

## Fuzzy Closure in $g$ -Fuzzy Topological Spaces

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**Abstract:** The concept of fuzzy sets was introduced by L. A. Zadeh<sup>1</sup> in 1965 and investigates various properties. Csaszar<sup>2</sup> introduced and studied the concept of generalized open sets in generalized topologies. In this paper we have obtained significant properties of  $g$  – fuzzy closure in generalized fuzzy topological spaces simply called  $g$  – fuzzy topology.

**Keywords:** Fuzzy sets, fuzzy topology,  $g$  – fuzzy topology  $g$  – fuzzy closure.

### 1. Introduction

The concept of fuzzy topological space was first introduced in 1968 by C.L. Chang<sup>3</sup>. In 1981 K.K. Azad<sup>4</sup> has introduced the concept of fuzzy semi-open sets in fuzzy topological spaces. Bin Shahana<sup>5</sup> has introduced the concept of fuzzy pre-open sets and fuzzy  $\alpha$  – open sets in fuzzy topological space. S.S. Thakur<sup>6</sup> has introduced the concept of fuzzy semi pre-open sets in fuzzy topological spaces.

In this paper we have studied the concept of  $g$  – fuzzy closure in the generalized fuzzy topological spaces. Section 2 deals with the basic concepts of  $g$ -fuzzy topology. In section 3 the concept of  $g$  – fuzzy closure in generalized fuzzy topological spaces is introduced and studied and several relationships are established.

## 2. Preliminaries

**Definition 2.1:** Let  $X$  is non-empty universal crisp set. A fuzzy topology on  $X$  is a non empty collection  $\tau$  of fuzzy sets on  $X$  satisfies the following conditions

- (i) Fuzzy sets  $0$  and  $1$  belong to  $\tau$ .
- (ii) Any arbitrary union of members of  $\tau$  is in  $\tau$ .
- (iii) A finite intersection of members of  $\tau$  is in  $\tau$ .

Here  $0$  and  $1$  represent the Zero Fuzzy Set and the Whole Fuzzy set on  $X$ , defined as,  $1(x)=1$ . The pair  $(X, \tau)$  is called Fuzzy Topological Space on  $X$ , For Convenience, we shall denote the fuzzy topological space simply as  $X$ .

**Example 2.1:** Let  $X = \{x_1, x_2, x_3\}$  be the universal crisp set and  $\lambda$  be a fuzzy set defined on  $X$  as  $\lambda(x_1)=0.8, \lambda(x_2)=0.5, \lambda(x_3)=0.2$ . Then we can see that the collection  $\{1, \lambda, 0\}$  satisfies all the three conditions of fuzzy topology on  $X$ . Hence  $\tau = \{0, \lambda, 1\}$  is a fuzzy topology on  $X$  and  $\{X, \tau\}$  is a fuzzy topological space.

**Proposition 2.1:** Let  $\{X, \tau\}$  be a fuzzy topological space and  $\lambda$  let be a fuzzy set in  $X$ . Then

- (i)  $0$  and  $1$  are fuzzy closed set in  $X$ .
- (ii) Arbitrary intersection of fuzzy closed sets is a fuzzy closed set.
- (iii) Finite union of fuzzy closed sets is a fuzzy closed set.

**Definition 2.2:** Let  $(X, \tau)$  be a fuzzy topological space and let  $\lambda$  be a fuzzy set in  $X$ . Then closure of fuzzy set  $\lambda$  is denoted by  $Cl(\lambda)$  and is defined to the intersection of all fuzzy closed sets in  $X$  containing  $\lambda$ .

**Remark 2.1:** We note that  $Cl(\lambda) = \inf \{K : \lambda \leq K, K^c \in \tau\}$  Thus closure of a fuzzy set  $\lambda$  is the smallest fuzzy closed set containing.

**Proposition 2.2:** Let  $\lambda$  be a fuzzy set in a fuzzy topological space  $(X, \tau)$  then is  $\lambda$  a fuzzy closed if  $Cl(\lambda) = \lambda$ .

