

Hall Effect on Thermosolutal Instability of Couple-Stress Fluid with Suspended Particles in Porous Medium

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Abstract: The thermosolutal instability of a couple-stress fluid in porous medium is studied in the presence of uniform horizontal magnetic field to include Hall currents and suspended particles. The stable solute gradient, suspended particles, Hall currents, Couple-stress and magnetic field introduce oscillatory modes in the system, which were non-existent in their absence. For stationary convection, the stable solute gradient, magnetic field, couple-stress have stabilizing effects whereas suspended particles and Hall currents have a destabilizing effects on thermosolutal instability of couple-stress fluid. The effects of stable solute gradient, suspended particles, Hall currents, couple-stress and magnetic field have also been shown graphically.

Keywords: Thermosolutal Convection, Couple-Stress Fluid, Hall Effect, Suspended Particles, Porous Medium.

1. Introduction

A detailed knowledge of thermal instability of fluid layer with adverse temperature gradient by heating the underside plays a significant role in geophysics, interior of the Earth, oceanography and atmospheric physics. Chandrasekhar¹ has put forward a detailed account of the theoretical and experimental study of the onset of Benard convection in Newtonian fluid under varying assumption of hydrodynamics. The investigations on such fluids are gaining importance with growing desirability of Non-Newtonian fluids in modern technology and industries.

Stokes² proposed and postulated the theory of couple-stress fluid. Synovial joints have become the object of scientific research which involves the application of couple-stress fluid. According to the theory of Stokes², couple-stresses are found to appear in noticeable magnitude in fluids with very large molecules. The Hall effect is possibly important in the flows of

laboratory plasma, geophysical and astrophysical situations. Sherman and Sutton³ have put forward the theory of effect of Hall currents on the efficiency of a magneto-fluid-dynamic generator. Gupta⁴ studied the thermal instability of fluid in the presence of Hall currents. The unsteady flow of a dusty elasto-viscous Rivlin-Ericksen fluid through channel of different cross-sections in the presence of time dependent pressure gradient was studied by Srivastava and Singh⁵. The thermal instability of the fluid layer under varying gravitational field have been studied by Pradhan, Samal and Tripathy⁶. Sharma and Thakur⁷ have studied the thermal convection in couple-stress fluid in porous medium in hydromagnetics. Sharma and Sharma have studied the effect of suspended particles on couple-stress fluid heated from below in the presence of rotation and magnetic field. The thermosolutal instability of Walters (Model B') visco-elastic rotating fluid permeated with suspended particles and variable gravity field in porous medium has been studied by Sharma and Rana.

In this paper we have considered the Hall effect on thermosolutal instability of couple-stress fluid with suspended particles in porous medium following Sharma and Sharma⁸ and Sharma and Rana⁹.

2. Formulation of the Problem and Perturbation Equations

Consider an infinite, horizontal, incompressible couple-stress fluid layer of thickness d , heated and soluted from below so that, the temperatures, densities and solute concentrations at the bottom surface $z = 0$ are T_0 , ρ_0 and C_0 and at the upper surface $z = d$ are T_d , ρ_d and C_d respectively, and that a uniform temperature gradient β ($= |dT/dz|$) and a uniform solute gradient β' ($= |dC/dz|$) are maintained. The gravity field $\vec{g}(0,0,-g)$ and a uniform horizontal magnetic field $\vec{H}(0,0,H)$ pervade the system.

Let p , ρ , T , C , α , α' , ν , μ' , g , η , μ_e and $\vec{q}$ (u,v,w) denote, respectively, pressure, density, temperature, solute concentration, thermal coefficient of expansion, an analogous solvent coefficient of expansion, kinematic viscosity, couple-stress viscosity, gravitational acceleration, resistivity, magnetic permeability and velocity of the fluid. $K' = 6\pi\rho\nu\eta'$, η' being particle radius, is the stokes' drag coefficient, $\bar{x} = (x, y, z)$, $E = \varepsilon + (1 - \varepsilon)\left(\frac{\rho_s c_s}{\rho_0 c_f}\right)$ is constant and E' is a constant analogous to E but corresponding to solute rather than heat; k , k' are the thermal diffusivity and solute diffusivity respectively; ρ_s , c_s , ρ_0 , c_f denote the density and heat capacity of solid (porous) matrix and fluid, respectively; $q_d(\bar{x}, t)$ and

$N(\bar{x}, t)$ denote filter velocity and number density of the suspended particles, respectively, the suffix zero refers to the values at reference level $z=0$.

