

On a Tensor Quantity Involved in Harish-Chandra Formula for the Product of Dirac-Matrices

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Abstract: In his paper¹ Harish-Chandra gave the beautiful formula for the product of Dirac-matrices. His formula involves an anti-symmetric tensor quantity $\mathcal{E}_{\lambda\mu\nu\rho\sigma\tau}$ in which each index takes the values from 0 to 5. Each non-zero component of this tensor depends on the permutation of the set $\{0, 1, 2, 3, 4, 5\}$. To understand the analysis given in Harish-Chandra¹ it is essential to have an idea of such components of this tensor. For this purpose in this paper we have given a computer programme in 'C' language which first obtains all the permutations of the set $\{0, 1, 2, 3, 4, 5\}$, then arranges them in their increasing order, decides each whether it is an even or odd permutation and then prints the non-zero components of $\mathcal{E}_{\lambda\mu\nu\rho\sigma\tau}$ in the increasing order of their arguments. The output of the programme is also given.

Keywords: Dirac-matrices, Anti-symmetric tensor $\mathcal{E}_{\lambda\mu\nu\rho\sigma\tau}$, Components, Even and odd permutation, 'C' language program.

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1. Introduction

Harish-Chandra Formula¹ for the product of Dirac-matrices is given as-

$$(1.1) \quad E_{\lambda\mu} E_{\nu\rho} = \left\{ \begin{array}{l} -\delta_{\lambda\nu} \delta_{\mu\rho} + \delta_{\mu\nu} \delta_{\lambda\rho} + E_{\lambda\nu} \delta_{\mu\rho} - E_{\mu\nu} \delta_{\lambda\rho} \\ -E_{\lambda\rho} \delta_{\mu\nu} + E_{\mu\rho} \delta_{\lambda\nu} - \frac{i}{2} \mathcal{E}_{\lambda\mu\nu\rho\sigma\tau} E^{\sigma\tau} \end{array} \right\}.$$

Here $\delta_{\mu\nu}$ is a Kronecker delta function defined as

$$(1.2) \quad \delta_{\mu\nu} = \begin{cases} 1, \mu = \nu \\ 0, \mu \neq \nu \end{cases},$$

and $\varepsilon_{\lambda\mu\nu\rho\sigma\tau}$ is anti-symmetric tensor in all six indices, and all indices $\lambda, \mu, \nu, \rho, \sigma, \tau$ run from 0 to 5 and $\varepsilon_{012345} = 1$. Since $\varepsilon_{\lambda\mu\nu\rho\sigma\tau}$ is anti-symmetric tensor in all six indices so if we change the pair of indices in $\varepsilon_{\lambda\mu\nu\rho\sigma\tau}$ in an even or odd number of times the value of $\varepsilon_{\lambda\mu\nu\rho\sigma\tau}$ function is non-zero (+1 or -1 respectively) and also if two or more indices are same the value of $\varepsilon_{\lambda\mu\nu\rho\sigma\tau}$ function is zero.

Dirac-matrices γ_a ($a = 1, 2, 3, 4$) are the four matrices each of order four with complex entries. These matrices are characterized by the relation -

$$(1.3) \quad \gamma_a \gamma_b + \gamma_b \gamma_a = 2\delta_{ab}.$$

This relation shows that matrices anti-commute with each other and square of each is an identity matrix. These four matrices gives rise to a set of 16 quantities

$$\begin{aligned} &1; \{\text{Identity matrix}\} \\ &\gamma_a; \{\gamma_1, \gamma_2, \gamma_3, \gamma_4\} \\ &i\gamma_a \gamma_b, (a \neq b); \{i\gamma_1 \gamma_2, i\gamma_1 \gamma_3, i\gamma_1 \gamma_4, i\gamma_2 \gamma_3, i\gamma_2 \gamma_4, i\gamma_3 \gamma_4\} \\ &i\gamma_a \gamma_b \gamma_c, (a, b, c \text{ all different}); \{i\gamma_1 \gamma_2 \gamma_3, i\gamma_1 \gamma_2 \gamma_4, i\gamma_1 \gamma_3 \gamma_4, i\gamma_2 \gamma_3 \gamma_4\} \\ &\gamma_1 \gamma_2 \gamma_3 \gamma_4 \end{aligned}$$