**Proposition 2.3:** Let  $(X, \tau)$  be a fuzzy topological space and  $\lambda_1, \lambda_2$  be a two fuzzy sets in  $X$ . Then

- (i)  $Cl(0) = 0$ ,

- (ii)  $Cl(1)=1$ ,
- (iii) If  $\lambda_1 \subseteq \lambda_2$  then  $Cl(\lambda_1) \subseteq Cl(\lambda_2)$ ,
- (iv)  $Cl(\lambda_1 \cup \lambda_2) = Cl(\lambda_1) \cup Cl(\lambda_2)$ ,
- (v)  $Cl(Cl(\lambda_1)) = Cl(\lambda_1)$ .

**Proposition 2.4:** Let  $X$  be a topological space and  $\{\lambda_j\}_{j \in J}$  be a family of subsets of  $X$ . Then

- (i)  $\bigcup_{j \in J} Cl(\lambda_j) \subseteq Cl(\bigcup_{j \in J} \lambda_j)$ ,
- (ii)  $Cl(\bigcup_{j \in J} \lambda_j) \subseteq \bigcap_{j \in J} Cl(\lambda_j)$ .

### 3. Fuzzy $g$ -Closure

**Definition 3.1:** Let  $X$  be a (non-empty) universal set. A fuzzy topology on  $X$  is a non empty collection  $\tau_g$  of fuzzy sets on  $X$  satisfying the conditions

- (i) Fuzzy sets 0 and 1 belong to  $\tau$ .
- (ii) if  $\{\lambda_j\}$  for  $j \in J$  is any family of fuzzy sets on  $X$  and  $\lambda_j \in \tau_g, \forall j \in J$  then  $\bigcup_{j \in J} \lambda_j \in \tau_g$ .

The pair  $\{X, \tau_g\}$  is called fuzzy generalized topological space. The elements of family  $\tau_g$  are called fuzzy  $g$ -open sets and their complements are called fuzzy  $g$ -closed sets.

**Example 3.1:** Let  $X = \{x_1, x_2\}$  and the fuzzy set  $\lambda_1, \lambda_2$  be define follow as  $\lambda_1 = \{(x_1, 0.3), (x_2, 0.6)\}$ ,  $\lambda_2 = \{(x_1, 0.5), (x_2, 0.4)\}$ ,  $\lambda_3 = \{(x_1, 0.5), (x_2, 0.6)\}$ . Let  $\tau_g = \{0, \lambda_1, \lambda_2, \lambda_3, 1\}$  is a generalized fuzzy topology on  $X$ , but not fuzzy topology on  $X$ .

**Proposition 3.1:** let  $\{X, \tau_g\}$  be a generalized fuzzy topological space then

- (i) 0 and 1 are  $g$ -fuzzy closed set in  $X$ .
- (ii) Arbitrary intersection of  $g$ -fuzzy closed sets is a  $g$ -fuzzy closed set is  $g$ -fuzzy closed in  $X$ .

**Proof:** Let  $0$  and  $1$  are  $g$ -fuzzy closed set, therefore their complements  $1$  and  $0$  are  $g$ -fuzzy closed set in  $X$ . Similarly let  $\{X, \tau_g\}$  be a generalized fuzzy topological space and let  $\{\lambda_j\}_{j \in J}$  be a collection of  $g$ -fuzzy closed sets in  $X$ , where  $J$  is index set then  $\{\lambda_j^c\}_{j \in J}$  is collection of  $g$ -fuzzy open sets in  $X$ . This implies  $\bigcup_{j \in J} \lambda_j^c$  is fuzzy  $g$ -open set in  $X$ . Hence  $(\bigcup_{j \in J} \lambda_j^c)^c = (\bigcap_{j \in J} \lambda_j)$  is  $g$ -fuzzy closed set in  $X$ .

**Remark 3.1:** We note that union of two  $g$ -fuzzy closed set in  $X$  may not be a  $g$ -fuzzy closed set in  $X$ .

**Example 3.2:** let  $X = \{x_1, x_2\}$  and the fuzzy set  $\lambda_1, \lambda_2$  be define follow as  $\lambda_1 = \{(x_1, 0.5), (x_2, 0.3)\}$ ,  $\lambda_2 = \{(x_1, 0.3), (x_2, 0.4)\}$  and  $\lambda_1 = \{(x_1, 0.5), (x_2, 0.4)\}$ . Let  $\tau = \{0, \lambda_1, \lambda_2, \lambda_3, 1\}$  be a  $g$ -fuzzy topology on  $X$ . Then we see union of  $g$ -fuzzy closed sets is not  $g$ -fuzzy closed.