The equations expressing the conservation of momentum, temperature mass balance equations of the couple-stress fluid (Stokes² and Chandrasekhar¹) are

$$(2.1) \quad \left[\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla) \vec{q} \right] = - \left(\frac{1}{\rho_0} \right) \nabla p + \vec{g} \left(1 + \frac{\delta \rho}{\rho_0} \right) + \frac{KN}{\rho_0} (\vec{q}_d - \vec{q}) \\ + \left(\nu - \frac{\mu'}{\rho_0} \nabla^2 \right) \nabla^2 \vec{q} + \frac{\mu_e}{4\pi\rho_0} (\nabla \times \vec{h}) \times \vec{H}.$$

$$(2.2) \quad \nabla \cdot \vec{q} = 0,$$

$$(2.3) \quad E \frac{\partial}{\partial t} T + (\vec{q} \cdot \nabla) T + \frac{mN C_{pt}}{\rho_0 c_f} \left[\varepsilon \frac{\partial}{\partial t} + \vec{q}_d \cdot \nabla \right] T = k \nabla^2 T,$$

$$(2.4) \quad E' \frac{\partial}{\partial t} C + (\vec{q} \cdot \nabla) C + \frac{mN' C_{pt}}{\rho_0 c_f} \left[\varepsilon \frac{\partial}{\partial t} + \vec{q}_d \cdot \nabla \right] C = k \nabla^2 C,$$

$$(2.5) \quad \rho = \rho_0 [1 - \alpha (T - T_0) + \alpha' (C - C_0)].$$

Assuming uniform particles size, spherical shape and small relative velocities between the fluid and particles, the presence of particles adds an extra force term, in the equation of motion (2.1), proportional to the velocity difference between particles and fluid.

The force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid. Interparticle reactions are ignored for we assume that the distances between the particles are large compared with their diameters. If mN is the mass of the particles per unit volume, then the equations of motion and continuity for the particles are

$$(2.6) \quad mN \left[\frac{\partial \vec{q}_d}{\partial t} + (\vec{q}_d \cdot \nabla) \vec{q}_d \right] = KN (\vec{q} - \vec{q}_d),$$

$$(2.7) \quad \frac{\partial N}{\partial t} + \nabla \cdot (N \vec{q}_d) = 0.$$

The Maxwell's equations yield

$$(2.8) \quad \frac{d\vec{H}}{dt} = (\vec{H} \cdot \nabla) \vec{q} + \eta \nabla^2 \vec{H} - \frac{c}{4\pi N e} \nabla \times [(\nabla \times \vec{H}) \times \vec{H}]$$

and

$$(2.9) \quad \nabla \cdot \vec{H} = 0,$$

where $\frac{d}{dt} = \frac{\partial}{\partial t} + \vec{q} \cdot \nabla$ stands for the convective derivative.

The initial state of the system is taken to be a quiescent layer (no settling) with a uniform particle distribution N_0 . The initial state is

$$(2.10) \quad \vec{q} = (0,0,0), \quad \vec{q}_d = (0,0,0), \quad T = -\beta z + T_0, \quad C = -\beta' z + C_0 \\ \rho = \rho_0 (1 + \alpha \beta z - \alpha' \beta' z), \quad N_0 = \text{constant}$$

is an exact solution to the governing equations.

Let $\delta \rho, \delta p, \theta, \gamma, \vec{q}(u, v, w)$ and $\vec{q}_d(l, r, s)$ denote, respectively, the perturbations in density ρ , pressure p , temperature T , solute concentration C , fluid velocity $(0,0,0)$ and particle velocity $(0,0,0)$. The change in density $\delta \rho$ caused by perturbation θ and γ in temperature and solute concentration is given by

$$(2.11) \quad \delta \rho = -\rho_0(\alpha \theta - \alpha' \gamma).$$

Then the linearized perturbation equations become

$$(2.12) \quad \frac{\partial \vec{q}}{\partial t} = -\frac{1}{\rho_0} \nabla \delta p - \vec{g}(\alpha \theta - \alpha' \gamma) + \frac{KN_0}{\rho_0} (\vec{q}_d - \vec{q}) \\ + \left(\nu - \frac{\mu'}{\rho_0} \nabla^2 \right) \nabla^2 \vec{q} + \frac{\mu_e}{4\pi \rho_0} (\nabla \times \vec{h}) \times \vec{H},$$

$$(2.13) \quad \nabla \cdot \vec{q} = 0 ,$$

$$(2.14) \quad \left(\frac{m}{K} \frac{\partial}{\partial t} + 1 \right) \vec{q}_d = \vec{q} ,$$

$$(2.15) \quad (E + b\varepsilon) \frac{\partial \theta}{\partial t} = \beta(w + bs) + k \nabla^2 \theta ,$$

$$(2.16) \quad (E' + b'\varepsilon) \frac{\partial \gamma}{\partial t} = \beta'(w + b's) + k' \nabla^2 \gamma ,$$

$$(2.17) \quad \nabla \cdot \vec{h} = 0$$

and

$$(2.18) \quad \frac{d\vec{h}}{dt} = (\vec{H} \cdot \nabla) \vec{q} + \eta \nabla^2 \vec{H} - \frac{c}{4\pi Ne} \nabla \times [(\nabla \times \vec{h}) \times \vec{H}] .$$

3. The Dispersion Relation

Analyzing the disturbances into normal modes, we assume that the perturbation quantities are of the form

$$(3.1) \quad [w, \theta, \gamma, h_z, \zeta, \xi] = [W(z), \Theta(z), \Gamma(z), K(z), Z(z), X(z)] \\ \times \exp[ik_x x + ik_y y + nt] ,$$

where k_x and k_y are the wave numbers along the x - and y - directions, respectively. $k = (k_x^2 + k_y^2)^{1/2}$ is the resultant wave number and n is the growth rate, which is, in general, a complex constant $\zeta = (\partial v / \partial x - \partial u / \partial y)$ and $\xi = (\partial h_y / \partial x - \partial h_x / \partial y)$ are the z -components of the vorticity and current density respectively.