This set of matrices is closed under multiplication if we consider two quantities, which differ by a numerical factor 1, -1, i or $-i$ as essentially the same. The advantage of multiplication by ' i ' is that the square of each matrix becomes unit matrix 1 and all matrices (excluding 1) become anti-commuting to each other.

In¹ the quantity E_a is obtained by multiplying i in γ_a .

So

$$(1.4) \quad E_a \equiv i \gamma_a.$$

So equation (1.3) becomes

$$(1.5) \quad E_a E_b + E_b E_a = -2\delta_{ab},$$

and E_5 is obtain as

$$E_5 = i\gamma_5 = i\gamma_1\gamma_2\gamma_3\gamma_4 .$$

So

$$(1.6) \quad E_5 = iE_1E_2E_3E_4 .$$

Hence

$$E_5^2 = -1 , E_a E_5 = -E_5 E_a \quad (a = 1, 2, 3, 4)$$

The matrices E_1, E_2, E_3, E_4, E_5 anti-commute with each other and squares equal to -1 .

In Harish-Chandra¹ also defined

$$E_{\lambda\mu} \equiv E_\lambda E_\mu \quad (\lambda, \mu = 1, 2, 3, 4, 5; \lambda \neq \mu)$$

$$E_{0\lambda} \equiv E_\lambda$$

$$E_{\lambda 0} \equiv -E_{0\lambda} = -E_\lambda$$

$$E_{\lambda\lambda} \equiv 0 \quad (\lambda = 0, 1, 2, 3, 4, 5).$$

So $E_{\lambda\mu}$ is product of two quantity E_λ, E_μ and $E_\lambda = i\gamma_\lambda, E_\mu = i\gamma_\mu$.

2. The Tensor Identities

In section 3 of Prof. Harish-Chandra¹ using the following tensor identities to calculate the matrix determinant of T.

$$(2.1) \quad \epsilon_{\alpha\beta\gamma\delta\lambda\rho} \epsilon^{\lambda\rho\alpha'\beta'\gamma'\delta'} t_{\alpha'\beta'} t_{\gamma'\delta'} = 16(t_{\alpha\beta} t_{\gamma\delta} - t_{\alpha\gamma} t_{\beta\delta} - t_{\alpha\delta} t_{\gamma\beta}).$$

In L.H.S. there are 4 free indices $\alpha, \beta, \gamma, \delta$ and 6 summation indices $\lambda, \rho, \alpha', \beta', \gamma', \delta'$. Each summation index run from 0 to 5. If we vary all summation index we get only 48 non-zero term of type $t_{\mu\nu} t_{\sigma\tau}$ in R.H.S. In this there are 16 positive terms of $t_{\alpha\beta} t_{\gamma\delta}$ as $(\alpha, \beta, \gamma, \delta)$ is even permutation $(\alpha, \beta, \gamma, \delta)$, 16 negative term of $t_{\alpha\gamma} t_{\beta\delta}$ as $(\alpha, \gamma, \beta, \delta)$ is odd permutation of $(\alpha, \beta, \gamma, \delta)$ and 16 negative term of $t_{\alpha\delta} t_{\gamma\beta}$ as $(\alpha, \delta, \gamma, \beta)$ is odd permutation of $(\alpha, \beta, \gamma, \delta)$. The positive or negative sign is choosen according as even or odd permutation of $(\alpha, \beta, \gamma, \delta)$.


```

for(l=0;l<6;l++)
{
  if(l!=i && l!=j && l!=k)
  {
    for(m=0;m<6;m++)
    {
      if(m!=i && m!=j && m!=k && m!=l)
      {
        for(n=0;n<6;n++)
        {
          if(n!=i && n!=j && n!=k && n!=l && n!=m)
          {
            if(i>j)
              a=1;
            else
              a=0;
            if(i>k)
              b=1;
            else
              b=0;
            if(i>l)
              c=1;
            else
              c=0;
            if(i>m)
              d=1;
            else
              d=0;
            if(i>n)
              e=1;
            else
              e=0;
            if(j>k)

