

Chattering Reduction in Variable Structure Controlled DC Motor

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Abstract: Variable structure control (VSC) is a controlling method in which the structure of feedback is altered to get desired output for DC motor. However, the technique has not had wide spread acceptance due to some limitations in form of chattering which is ever-existing in any VSC system. The chattering problem is much more serious in low moment of inertia systems because of finite amount of discontinuous surface. The DC motors have not only low inertia but also small time constants. The chattering is main problem for such type motor. To reduce this problem, the concept of switching lines is used. The basic objective of using single line switching technique is to improve the responses of the system by combining the features of under damped and over damped response. The biggest advantage of this technique is that it results in invariant systems and makes the system total robust.

Keyword: Variable structure control (VSC), Chattering, Representing point (RP), Single line switching technique (SLST), Sliding mode control (SLMC).

1. Introduction

Variable structure control consists of a set of continuous sub-systems together with the suitable switching logics¹⁻³. Advantageous property of such system is that the output of the system can be changed according to switching logics. It means plant remains same but the characteristics of system are changed. For this, the gains and other parameters are changed to alter the characteristics of system or a program of a model is divided into

switching as positive or negative according to required condition, then change in subprogram is called VSC.

By changing the structure of this system one can resolve the conflict between static accuracy and speed of response. The motion of the representing point (RP) along a line (switching line), which is not a trajectory of any of these structures, is sliding mode. Variable structure system operating in sliding mode has much advantages⁴⁻⁶. The motion in sliding mode is insensitive to variation in the plant parameters and external disturbances. However, these invariance properties are exhibited only during the so-called sliding mode. The control laws are so designed that the system trajectory always reaches the sliding mode.

In the design of VSC system we assume that the control can be switched from one value to another and will be infinitely fast. Practically it is impossible to achieve high switching control due to presence of finite time delays for control computations, limitations of physical actuator, switching delays due to non-idealities such as hysteresis, and neglected time constants etc. So system trajectory oscillates about the switching line. These high frequency oscillations about the switching line in sliding mode and about the desired equilibrium point in the steady state mode of VSC system is termed as chattering⁷. In many VSC systems chattering causes harmful effects, such as torque pulsation, unmodelled undesired high frequency dynamics and acoustic noise. Such problem is much more serious in low moment of inertia system because of finite amount of discontinuous surface, and thus helps to smoothen the movement of the RP around the switching line. The chattering affects the efficiency of system (DC motor). It is essential to minimize the chattering problem in VSC controlled DC motor. For reducing the chattering, a chattering reduction technique is used that is single line switching technique.

Nomenclature: U : Control input, K_a : Power amplifier gain, r : Reference input, X_1 : Angular position measured by transducer, K_1 : Position switching gain, K_p : Position constant, K_2 : Velocity switching gain, K_v : Velocity constant, θ : Angular position of output shaft, E_a : voltage input to the motor, s : Laplace variable, K_m : Motor gain constant, T_m : Motor time constant, X_2 : Angular velocity measured by transducer, α : Position transducer gain, β : Velocity transducer gain, w : Angular velocity, S : Switching variable, t_h : Hitting time, C : Slope of the switching line, t : Time,

2. Variable Structure Control

The block diagram of VSC in sliding mode for the position controlled DC motor is shown in Fig.1, which is based on the control laws⁴⁻⁶. The output from position tracking is fed back to reference input. The output of summation is given to variable structure control. The control input U after variable structure control is fed to DC motor can be written as:

$$(2.1) \quad U = K_a \left[(r - X_1) K_1 K_p + \frac{d}{dt} (r - X_1) K_2 K_v \right]$$

The DC motor dynamics can be written as⁸

$$\frac{\theta(s)}{E_a(s)} = \frac{K_m}{s(1 + sT_m)}$$

$$s^2 T_m \theta(s) + s \theta(s) = K_m E_a(s)$$

Taking inverse Laplace

$$(2.2) \quad \ddot{\theta} T_m + \dot{\theta} = K_m E_a$$

To convert this equation into State Space, we get

$$\theta = X_1$$

$$\dot{\theta} = X_2 = \dot{X}_1$$

$$\ddot{\theta} = \dot{X}_2$$

Output of transducers are used for position and velocity measurement is

$$X_1 = \alpha \theta, \quad X_2 = \beta \dot{\theta}$$

$$\theta = \frac{X_1}{\alpha} \quad \text{and}$$

$$(2.3) \quad \dot{\theta} = \frac{X_2}{\beta} = \frac{\dot{X}_1}{\alpha}$$

From equation (2.2) and (2.3), we get

$$T_m \left(\frac{\dot{X}_2}{\beta} \right) + \left(\frac{X_2}{\beta} \right) = K_m U$$

Where $U (= E_a)$ is control input voltage to the motor.

