. In order to solve these equations, the u_0 , u_1 , θ_0 and θ_1 can be expanded in the powers of Gr (when $\text{Gr} \ll 1$) as given below

$$(16) \quad F(y) = F_0(y) + \text{Gr} F_1(y) + O(\text{Gr}^2) + \dots,$$

where F stands for any u_0 , u_1 , θ_0 or θ_1 .

Substituting (16) into the equations (11) to (14) and equating the coefficients of like powers of Gr , we get

$$(17) \quad u_{00}'' - M u_{00} = -\theta_{00} \sin \psi,$$

$$(18) \quad u_{01}'' - M u_{01} = -\theta_{01} \sin \psi,$$

$$(19) \quad u_{10}'' - (M + i\omega)u_{10} = -\theta_{10} \sin \psi,$$

$$(20) \quad u_{11}'' - (M + i\omega)u_{11} = -\theta_{11} \sin \psi,$$

$$(21) \quad \theta_{00}'' - N\theta_{00} = 0,$$

$$(22) \quad \theta_{01}'' - N\theta_{01} = -M u_{00}^2 - u_{00}'^2,$$

$$(23) \quad \theta_{10}'' - (N + i\omega \text{Pr})\theta_{10} = 0,$$

$$(24) \quad \theta_{11}'' - (N + i\omega \text{Pr})\theta_{11} = -2M u_{00} u_{10} - 2 u_{00}' u_{10}'.$$

Now, the corresponding boundary conditions are reduced to

$$(25) \quad y=0: u_{00}' = 0, u_{01}' = 0, u_{10}' = 0, u_{11}' = 0,$$

$$\theta_{00}' = 0, \theta_{01}' = 0, \theta_{10}' = 0, \theta_{11}' = 0;$$

$$y=1: u_{00} = U_d, u_{01} = 0, u_{10} = U_d, u_{11} = 0,$$

$$\theta_{00} = 1, \theta_{01} = 0, \theta_{10} = T_d, \theta_{11} = 0.$$

Now, the equations (17) to (24) are ordinary second order linear coupled differential equations and solved under the boundary conditions (25). Through straight forward algebra, the solutions of $u_{00}(y)$, $u_{01}(y)$, $u_{10}(y)$, $u_{11}(y)$, $\theta_{00}(y)$, $\theta_{01}(y)$, $\theta_{10}(y)$ and $\theta_{11}(y)$ are known but their expressions are not given here due to sake of brevity.

4. Skin-friction Coefficient

The coefficient of skin-friction at the upper plate is given by

$$(26) \quad C_f = \left(\frac{\partial u}{\partial y} \right)_{y=1} = \left(\frac{\partial u_0}{\partial y} + \epsilon e^{i\omega t} \frac{\partial u_1}{\partial y} \right)_{y=1} \\ = \left\{ \frac{\partial u_{00}}{\partial y} + Gr \frac{\partial u_{01}}{\partial y} + \epsilon e^{i\omega t} \left(\frac{\partial u_{10}}{\partial y} + Gr \frac{\partial u_{11}}{\partial y} \right) \right\}_{y=1},$$

Table- 1. Numerical values of skin-friction coefficient at the upper plate for various values of physical parameters

ϵ	Gr	ω	ωt	T_d	U_d	N	M	ψ	Pr	C_f
0.00	0.1	2	$\pi/6$	2	1	3	4	$\pi/3$	0.71	1.614267
0.01	0.1	2	$\pi/6$	2	1	3	4	$\pi/3$	0.71	1.636588
0.00	0.2	2	$\pi/6$	2	1	3	4	$\pi/3$	0.71	1.583233
0.00	0.1	3	$\pi/6$	2	1	3	4	$\pi/3$	0.71	1.636947
0.00	0.1	2	$\pi/3$	2	1	3	4	$\pi/3$	0.71	1.627559
0.00	0.1	2	$\pi/6$	3	1	3	4	$\pi/3$	0.71	1.644593
0.00	0.1	2	$\pi/6$	2	1.5	3	4	$\pi/3$	0.71	2.913556
0.00	0.1	2	$\pi/6$	2	1	5	4	$\pi/3$	0.71	5.893379
0.00	0.1	2	$\pi/6$	2	1	3	5	$\pi/3$	0.71	2.432173
0.00	0.1	2	$\pi/6$	2	1	3	4	$\pi/2$	0.71	1.560011
0.00	0.1	2	$\pi/6$	2	1	3	4	$\pi/3$	7.0	1.643126