```

```
f=1;
else
    f=0;
if(j>l)
    g=1;
else
    g=0;
if(j>m)
    h=1;
else
    h=0;
if(j>n)
    u=1;
else
    u=0;
if(k>l)
    v=1;
else
    v=0;
if(k>m)
    w=1;
else
    w=0;
if(k>n)
    x=1;
else
    x=0;
if(l>m)
    y=1;
else
    y=0;
if(l>n)
    z=1;
```

4. The Output of Programme

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Output of program page-1/6

```

e(012345) ==+1 e(012354) ==-1 e(012435) ==-1 e(012453) ==+1
e(012534) ==+1 e(012543) ==-1 e(013245) ==-1 e(013254) ==+1
e(013425) ==+1 e(013452) ==-1 e(013524) ==-1 e(013542) ==+1
e(014235) ==+1 e(014253) ==-1 e(014325) ==-1 e(014352) ==+1
e(014523) ==+1 e(014532) ==-1 e(015234) ==-1 e(015243) ==+1
e(015324) ==+1 e(015342) ==-1 e(015423) ==-1 e(015432) ==+1
e(021345) ==-1 e(021354) ==+1 e(021435) ==+1 e(021453) ==-1
e(021534) ==-1 e(021543) ==+1 e(023145) ==+1 e(023154) ==-1
e(023415) ==-1 e(023451) ==+1 e(023514) ==+1 e(023541) ==-1
e(024135) ==-1 e(024153) ==+1 e(024315) ==+1 e(024351) ==-1
e(024513) ==-1 e(024531) ==+1 e(025134) ==+1 e(025143) ==-1
e(025314) ==-1 e(025341) ==+1 e(025413) ==+1 e(025431) ==-1
e(031245) ==+1 e(031254) ==-1 e(031425) ==-1 e(031452) ==+1
e(031524) ==+1 e(031542) ==-1 e(032145) ==-1 e(032154) ==+1
e(032415) ==+1 e(032451) ==-1 e(032514) ==-1 e(032541) ==+1
e(034125) ==+1 e(034152) ==-1 e(034215) ==-1 e(034251) ==+1
e(034512) ==+1 e(034521) ==-1 e(035124) ==-1 e(035142) ==+1
e(035214) ==+1 e(035241) ==-1 e(035412) ==-1 e(035421) ==+1
e(041235) ==-1 e(041253) ==+1 e(041325) ==+1 e(041352) ==-1
e(041523) ==-1 e(041532) ==+1 e(042135) ==+1 e(042153) ==-1
e(042315) ==-1 e(042351) ==+1 e(042513) ==+1 e(042531) ==-1
e(043125) ==-1 e(043152) ==+1 e(043215) ==+1 e(043251) ==-1
e(043512) ==-1 e(043521) ==+1 e(045123) ==+1 e(045132) ==-1
e(045213) ==-1 e(045231) ==+1 e(045312) ==+1 e(045321) ==-1
e(051234) ==+1 e(051243) ==-1 e(051324) ==-1 e(051342) ==+1
e(051423) ==+1 e(051432) ==-1 e(052134) ==-1 e(052143) ==+1
e(052314) ==+1 e(052341) ==-1 e(052413) ==-1 e(052431) ==+1
e(053124) ==+1 e(053142) ==-1 e(053214) ==-1 e(053241) ==+1
e(053412) ==+1 e(053421) ==-1 e(054123) ==-1 e(054132) ==+1
e(054213) ==+1 e(054231) ==-1 e(054312) ==-1 e(054321) ==+1

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e(102345)=-1 e(102354)=+1 e(102435)=+1 e(102453)=-1
 e(102534)=-1 e(102543)=+1 e(103245)=+1 e(103254)=-1