Since $X_2 = \beta \dot{X}_1 / \alpha$, motor dynamics can be written as

3. Single Line Switching Technique

The concept of this technique improves the response and reducing chattering of the system by combining the features of under damped and over damped response⁷. The system also becomes sensitive for parameter variation. This concept completely eliminates the chattering and gives a very good response. The concept of such technique is explained on the basis of response characteristics of the system in terms of eigen values. These values are the function of gain values. The initial gains are decided by a reachability criterion and are generally taken very large to have complex poles that would minimize the hitting time. Soon after hitting the switching line, the system is switched to a new structure, where one pole is placed at a very large negative value so that its contribution in response is almost negligible, and the other pole is placed at $-c$ (or $-C$). Thus after hitting the switching line, the system order is reduced to first order, but with only one switching around the switching line. Since the inertia of the DC motor is small, it can follow changes instantaneously without any delay in torque production. This technique can be explained mathematically using the solution of equation (2.6), which is given by

$$(3.1) \quad X_1(t) = A_1 e^{(-b_1)t} + A_2 e^{(-b_2)t},$$

where $-b_1$ and $-b_2$ are two poles of equation, which are called eigen values. $-b_1$ & $-b_2$ are made conjugate to ensure lesser reaching time. On hitting the switching line, the two poles ($-b_1$ & $-b_2$) are placed at $-C$ and $-\infty$. The above equation becomes

$$(3.2) \quad \begin{aligned} X_1(t) &= B_1 e^{-c(t-t_h)} + B_2 e^{-\infty(t-t_h)} \\ X_1(t) &= B_1 e^{-c(t-t_h)}. \end{aligned}$$

Here B_1 depends on initial condition when the poles changed at $t = t_h$

$$X_1(t_h) = B_1,$$

Putting the values of B_1 in equation (3.2), we obtain

$$X_1(t) = X_1(t_h) e^{-c(t-t_h)},$$

Initially $t_h = 0$, so that

$$(3.3) \quad X_1(t) = X_1(0) e^{-ct}.$$

which is the same as the time domain solution of RP moving ideally on the switching line

$$S = CX_1 + X_2 = 0.$$

This concept is mainly a pole placement idea. After hitting the switching line, the system response is made to be directed by only one mode ($e^{\lambda t}$), while other mode is suppressed. Theoretically, the second pole is required to be placed at infinity (∞) soon after the RP hits the sliding line. But the second pole is placed at a distance 10 times C , then the characteristic equation becomes

$$(3.4) \quad s^2 + 11Cs + 10C^2 = 0.$$

The inverse laplace of equation (3.4) is written as

$$(3.5) \quad \ddot{X}_1 + 11C\dot{X}_1 + 10C^2 X_1 = 0.$$

Comparing the coefficient of equation (3.4) and (3.5), we get

$$10C^2 = 2242K_1,$$

$$(3.6) \quad K_1 = \frac{10C^2}{2242},$$

$$11C = 25 + 22.42K_2,$$

$$(3.7) \quad K_2 = \frac{11C - 25}{22.42}.$$

Simulation program consists of the solution of two equations; one before switching or above the switching line ($K_1 = +1$, $K_2 = -1$) and the other after switching or below the switching line ($K_1 = -1$, $K_2 = +1$).

4. Results and Analysis

4.1. Conventional VSC Simulation

In the simulation of conventional VSC, whatever combinations of slopes and gains are chosen, considerable amount of chattering still persists in simulation as shown in Figs. 2-5. Because of unreliability of infinitely fast switching, system trajectory oscillates about the switching line. These oscillations are termed as chattering.

The responses of Figs. 2-5 of system at different parameters are summarized as below:

The presence of chattering can be seen in response curves which exist because the RP cannot switch about the sliding line at infinite frequency. Larger the switching line slope C , the response is faster in sliding mode. But C cannot be increased as desired because of response capability of physical system. Smaller value of C result in small hitting time (t_h) and faster response in reaching mode but slower response in sliding mode. Therefore, rise time and setting time increases.