Equations (2.11)–(2.16) with the help of eq. (2.11) and eq. (3.1) in non-dimensional form become

$$(3.2) \quad \left[\sigma \left(1 + \frac{M}{\tau_1 \sigma + 1} \right) + F(D^2 - a^2)^2 - (D^2 - a^2) \right] (D^2 - a^2) W \\ + \frac{g a^2 d^2}{\nu} (\alpha \Theta - \alpha' \Gamma) - \frac{i k_x \mu_e H d^2}{4 \pi \rho_0 \nu} (D^2 - a^2) K = 0,$$

$$(3.3) \quad \left[\sigma \left(1 + \frac{M}{\tau_1 \sigma + 1} \right) + F(D^2 - a^2)^2 - (D^2 - a^2) \right] Z = \frac{i k_x \mu_e H d^2}{4 \pi \rho_0 \nu} X,$$

$$(3.4) \quad (D^2 - a^2 - E_1 p_1 \sigma) \Theta = - \frac{\beta d^2}{k} \left(\frac{B + \tau_1 \sigma}{1 + \tau_1 \sigma} \right) W,$$

$$(3.5) \quad (D^2 - a^2 - E'_1 q'_1 \sigma) \Gamma = - \left(\frac{\beta' d^2}{k'} \right) \left(\frac{B' + \tau_1 \sigma}{1 + \tau_1 \sigma} \right) W,$$

$$(3.6) \quad (D^2 - a^2 - p_2 \sigma) K = - \left(\frac{i k_x H d^2}{\eta} \right) W + \left(\frac{i k_x c H d^2}{4 \pi N e \eta} \right) X,$$

and

$$(3.7) \quad (D^2 - a^2 - p_2 \sigma) X = - \left(\frac{i k_x H d^2}{\eta} \right) Z - \left(\frac{i k_x c H}{4 \pi N e \eta} \right) (D^2 - a^2) K,$$

where we have put $a = kd$, $\sigma = nd^2/\nu$, $F = v'/d^2$, $x/d = x^*$, $y/d = y^*$, $z/d = z^*$, $\tau = \frac{m}{K}$,

$\tau_1 = \frac{\tau \nu}{d^2}$, $M = \frac{m N_0}{\rho_0}$, $B = b+1$, $B' = b' + 1$, $E_1 = E + b\varepsilon$, $E'_1 = E' + b'\varepsilon$,

$F = \frac{\mu'}{\rho_0 d^2 \nu}$, $D^* = d \frac{d}{dz} = dD$ and $p_1 = \nu/k$ is the thermal prandtl number

$p_2 = \nu/\eta$ is the magnetic prandtl number, $q'_1 = \nu/k'$ is the Schmidt number and the superscript * is suppressed.

Eliminating Θ , Γ , Z , X and K between eqs. (3.2) – (3.7), we have

$$(3.8) \quad W \left\{ k_x^2 d^2 Q(D^2 - a^2 - p_2 \sigma) + k_x^2 d^2 M(D^2 - a^2) \right\}$$

$$\begin{aligned}
& \left[\sigma \left(1 + \frac{M}{\tau_1 \sigma + 1} \right) + F(D^2 - a^2)^2 - (D^2 - a^2) \right] - \\
& (D^2 - a^2 - p_2 \sigma)^2 \left[\sigma \left(1 + \frac{M}{\tau_1 \sigma + 1} \right) + F(D^2 - a^2)^2 - (D^2 - a^2) \right] \Bigg\} \\
& \left\{ Ra^2 \left(\frac{\tau_1 \sigma + B}{1 + \tau_1 \sigma} \right) (D^2 - a^2 - p_2 \sigma) \right. \\
& \left. (D^2 - a^2 - E'_1 q'_1 \sigma) - (D^2 - a^2)(D^2 - a^2 - p_2 \sigma)(D^2 - a^2 - E_1 p_1 \sigma) \right. \\
& \left. \left[\sigma \left(1 + \frac{M}{1 + \tau_1 \sigma} \right) + F(D^2 - a^2)^2 - (D^2 - a^2) \right] \right. \\
& \left. (D^2 - a^2 - E'_1 q'_1 \sigma) - Sa^2 \left(\frac{B' + \tau_1 \sigma}{1 + \tau_1 \sigma} \right) (D^2 - a^2 - p_2 \sigma)(D^2 - a^2 - E_1 p_1 \sigma) \right\} - \\
& \left\{ k_x^2 d^2 QW(D^2 - a^2)(D^2 - a^2 - E_1 p_1 \sigma)(D^2 - a^2 - p_2 \sigma)(D^2 - a^2 - E'_1 q'_1 \sigma) \right\} \\
& \left\{ (D^2 - a^2 - p_2 \sigma) \left[\sigma \left(1 + \frac{M}{\tau_1 \sigma + 1} \right) + F(D^2 - a^2)^2 - (D^2 - a^2) \right] - Qk_x^2 d^2 \right\},
\end{aligned}$$

where, $Q = \left(\frac{\mu_e H^2 d^2}{4\pi \rho_0 \nu \eta} \right)$ is the Chandrasekhar number,

$R = \left(\frac{g \alpha \beta d^4}{\nu k} \right)$ is the thermal Rayleigh number,

$S = \left(\frac{g \alpha' \beta' d^4}{\nu k'} \right)$ is the analogous solute Rayleigh number,

and, $M = \left(\frac{cH}{4\pi Ne\eta} \right)^2$ is the non-dimensional number accounting for Hall currents.