 e(103425)=-1 e(103452)=+1 e(103524)=+1 e(103542)=-1
 e(104235)=-1 e(104253)=+1 e(104325)=+1 e(104352)=-1
 e(104523)=-1 e(104532)=+1 e(105234)=+1 e(105243)=-1
 e(105324)=-1 e(105342)=+1 e(105423)=+1 e(105432)=-1
 e(120345)=+1 e(120354)=-1 e(120435)=-1 e(120453)=+1
 e(120534)=+1 e(120543)=-1 e(123045)=-1 e(123054)=+1
 e(123405)=+1 e(123450)=-1 e(123504)=-1 e(123540)=+1
 e(124035)=+1 e(124053)=-1 e(124305)=-1 e(124350)=+1
 e(124503)=+1 e(124530)=-1 e(125034)=-1 e(125043)=+1
 e(125304)=+1 e(125340)=-1 e(125403)=-1 e(125430)=+1
 e(130245)=-1 e(130254)=+1 e(130425)=+1 e(130452)=-1
 e(130524)=-1 e(130542)=+1 e(132045)=+1 e(132054)=-1
 e(132405)=-1 e(132450)=+1 e(132504)=+1 e(132540)=-1
 e(134025)=-1 e(134052)=+1 e(134205)=+1 e(134250)=-1
 e(134502)=-1 e(134520)=+1 e(135024)=+1 e(135042)=-1
 e(135204)=-1 e(135240)=+1 e(135402)=+1 e(135420)=-1
 e(140235)=+1 e(140253)=-1 e(140325)=-1 e(140352)=+1
 e(140523)=+1 e(140532)=-1 e(142035)=-1 e(142053)=+1
 e(142305)=+1 e(142350)=-1 e(142503)=-1 e(142530)=+1
 e(143025)=+1 e(143052)=-1 e(143205)=-1 e(143250)=+1
 e(143502)=+1 e(143520)=-1 e(145023)=-1 e(145032)=+1
 e(145203)=+1 e(145230)=-1 e(145302)=-1 e(145320)=+1
 e(150234)=-1 e(150243)=+1 e(150324)=+1 e(150342)=-1
 e(150423)=-1 e(150432)=+1 e(152034)=+1 e(152043)=-1
 e(152304)=-1 e(152340)=+1 e(152403)=+1 e(152430)=-1
 e(153024)=-1 e(153042)=+1 e(153204)=+1 e(153240)=-1
 e(153402)=-1 e(153420)=+1 e(154023)=+1 e(154032)=-1
 e(154203)=-1 e(154230)=+1 e(154302)=+1 e(154320)=-1

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e(201345) ==+1 e(201354) ==-1 e(201435) ==-1 e(201453) ==+1
e(201534) ==+1 e(201543) ==-1 e(203145) ==-1 e(203154) ==+1
e(203415) ==+1 e(203451) ==-1 e(203514) ==-1 e(203541) ==+1
e(204135) ==+1 e(204153) ==-1 e(204315) ==-1 e(204351) ==+1
e(204513) ==+1 e(204531) ==-1 e(205134) ==-1 e(205143) ==+1
e(205314) ==+1 e(205341) ==-1 e(205413) ==-1 e(205431) ==+1
e(210345) ==-1 e(210354) ==+1 e(210435) ==+1 e(210453) ==-1
e(210534) ==-1 e(210543) ==+1 e(213045) ==+1 e(213054) ==-1
e(213405) ==-1 e(213450) ==+1 e(213504) ==+1 e(213540) ==-1
e(214035) ==-1 e(214053) ==+1 e(214305) ==+1 e(214350) ==-1
e(214503) ==-1 e(214530) ==+1 e(215034) ==+1 e(215043) ==-1
e(215304) ==-1 e(215340) ==+1 e(215403) ==+1 e(215430) ==-1
e(230145) ==+1 e(230154) ==-1 e(230415) ==-1 e(230451) ==+1