5. Nusselt Number

The rate of heat transfer in terms of Nusselt Number at the upper plate is given by

$$(27) \quad Nu = - \left(\frac{\partial \theta}{\partial y} \right)_{y=1} = - \left(\frac{\partial \theta_0}{\partial y} + \epsilon e^{i\omega t} \frac{\partial \theta_1}{\partial y} \right)_{y=1} \\ = - \left\{ \frac{\partial \theta_{00}}{\partial y} + Gr \frac{\partial \theta_{01}}{\partial y} + \epsilon e^{i\omega t} \left(\frac{\partial \theta_{10}}{\partial y} + Gr \frac{\partial \theta_{11}}{\partial y} \right) \right\}_{y=1},$$

Table- 2. Numerical values of Nusselt number at the plate for various values of physical parameters

ϵ	Gr	ω	ωt	T_d	U_d	N	M	ψ	Pr	Nu
0.00	0.1	2	$\pi/6$	2	1	3	4	$\pi/3$	0.71	-0.92057
0.01	0.1	2	$\pi/6$	2	1	3	4	$\pi/3$	0.71	-0.94624

0.00	0.2	2	$\pi/6$	2	1	3	4	$\pi/3$	0.71	-0.240730
0.00	0.1	3	$\pi/6$	2	1	3	4	$\pi/3$	0.71	-0.945253
0.00	0.1	2	$\pi/3$	2	1	3	4	$\pi/3$	0.71	-0.931376
0.00	0.1	2	$\pi/6$	3	1	3	4	$\pi/3$	0.71	-0.957797
0.00	0.1	2	$\pi/6$	2	1.5	3	4	$\pi/3$	0.71	0.121253
0.00	0.1	2	$\pi/6$	2	1	5	4	$\pi/3$	0.71	-1.829373
0.00	0.1	2	$\pi/6$	2	1	3	5	$\pi/3$	0.71	-0.482957
0.00	0.1	2	$\pi/6$	2	1	3	4	$\pi/2$	0.71	-1.003451
0.00	0.1	2	$\pi/6$	2	1	3	4	$\pi/3$	7.0	-0.946133

6. Results and Discussion

Unsteady free convective flow of a binary mixture of incompressible thermally and electrically conducting viscous fluids sheared between two electrically non-conducting inclined parallel flat plates in the presence of uniform magnetic field is investigated by applying Cogley et al.¹⁹ approximation for the radiative heat flux. This enables us to carry out the numerical computation for the velocity field, the temperature field, skin-friction and Nusselt number for the various values of physical parameters and shown through figures and presented through Tables.

Fig.2 shows that the fluid velocity of steady flow is less in comparison to unsteady flow. Further, it is observed that the fluid velocity of unsteady flow increases with the increase of frequency of vibration of fluid particles or Grashof number for heat transfer; while it decreases with the increase of phase angle keeping other parameters fixed.