Consider the case in which both the boundaries are free as well as maintained at constant temperatures while the adjoining medium is perfectly conducting. The case of two free boundaries is a little artificial but it enables us to find analytical solution and to make some qualitative conclusions. The appropriate boundary conditions, with respect to which the eqs. (3.2) – (3.7) must be solved are (Chandrasekhar¹)

$$(3.9) \quad W=D^2W=\Theta=\Gamma=DZ=0, \text{ at } z=0 \text{ and } l \text{ and } h_x, h_y, h_z \text{ are continuous.}$$

On the perfectly conducting boundaries $DX=0$ and $K=0$. The case of two free boundaries though little artificial, is the most appropriate case for stellar atmospheres (Spiegel¹⁰). Using the above boundary conditions, it can be shown that all the even order derivatives of W must vanish for $z = 0$ and $z = 1$ and hence proper solution of equation (26) characterizing the lowest mode is

$$(3.10) \quad W = W_0 \sin \pi z,$$

where W_0 is constant. Substituting the proper solution $W = W_0 \sin \pi z$ in eq. (3.8) and letting $x = a^2/\pi^2$, $R_1 = R/\pi^4$, $S_1 = S/\pi^4$, $Q_1 = Q/\pi^2$, $i\sigma_1 = \sigma/\pi^2$, $F = F_1/\pi^2$, and $k_x = k \cos \theta$, we obtain the dispersion relation

$$(3.11) \quad R_1 = \left(\frac{\tau_1 i \sigma_1 \pi^2 + 1}{\tau_1 i \sigma_1 \pi^2 + B} \right) \left[\frac{\left\{ (1+x)(1+x-E_1 p_1 i \sigma_1) \left[i \sigma_1 \left(1 + \frac{M}{\tau_1 i \sigma_1 \pi^2 + 1} \right) + F_1 (1+x)^2 + (1+x) \right] \right\}}{x} \right] \\ + \left[\frac{S_1 (B' + \tau_1 i \sigma_1 \pi^2) (1+x + E_1 p_1 i \sigma_1)}{(\tau_1 i \sigma_1 \pi^2 + B) (1+x + E_1' q' i \sigma_1)} \right]$$

$$\begin{aligned}
& + \left(\frac{\tau_1 i \sigma_1 \pi^2 + 1}{\tau_1 i \sigma_1 \pi^2 + B} \right) \left[\begin{aligned} & \left\{ Q_1 \cos^2 \theta (1+x)(1+x + E_1 p_1 i \sigma_1) \right\} \\ & \left[\left\{ (1+x + p_2 i \sigma_1) \left[i \sigma_1 \left(1 + \frac{M}{\tau_1 i \sigma_1 \pi^2 + 1} \right) + F_1 (1+x)^2 + (1+x) \right] \right\} \right. \\ & \left. + Q_1 x \cos^2 \theta \right] \\ & \left[\begin{aligned} & Q_1 x \cos^2 \theta (1+x + p_2 i \sigma_1) + M x \cos^2 \theta (1+x) \\ & \left[i \sigma_1 \left(1 + \frac{M}{\tau_1 i \sigma_1 \pi^2 + 1} \right) + F_1 (1+x)^2 + (1+x) \right] + \\ & (1+x + p_2 i \sigma_1)^2 \left[i \sigma_1 \left(1 + \frac{M}{1 + i \sigma_1 \tau_1 \pi^2} \right) + F_1 (1+x)^2 + (1+x) \right] \end{aligned} \right] \end{aligned} \right]
\end{aligned}$$

Equation (3.11) is the required dispersion relation studying the effects of suspended particles, couple-stress fluid, magnetic field, stable solute gradient and Hall current of the system.

4. Stability of the System and Oscillatory Modes

In this section, we consider the possibility of oscillatory modes, if any, on the Couple-Stress fluid due to the presence of suspended particles, magnetic field, stable solute gradient and Hall current.