**Definition 3.2:** Let  $\{X, \tau_g\}$  be a generalized fuzzy topological space and let  $\lambda$  be fuzzy set in  $X$ . Then the  $g$ -fuzzy closure of  $\lambda$  is defined as the intersection of all  $g$ -fuzzy closed sets in  $X$  containing  $\lambda$ . The  $g$ -fuzzy closure of  $\lambda$  is denoted by  $C_g(\lambda)$ .

**Remark 3.2:** We note that  $C_g(\lambda)$  is the smallest fuzzy  $g$ -closed set in  $\{X, \tau_g\}$  containing  $\lambda$ .

**Proposition 3.2:** Let  $\{X, \tau_g\}$  be a generalized fuzzy topological space and let  $\lambda$  be a fuzzy set in  $X$ . Then  $\lambda$  is  $g$ -fuzzy closed set if and only if  $C_g(\lambda) = \lambda$ .

**Proof:** Suppose that  $\lambda$  is a  $g$ -fuzzy closed set in  $X$ . Then clearly the smallest  $g$ -fuzzy closed set containing  $\lambda$  is itself  $\lambda$ . Hence  $C_g(\lambda) = \lambda$  conversely suppose  $C_g(\lambda) = \lambda$  then by definition of  $g$ -fuzzy closure it follow that  $C_g(\lambda)$  is a  $g$ -fuzzy closed set.

**Proposition 3.3:** Let  $\{X, \tau_g\}$  be a generalized fuzzy topological space and let  $\lambda$  be a fuzzy set in  $X$  and let  $\lambda_1, \lambda_2$  be a two fuzzy sets on  $X$ . Then following properties holds.

- (i)  $C_g(0)=0$ ,
- (ii)  $C_g(1)=1$ ,
- (iii) If  $\lambda_1 \subseteq \lambda_2$  then  $C_g(\lambda_1) \subseteq C_g(\lambda_2)$ ,
- (iv)  $C_g(\lambda_1 \cup \lambda_2) \subseteq C_g(\lambda_1) \cup C_g(\lambda_2)$ ,
- (v)  $C_g(C_g(\lambda_1)) = C_g(\lambda_1)$ .

**Proof:** Since 0 and 1 are fuzzy  $g$ -closed sets from let  $\{X, \tau_g\}$  be a generalized fuzzy topological space and let  $\lambda$  be a fuzzy set in  $X$ . then  $\lambda$  is fuzzy  $g$ -closed set if and only if  $C_g(\lambda) = \lambda$  we have  $C_g(0) = 0$  and  $C_g(1) = 1$ . Suppose  $\lambda_1 \subseteq \lambda_2$  in  $X$ , since  $\lambda_2 \subseteq C_g(\lambda_2)$  and  $\lambda_1 \subseteq \lambda_2$  we have  $\lambda_1 \subseteq C_g(\lambda_2)$ . Now  $C_g(\lambda_2)$  is a fuzzy  $g$ -closed set and  $C_g(\lambda_1) \subseteq C_g(\lambda_2)$  where  $C_g(\lambda_1)$  is the smallest fuzzy  $g$ -closed set containing  $\lambda_1$ . As  $\lambda_1 \subseteq \lambda_1 \cup \lambda_2$ ,  $\lambda_2 \subseteq \lambda_1 \cup \lambda_2$  we have  $C_g(\lambda_1) \subseteq C_g(\lambda_1 \cup \lambda_2)$  and  $C_g(\lambda_2) \subseteq C_g(\lambda_1 \cup \lambda_2)$  this implies  $C_g(\lambda_1) \cup C_g(\lambda_2) \subseteq C_g(\lambda_1 \cup \lambda_2)$ . Since  $C_g(\lambda_1)$  is a fuzzy  $g$ -closed set in  $X$ . if follow that  $C_g(C_g(\lambda_1)) = C_g(\lambda_1)$ .

**Proposition 3.4:** Let  $X$  be a generalized topological space and  $\{\lambda_j\}_{j \in J}$  be a family of subsets of  $X$ . Then

- (i)  $\bigcup_{j \in J} C_g(\lambda_j) \subseteq C_g(\bigcup_{j \in J} \lambda_j)$ ,
- (ii)  $C_g(\bigcap_{j \in J} \lambda_j) \subseteq C_g(\bigcap_{j \in J} C_g(\lambda_j))$ .

**Proof:** Quite easy

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