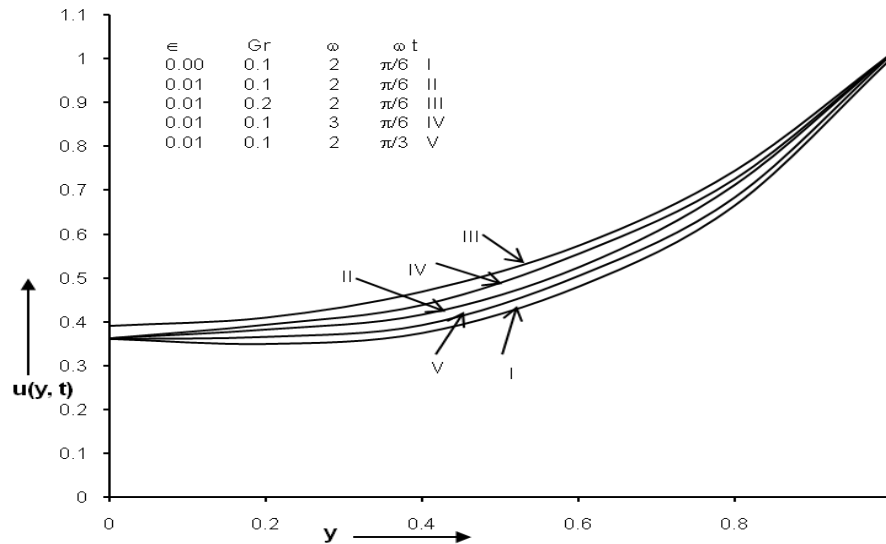


Fig. 2. Velocity distribution versus y when $T_d = 2$, $U_d = 1$, $N = 3$, $\psi = \pi/3$, $Pr = 0.71$ and $M = 4$.

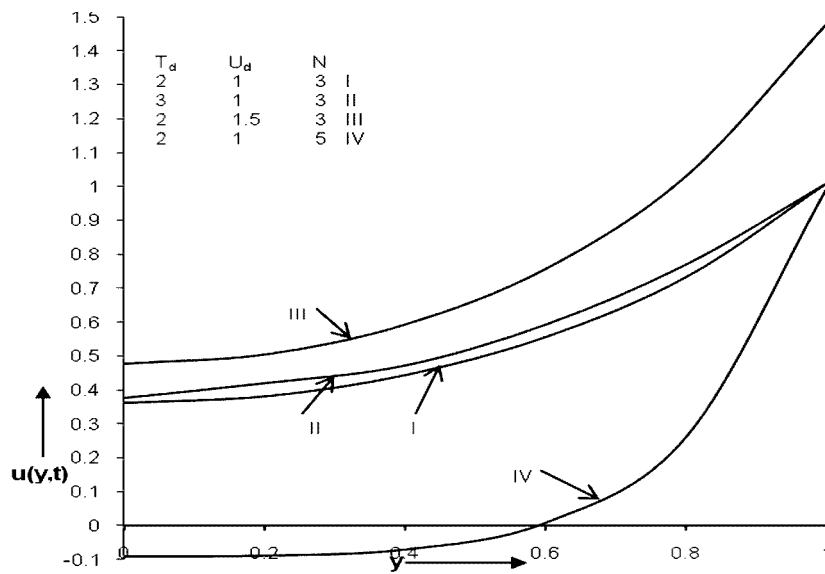


Fig.3. Velocity distribution versus y when $M=4$, $\psi = \pi/3$, $Pr = 0.71$, $\omega = 2$, $\omega t = \pi/6$, $Gr=0.1$ and $\epsilon=0.01$

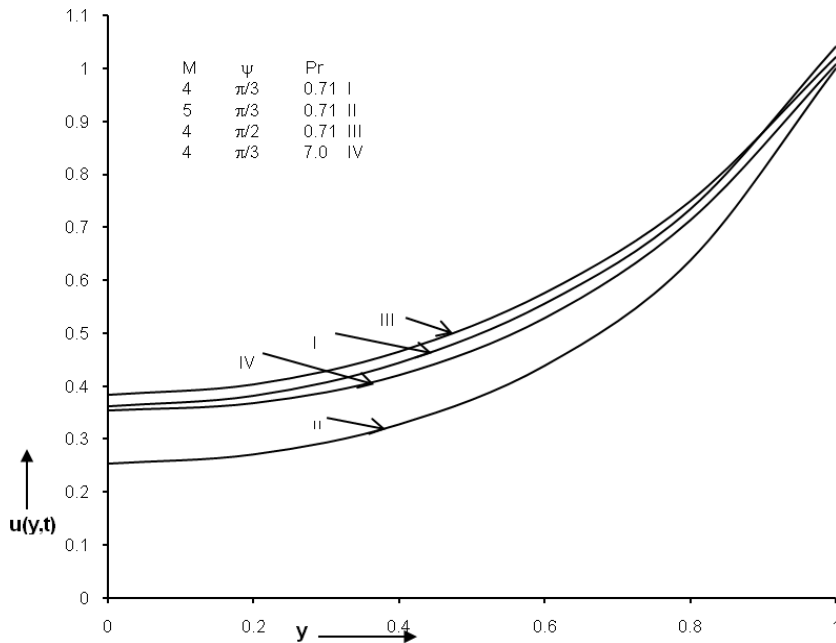


Fig.4. Velocity distribution versus y when $N=3$, $T_d = 2$, $U_d = 1$, $\omega = 2$, $\omega t = \pi/6$, $Gr=0.1$ and $\epsilon = 0.01$.