Multiplying eq. (3.2) by W^* , the complex conjugate of W , and using equations (3.3)-(3.7) together with the boundary conditions (3.9), we obtain

$$(4.1) \left[\sigma \left(1 + \frac{M}{1 + \tau_1 \sigma} \right) \right] [I_1] + F[I_2] + [I_3] - \frac{ga^2 \alpha k}{\nu \beta} \left(\frac{1 + \tau_1 \sigma}{B + \tau_1 \sigma} \right) [I_4 + E_1 p_1 \sigma^* I_5] +$$

$$\begin{aligned} & \frac{g\alpha'k'a^2}{v\beta'} \left(\frac{1+\tau_1\sigma}{B'+\tau_1\sigma} \right) [I_6 + E'_1 q'_1 \sigma^* I_7] + d^2 \left[\sigma^* \left(1 + \frac{M}{1+\tau_1\sigma^*} \right) \right] [I_8] + d^2 F[I_9] \\ & + d^2 [I_{10}] + \left(\frac{\mu_e \eta d^2}{4\pi\rho_0 v} \right) [I_{11} + p_2 \sigma I_{12}] + \left(\frac{\mu_e \eta}{4\pi\rho_0 v} \right) [I_{13} + p_2 \sigma^* I_{14}] = 0, \end{aligned}$$

where,

$$\begin{aligned} (4.2) \quad I_1 &= \int_0^1 \left(|DW|^2 + a^2 |W|^2 \right) dz, \\ I_2 &= \int_0^1 \left(|D^3 W|^2 + 3a^2 |D^2 W|^2 + 3a^4 |DW|^2 + a^6 |W|^2 \right) dz, \\ I_3 &= \int_0^1 \left(|D^2 W|^2 + 2a^2 |DW|^2 + a^4 |W|^2 \right) dz, \\ I_4 &= \int_0^1 \left(|D\Theta|^2 + a^2 |\Theta|^2 \right) dz, \quad I_5 = \int_0^1 \left(|\Theta|^2 \right) dz, \\ I_6 &= \int_0^1 \left(|D\Gamma|^2 + a^2 |\Gamma|^2 \right) dz, \quad I_7 = \int_0^1 \left(|\Gamma|^2 \right) dz, \quad I_8 = \int_0^1 \left(|Z|^2 \right) dz, \\ I_9 &= \int_0^1 \left(|D^2 Z|^2 + 2a^2 |DZ|^2 + a^4 |Z|^2 \right) dz, \\ I_{10} &= \int_0^1 \left(|DZ|^2 + a^2 |Z|^2 \right) dz, \quad I_{11} = \int_0^1 \left(|DX|^2 + a^2 |X|^2 \right) dz, \\ I_{12} &= \int_0^1 \left(|X|^2 \right) dz, \\ I_{13} &= \int_0^1 \left(|D^2 K|^2 + 2a^2 |DK|^2 + a^4 |K|^2 \right) dz, \\ I_{14} &= \int_0^1 \left(|DK|^2 + a^2 |K|^2 \right) dz, \end{aligned}$$

which are all positive definite.

Substituting $\sigma = i\sigma_0$ where σ_0 is real in equation (4.1), and equating the imaginary parts, we obtain

$$\sigma_0 \left[\frac{M+1+(\tau_1\sigma_0)^2}{1+(\tau_1\sigma_0)^2} (I_1 - d^2 I_8) - \frac{g\alpha k a^2}{v\beta} \right]$$

$$(4.3) \quad \left\{ \frac{\tau_1(B-1)}{B^2 + (\tau_1\sigma_0)^2} I_4 - E_1 p_1 \frac{B + (\tau_1\sigma_0)^2}{B^2 + (\tau_1\sigma_0)^2} I_5 \right\} + \frac{g\alpha'k'a^2}{v\beta'} \\ \left\{ \frac{\tau_1(B'-1)}{B'^2 + (\tau_1\sigma_0)^2} I_6 - E'_1 q'_1 \frac{B' + (\tau_1\sigma_0)^2}{B'^2 + (\tau_1\sigma_0)^2} I_7 \right\} + \left(\frac{\mu_e \eta d^2}{4\pi\rho_0 v} \right) P_2 I_{12} \\ - \frac{\mu_e \eta}{4\pi\rho_0 v} P_2 I_{14} \Big] = 0 .$$

Equation (4.3) yields $\sigma_0 = 0$ or $\sigma_0 \neq 0$, which means that modes may be non-oscillatory or oscillatory. In the absence of suspended particles, stable solute gradient, magnetic field, Hall currents and couple-stress, eqn.(4.3) reduces to

$$\sigma_0 \left(I_1 + \frac{g\alpha k a^2}{v\beta} E_1 p_1 I_5 \right) = 0$$

and terms in bracket are positive definite. Thus $\sigma_0 = 0$, which means the modes are non-oscillatory and the principle of exchange of stabilities is satisfied for a porous medium in the absence of suspended particles, magnetic field, stable solute gradient, couple-stress and Hall currents. The oscillatory modes are introduced due to the presence of suspended particles, magnetic field, stable solute gradient, couple-stress and Hall currents, which were non-existent in their absence.