e(230514) ==+1 e(230541) ==-1 e(231045) ==-1 e(231054) ==+1
e(231405) ==+1 e(231450) ==-1 e(231504) ==-1 e(231540) ==+1
e(234015) ==+1 e(234051) ==-1 e(234105) ==-1 e(234150) ==+1
e(234501) ==+1 e(234510) ==-1 e(235014) ==-1 e(235041) ==+1
e(235104) ==+1 e(235140) ==-1 e(235401) ==-1 e(235410) ==+1
e(240135) ==-1 e(240153) ==+1 e(240315) ==+1 e(240351) ==-1
e(240513) ==-1 e(240531) ==+1 e(241035) ==+1 e(241053) ==-1
e(241305) ==-1 e(241350) ==+1 e(241503) ==+1 e(241530) ==-1
e(243015) ==-1 e(243051) ==+1 e(243105) ==+1 e(243150) ==-1
e(243501) ==-1 e(243510) ==+1 e(245013) ==+1 e(245031) ==-1
e(245103) ==-1 e(245130) ==+1 e(245301) ==+1 e(245310) ==-1
e(250134) ==+1 e(250143) ==-1 e(250314) ==-1 e(250341) ==+1
e(250413) ==+1 e(250431) ==-1 e(251034) ==-1 e(251043) ==+1
e(251304) ==+1 e(251340) ==-1 e(251403) ==-1 e(251430) ==+1
e(253014) ==+1 e(253041) ==-1 e(253104) ==-1 e(253140) ==+1
e(253401) ==+1 e(253410) ==-1 e(254013) ==-1 e(254031) ==+1
e(254103) ==+1 e(254130) ==-1 e(254301) ==-1 e(254310) ==+1

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Output of program page-4/6

e(301245)=-1 e(301254)=+1 e(301425)=+1 e(301452)=-1
 e(301524)=-1 e(301542)=+1 e(302145)=+1 e(302154)=-1
 e(302415)=-1 e(302451)=+1 e(302514)=+1 e(302541)=-1
 e(304125)=-1 e(304152)=+1 e(304215)=+1 e(304251)=-1
 e(304512)=-1 e(304521)=+1 e(305124)=+1 e(305142)=-1
 e(305214)=-1 e(305241)=+1 e(305412)=+1 e(305421)=-1
 e(310245)=+1 e(310254)=-1 e(310425)=-1 e(310452)=+1
 e(310524)=+1 e(310542)=-1 e(312045)=-1 e(312054)=+1
 e(312405)=+1 e(312450)=-1 e(312504)=-1 e(312540)=+1
 e(314025)=+1 e(314052)=-1 e(314205)=-1 e(314250)=+1
 e(314502)=+1 e(314520)=-1 e(315024)=-1 e(315042)=+1
 e(315204)=+1 e(315240)=-1 e(315402)=-1 e(315420)=+1
 e(320145)=-1 e(320154)=+1 e(320415)=+1 e(320451)=-1
 e(320514)=-1 e(320541)=+1 e(321045)=+1 e(321054)=-1
 e(321405)=-1 e(321450)=+1 e(321504)=+1 e(321540)=-1
 e(324015)=-1 e(324051)=+1 e(324105)=+1 e(324150)=-1
 e(324501)=-1 e(324510)=+1 e(325014)=+1 e(325041)=-1
 e(325104)=-1 e(325140)=+1 e(325401)=+1 e(325410)=-1
 e(340125)=+1 e(340152)=-1 e(340215)=-1 e(340251)=+1
 e(340512)=+1 e(340521)=-1 e(341025)=-1 e(341052)=+1
 e(341205)=+1 e(341250)=-1 e(341502)=-1 e(341520)=+1
 e(342015)=+1 e(342051)=-1 e(342105)=-1 e(342150)=+1
 e(342501)=+1 e(342510)=-1 e(345012)=-1 e(345021)=+1