Fig.3 depicts that the fluid velocity increases with the increase of characteristic temperature or characteristic velocity; while it decreases with the increase of radiation parameter. It is observed from fig.4 that the amplitude of fluid velocity increases with the increase of inclination angle

of plates with horizon; while it decreases with the increase of Prandtl number or the Hartmann number.

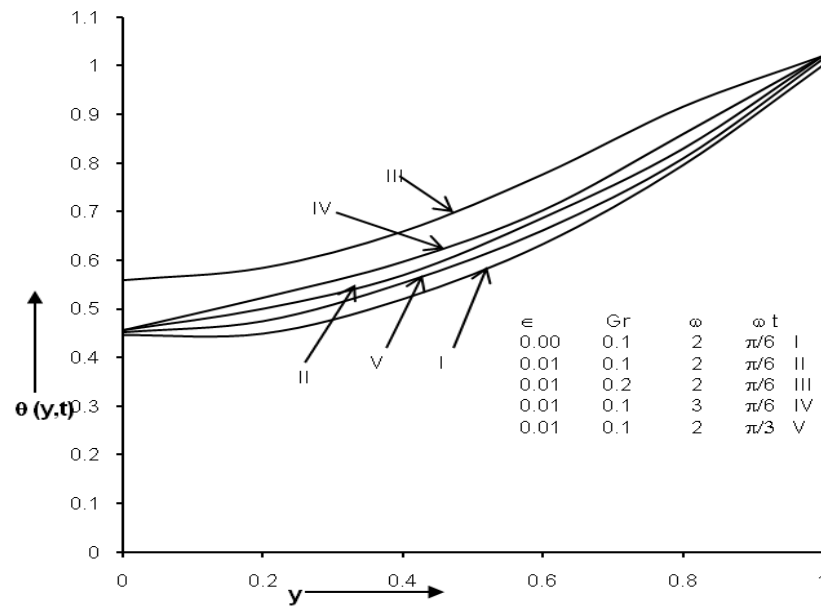


Fig.5. Temperature distribution versus y when $T_d = 2$, $U_d = 1$, $N = 3$, $\psi = \pi/3$, $Pr = 0.71$ and $M = 4$

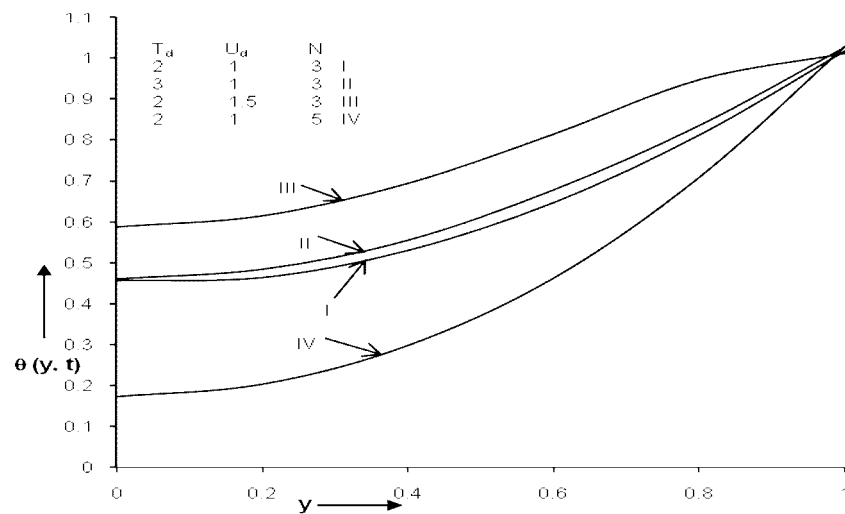


Fig. 6. Temperature distribution versus y when $M = 4$, $\psi = \pi/3$, $Pr = 0.71$, $\omega = 2$, $\omega t = \pi/6$, $Gr = 0.1$ and $\epsilon = 0.01$

It is noted from fig. 5 that the amplitude of fluid temperature of steady flow is less in comparison to unsteady flow. Further, it is observed for

unsteady flow that the fluid temperature increases with the increase of frequency of vibration of fluid particles or Grashof number for heat transfer; while it decreases with the increase of phase angle.