5. The Stationary Convection

For stationary convection, the marginal state will be characterized by $\sigma = 0$; substituting $\sigma = 0$, the dispersion relation (3.11) reduced to

$$(5.1) \quad R_1 = \frac{1}{B} \left[\frac{F_1(1+x)^4 + (1+x)^3}{x} + S_1 B' \right. \\ \left. + \frac{Q_1 F_1(1+x)^5 \cos^2 \theta + (1+x)^4 Q_1 \cos^2 \theta + x(1+x)^2 Q_1^2 \cos^4 \theta}{x(1+x) Q_1 \cos^2 \theta + x(1+x)^3 M F_1 \cos^2 \theta + x(1+x)^2 M \cos^2 \theta + (1+x)^4 F_1 + (1+x)^3} \right]$$

To investigate the effects of suspended particles, stable solute gradient, Hall currents, magnetic field and couple-stress, we examine the behaviour of $\frac{dR_1}{dB}$, $\frac{dR_1}{dS_1}$, $\frac{dR_1}{dM}$, $\frac{dR_1}{dQ}$ and $\frac{dR_1}{dF_1}$ analytically. Eq. 5.1 yields

$$(5.2) \quad \frac{dR_1}{dB} = -\frac{1}{B^2} \left[\frac{(1+x)^4 F_1 + (1+x)^3}{x} + S_1 B' + \frac{(1+x)^5 Q_1 F_1 \cos^2 \theta + (1+x)^4 Q_1 \cos^2 \theta + x(1+x)^2 Q_1^2 \cos^4 \theta}{x(1+x)Q_1 \cos^2 \theta + x(1+x)^3 F_1 M \cos^2 \theta + x(1+x)^2 M \cos^2 \theta + (1+x)^4 F_1 + (1+x)^3} \right]$$

$$(5.3) \quad \frac{dR_1}{dS_1} = \left[\frac{B'}{B} \right],$$

$$(5.4) \quad \frac{dR_1}{dM} = -\frac{1}{B} \left[\frac{x(1+x)^8 F_1^2 Q_1 \cos^4 \theta + 2x(1+x)^7 F_1 Q_1 \cos^4 \theta + x^2(1+x)^5 F_1 Q_1^2 \cos^6 \theta + x(1+x)^6 Q_1 \cos^4 \theta + x^2(1+x)^4 Q_1^2 \cos^6 \theta}{\left\{ x(1+x)Q_1 \cos^2 \theta + x(1+x)^3 F_1 M \cos^2 \theta + x(1+x)^2 M \cos^2 \theta + (1+x)^4 F_1 + (1+x)^3 \right\}^2} \right]$$

$$(5.5) \quad \frac{dR_1}{dQ_1} = \frac{1}{B} \left[\frac{\left\{ x^2(1+x)^3 Q_1^2 \cos^6 \theta + x(1+x)^8 F_1^2 M \cos^4 \theta + 2x(1+x)^7 F_1 M \cos^4 \theta + 2x^2(1+x)^5 F_1 Q_1 M \cos^6 \theta + x(1+x)^6 M \cos^4 \theta + 2x^2(1+x)^4 Q_1 M \cos^6 \theta + (1+x)^9 F_1^2 \cos^2 \theta + 2(1+x)^8 F_1 \cos^2 \theta + 2x(1+x)^6 Q_1 F_1 \cos^4 \theta + (1+x)^7 \cos^2 \theta + 2x(1+x)^5 Q_1 \cos^4 \theta \right\}}{\left\{ x(1+x)Q_1 \cos^2 \theta + x(1+x)^3 F_1 M \cos^2 \theta + x(1+x)^2 M \cos^2 \theta + (1+x)^4 F_1 + (1+x)^3 \right\}^2} \right]$$

and

$$(5.6)$$

$$\frac{dR_1}{dF_1} = \frac{1}{B} \left[\frac{\left\{ \begin{aligned} &x^3(1+x)^6 Q_1^2 \cos^4 \theta + 2x^3(1+x)^8 Q_1 F_1 M \cos^4 \theta + x^3(1+x)^{10} F_1^2 M^2 \cos^4 \theta + \\ &2x^3(1+x)^7 Q_1 M \cos^4 \theta + 2x^3(1+x)^9 F_1 M^2 \cos^4 \theta + x^3(1+x)^8 M^2 \cos^4 \theta + \\ &2x^2(1+x)^9 F_1 Q_1 \cos^2 \theta + 2x^2(1+x)^{11} F_1^2 M \cos^2 \theta + 4x^2(1+x)^{10} F_1 M \cos^2 \theta + \\ &x(1+x)^{12} F_1^2 + 2x^2(1+x)^8 Q_1 \cos^2 \theta + 2x^2(1+x)^9 M \cos^2 \theta + 2x(1+x)^{11} F_1 + \\ &x(1+x)^{10} - x^4(1+x)^5 Q_1^2 M \cos^6 \theta \end{aligned} \right\}}{\left\{ \begin{aligned} &x^2(1+x) Q_1 \cos^2 \theta + x^2(1+x)^3 F_1 M \cos^2 \theta + x^2(1+x)^2 M \cos^2 \theta + \\ &x(1+x)^4 F_1 + x(1+x)^3 \end{aligned} \right\}^2} \right]$$

For stationary convection, magnetic field, stable solute gradient and couple-stress have a stabilizing effect whereas the Hall current, and suspended particles have a destabilizing effect on thermosolutal instability of couple-stress fluid with suspended particles.