 e(345102)=+1 e(345120)=-1 e(345201)=-1 e(345210)=+1
 e(350124)=-1 e(350142)=+1 e(350214)=+1 e(350241)=-1
 e(350412)=-1 e(350421)=+1 e(351024)=+1 e(351042)=-1
 e(351204)=-1 e(351240)=+1 e(351402)=+1 e(351420)=-1
 e(352014)=-1 e(352041)=+1 e(352104)=+1 e(352140)=-1
 e(352401)=-1 e(352410)=+1 e(354012)=+1 e(354021)=-1
 e(354102)=-1 e(354120)=+1 e(354201)=+1 e(354210)=-1

Output of program page-5/6

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e(401235) ==+1 e(401253) ==-1 e(401325) ==-1 e(401352) ==+1
e(401523) ==+1 e(401532) ==-1 e(402135) ==-1 e(402153) ==+1
e(402315) ==+1 e(402351) ==-1 e(402513) ==-1 e(402531) ==+1
e(403125) ==+1 e(403152) ==-1 e(403215) ==-1 e(403251) ==+1
e(403512) ==+1 e(403521) ==-1 e(405123) ==-1 e(405132) ==+1
e(405213) ==+1 e(405231) ==-1 e(405312) ==-1 e(405321) ==+1
e(410235) ==-1 e(410253) ==+1 e(410325) ==+1 e(410352) ==-1
e(410523) ==-1 e(410532) ==+1 e(412035) ==+1 e(412053) ==-1
e(412305) ==-1 e(412350) ==+1 e(412503) ==+1 e(412530) ==-1
e(413025) ==-1 e(413052) ==+1 e(413205) ==+1 e(413250) ==-1
e(413502) ==-1 e(413520) ==+1 e(415023) ==+1 e(415032) ==-1
e(415203) ==-1 e(415230) ==+1 e(415302) ==+1 e(415320) ==-1
e(420135) ==+1 e(420153) ==-1 e(420315) ==-1 e(420351) ==+1
e(420513) ==+1 e(420531) ==-1 e(421035) ==-1 e(421053) ==+1
e(421305) ==+1 e(421350) ==-1 e(421503) ==-1 e(421530) ==+1
e(423015) ==+1 e(423051) ==-1 e(423105) ==-1 e(423150) ==+1
e(423501) ==+1 e(423510) ==-1 e(425013) ==-1 e(425031) ==+1
e(425103) ==+1 e(425130) ==-1 e(425301) ==-1 e(425310) ==+1
e(430125) ==-1 e(430152) ==+1 e(430215) ==+1 e(430251) ==-1
e(430512) ==-1 e(430521) ==+1 e(431025) ==+1 e(431052) ==-1
e(431205) ==-1 e(431250) ==+1 e(431502) ==+1 e(431520) ==-1
e(432015) ==-1 e(432051) ==+1 e(432105) ==+1 e(432150) ==-1
e(432501) ==-1 e(432510) ==+1 e(435012) ==+1 e(435021) ==-1
e(435102) ==-1 e(435120) ==+1 e(435201) ==+1 e(435210) ==-1
e(450123) ==+1 e(450132) ==-1 e(450213) ==-1 e(450231) ==+1
e(450312) ==+1 e(450321) ==-1 e(451023) ==-1 e(451032) ==+1
e(451203) ==+1 e(451230) ==-1 e(451302) ==-1 e(451320) ==+1
e(452013) ==+1 e(452031) ==-1 e(452103) ==-1 e(452130) ==+1
e(452301) ==+1 e(452310) ==-1 e(453012) ==-1 e(453021) ==+1
e(453102) ==+1 e(453120) ==-1 e(453201) ==-1 e(453210) ==+1

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Output of program page-6/6