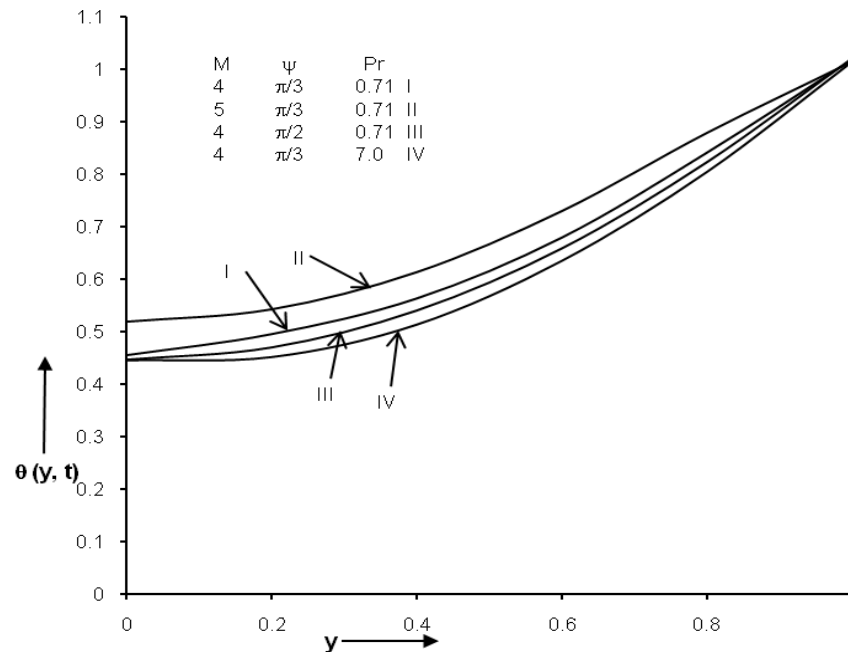


Fig.7. Temperature distribution versus y when $N=3$, $T_d=2$, $U_d=1$, $\omega=2$, $\omega t=\pi/6$, $Gr=0.1$ and $\epsilon=0.01$

Fig.6 reflects that the fluid temperature increases with the increase of characteristic temperature or characteristic velocity; while it decreases with the increase of radiation parameter. Fig.7 shows that the fluid temperature increases with the increase of Hartmann number; while it decreases with the increase of inclination angle of plates with horizon or Prandtl number.

It is inferred for the Table-1 that the skin-friction coefficient at the upper plate increases with the increase of frequency of vibration of fluid particles, characteristic temperature, characteristic velocity, radiation parameter, Hartmann number or Prandtl number; while it decreases with the increase of Grashof number for heat transfer, phase angle or inclination angle of plates with the horizon.

It is noted from Table-2 that the Nusselt number at the upper plate increases with the increase of Grashof number for heat transfer, frequency of vibration of fluid particles, phase angle, characteristic velocity or Hartmann number; while it decreases with the increase of characteristic temperature, radiation parameter, inclination angle of plates with the horizon or Prandtl number.

7. Conclusions

The unsteady MHD natural convection flow of a binary mixture and heat transfer between heated inclined plates under the influence of magnetic field and in the presence of radiation effect is investigated numerically. Neglecting the induced magnetic field, the equations governing the motion and energy are solved by perturbation technique taking buoyancy force parameter as a perturbation parameter and the conclusions are summarized below

- (i) An increase in buoyancy parameter, leads to rise in the magnitude of fluid velocity and fluid temperature in the presence of radiation effects.
- (ii) The magnitude of fluid velocity decreases with the increase of intensity of magnetic field.
- (iii) An increase in the radiation heat transfer leads to decrease rapidly in the magnitude of fluid velocity and fluid temperature within the boundary layer.
- (iv) The magnitude of fluid velocity and fluid temperature are greater for air in comparison to water.
- (v) The magnitude of fluid velocity increases with the increase of the angle ψ of inclination of the plates with horizon.
- (vi) An increase in the intensity in the magnetic field causes a rise in the magnitude of fluid temperature.
- (vii) An increase in the inclination angle of plates with the horizon leads to decrease in the magnitude of fluid temperature.

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