The dispersion relation (5.1) is analysed numerically. Graphs have been plotted by giving some numerical values to the parameters, to depict the stability characteristics. In Fig. 1, R_1 is plotted against B for $F_1 = 0.6$, $B' = 2$, $S_1 = 6$, $Q_1 = 30$, $\theta = 45^\circ$, $M = 30$ for fixed wave numbers $x = 0.5$ and $x = 1.0$. For the wave numbers $x = 0.5$ and $x = 1.0$, suspended particles have a destabilizing effect. In figures 2, R_1 is plotted against S_1 for $F_1 = 0.6$, $B' = 2$, $Q_1 = 30$, $\theta = 45^\circ$, $M = 30$ and $B = 3$ for fixed wave numbers $x = 0.5$ and $x = 1.0$. For the wave numbers $x = 0.5$ and $x = 1.0$, stable solute gradient has a stabilizing effect. In figures 3, R_1 is plotted against M for $F_1 = 0.6$, $B' = 2$, $S_1 = 6$, $Q_1 = 30$, $\theta = 45^\circ$, $B = 3$ for fixed wave numbers $x = 0.5$ and $x = 1.0$. This shows that the Hall current has a destabilizing effect. In figures 4, R_1 is plotted against Q_1 for $F_1 = 0.6$, $B' = 2$, $S_1 = 6$, $\theta = 45^\circ$, $M = 30$, $B = 3$ for fixed wave numbers $x = 0.5$ and $x = 1.0$. For both the wave numbers R_1 increases as Q_1 increases implying the stabilizing effect of magnetic field.

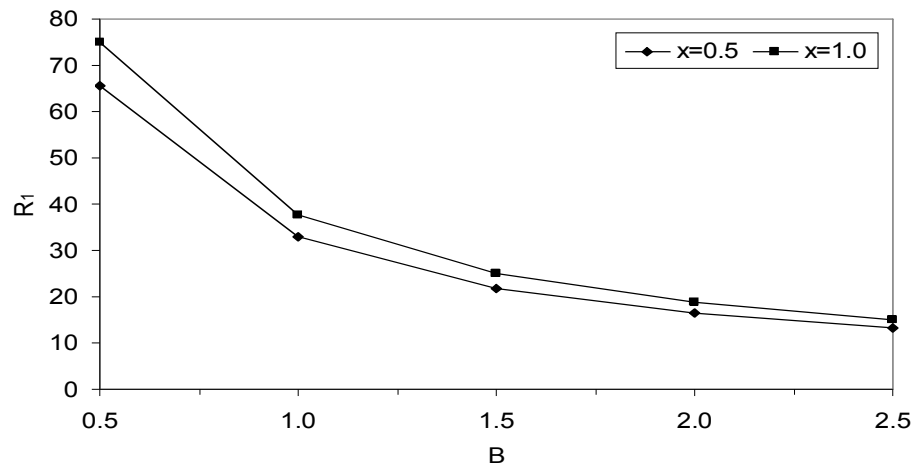


Fig. 1 The variation of R_1 with B for a fixed $F_1=0.6$, $B'=2$, $S_1=6$, $Q_1=30$, $\theta=45^\circ$, $M=30$ and for a fixed wave number $x=0.5$ and $x=1.0$

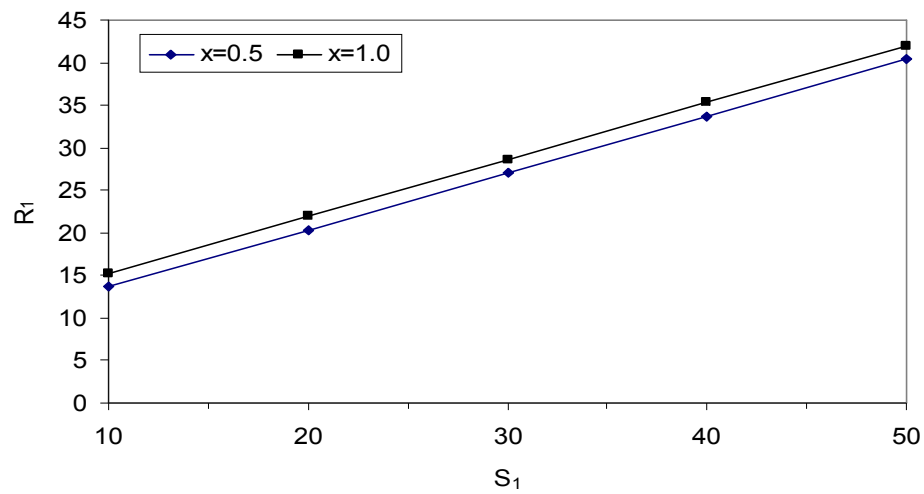


Fig. 2 The variation of R_1 with S_1 for a fixed $F_1=0.6$, $B'=2$, $Q_1=30$, $\theta=45^\circ$, $M=30$, $B=3$ and for a fixed wave number $x=0.5$ and $x=1.0$