$e(501234) = -1$ $e(501243) = +1$ $e(501324) = +1$ $e(501342) = -1$
 $e(501423) = -1$ $e(501432) = +1$ $e(502134) = +1$ $e(502143) = -1$
 $e(502314) = -1$ $e(502341) = +1$ $e(502413) = +1$ $e(502431) = -1$
 $e(503124) = -1$ $e(503142) = +1$ $e(503214) = +1$ $e(503241) = -1$
 $e(503412) = -1$ $e(503421) = +1$ $e(504123) = +1$ $e(504132) = -1$
 $e(504213) = -1$ $e(504231) = +1$ $e(504312) = +1$ $e(504321) = -1$
 $e(510234) = +1$ $e(510243) = -1$ $e(510324) = -1$ $e(510342) = +1$
 $e(510423) = +1$ $e(510432) = -1$ $e(512034) = -1$ $e(512043) = +1$
 $e(512304) = +1$ $e(512340) = -1$ $e(512403) = -1$ $e(512430) = +1$
 $e(513024) = +1$ $e(513042) = -1$ $e(513204) = -1$ $e(513240) = +1$
 $e(513402) = +1$ $e(513420) = -1$ $e(514023) = -1$ $e(514032) = +1$
 $e(514203) = +1$ $e(514230) = -1$ $e(514302) = -1$ $e(514320) = +1$
 $e(520134) = -1$ $e(520143) = +1$ $e(520314) = +1$ $e(520341) = -1$
 $e(520413) = -1$ $e(520431) = +1$ $e(521034) = +1$ $e(521043) = -1$
 $e(521304) = -1$ $e(521340) = +1$ $e(521403) = +1$ $e(521430) = -1$
 $e(523014) = -1$ $e(523041) = +1$ $e(523104) = +1$ $e(523140) = -1$
 $e(523401) = -1$ $e(523410) = +1$ $e(524013) = +1$ $e(524031) = -1$
 $e(524103) = -1$ $e(524130) = +1$ $e(524301) = +1$ $e(524310) = -1$
 $e(530124) = +1$ $e(530142) = -1$ $e(530214) = -1$ $e(530241) = +1$
 $e(530412) = +1$ $e(530421) = -1$ $e(531024) = -1$ $e(531042) = +1$
 $e(531204) = +1$ $e(531240) = -1$ $e(531402) = -1$ $e(531420) = +1$
 $e(532014) = +1$ $e(532041) = -1$ $e(532104) = -1$ $e(532140) = +1$
 $e(532401) = +1$ $e(532410) = -1$ $e(534012) = -1$ $e(534021) = +1$
 $e(534102) = +1$ $e(534120) = -1$ $e(534201) = -1$ $e(534210) = +1$
 $e(540123) = -1$ $e(540132) = +1$ $e(540213) = +1$ $e(540231) = -1$
 $e(540312) = -1$ $e(540321) = +1$ $e(541023) = +1$ $e(541032) = -1$
 $e(541203) = -1$ $e(541230) = +1$ $e(541302) = +1$ $e(541320) = -1$
 $e(542013) = -1$ $e(542031) = +1$ $e(542103) = +1$ $e(542130) = -1$
 $e(542301) = -1$ $e(542310) = +1$ $e(543012) = +1$ $e(543021) = -1$
 $e(543102) = -1$ $e(543120) = +1$ $e(543201) = +1$ $e(543210) = -1$

5. Conclusion

The values obtained in this paper are useful in understanding the analysis¹ and verifying the tensor identities (2.1), (2.2) and others given¹.

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