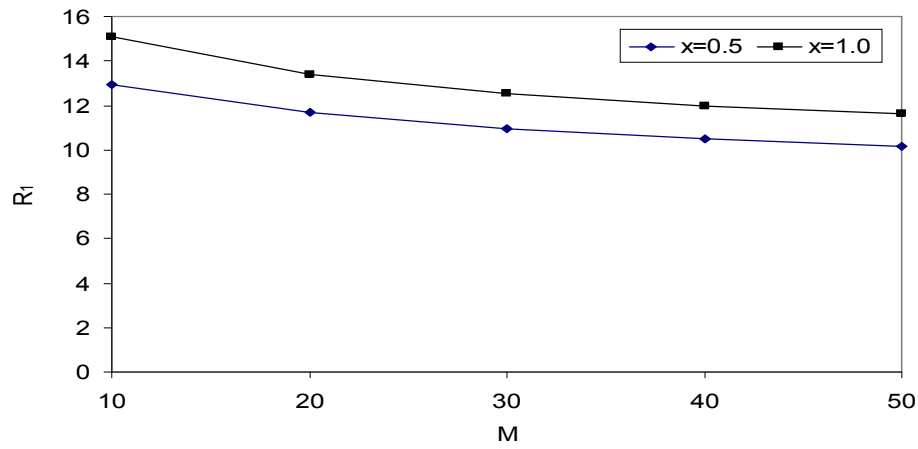


Fig. 3 The variation of R_1 with M for a fixed $F_1=0.6$, $B'=2$, $S_1=6$, $Q_1=30$, $\theta=45^\circ$, $B=3$ and for a fixed wave number $x=0.5$ and $x=1.0$

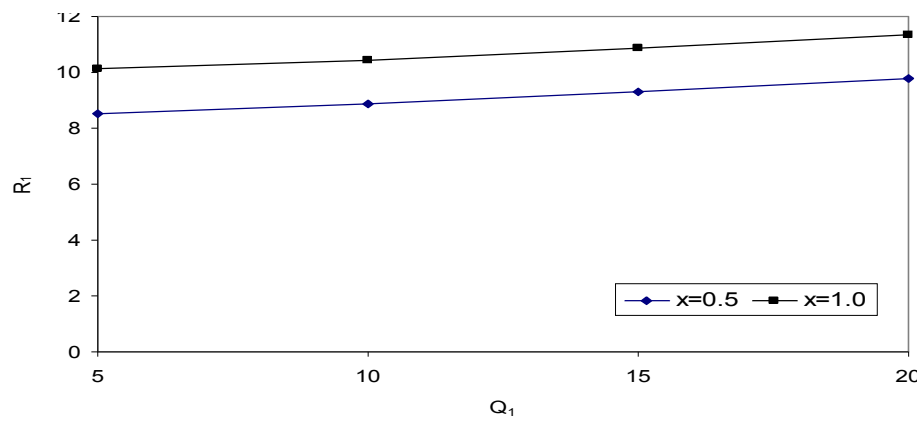


Fig. 4 The variation of R_1 with Q_1 for a fixed $F_1=0.6$, $B'=2$, $S_1=6$, $\theta=45^\circ$, $M=30$, $B=3$ and for a fixed wave number $x=0.5$ and $x=1.0$

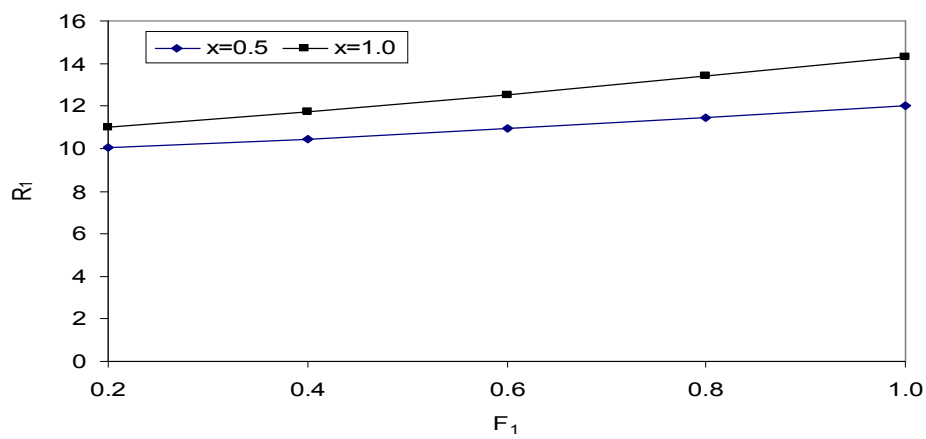


Fig. 5 The variation of R_1 with F_1 for a fixed $B'=2$, $S_1=6$, $Q_1=30$,

$\theta=45^\circ$, $M=30$, $B=3$ and for a fixed wave number $x=0.5$ and $x=1.0$

In figures 5, R_1 is plotted against F_1 for $B'=2$, $S_1 = 6$, $Q_1 = 30$, $\theta = 45^\circ$, $M=30$, $B = 3$ for fixed wave numbers $x = 0.5$ and $x = 1.0$. This shows that couple-stress has a stabilizing effect.

6. Summary

For stationary convection, the magnetic field has a stabilizing effect whereas the Hall current has a destabilizing effect on the system. The suspended particles destabilize the layer. The stable solute gradient and couple-stress are found to have stabilizing effects on the system. In the presence of magnetic field, stable solute gradient, suspended particles, Hall currents and Couple-stress oscillatory modes are introduced in the system. The effect on motion of the particles is shown graphically.

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