Increase in gains (k_p and k_v) implies considerable decrease in hitting time (t_h) and makes the overall response faster. But increase in gains, proportionately increase the control input required and may exceed the rated value. For the same gain values, chattering is more at lower value of C . This is so because at low values of slope, the response time in the sliding mode is more and control input required is much less. In this an optimum value of gain can be chosen that would meet the existence condition and also minimize chattering for a definite value of system parameters. However, the chattering presents in the system. Even with 100% variation on either side, in the parameters of the system, there is no significant change in the response on the sliding line. Response on hitting zone however, remains sensitive to the parameter variations.

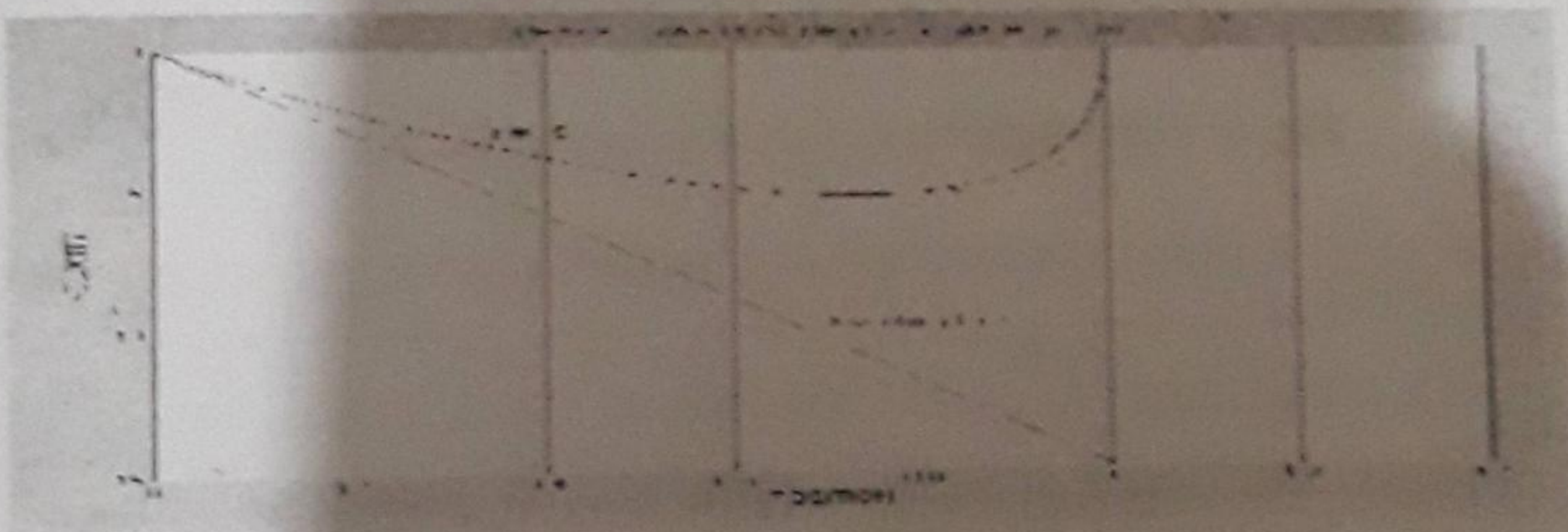


Fig. 2: Phase trajectory of conventional VSC at $C = 15$

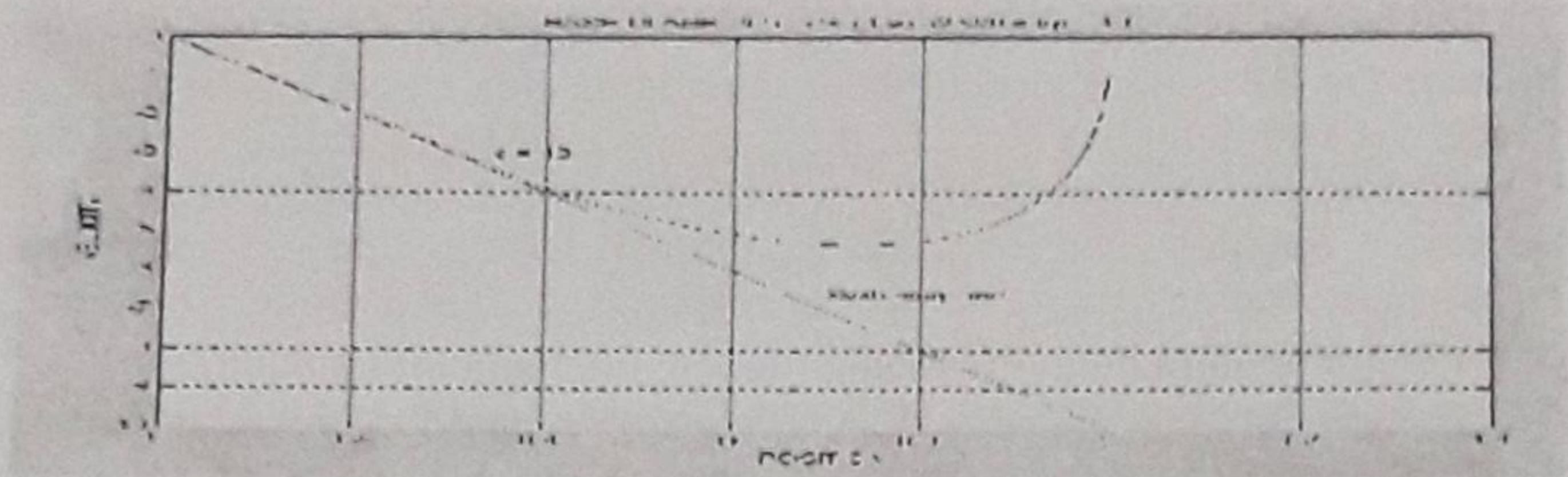


Fig. 3: Phase trajectory of conventional VSC at $C = 10$

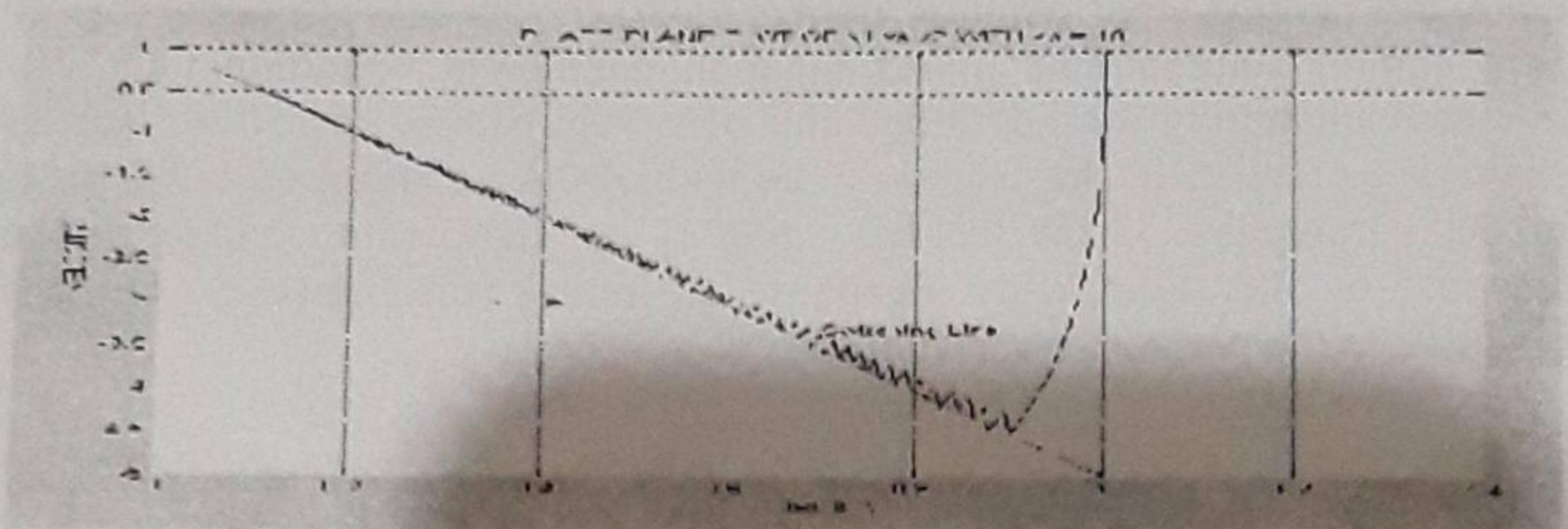


Fig. 4: Phase trajectory of conventional VSC at $C = 05$

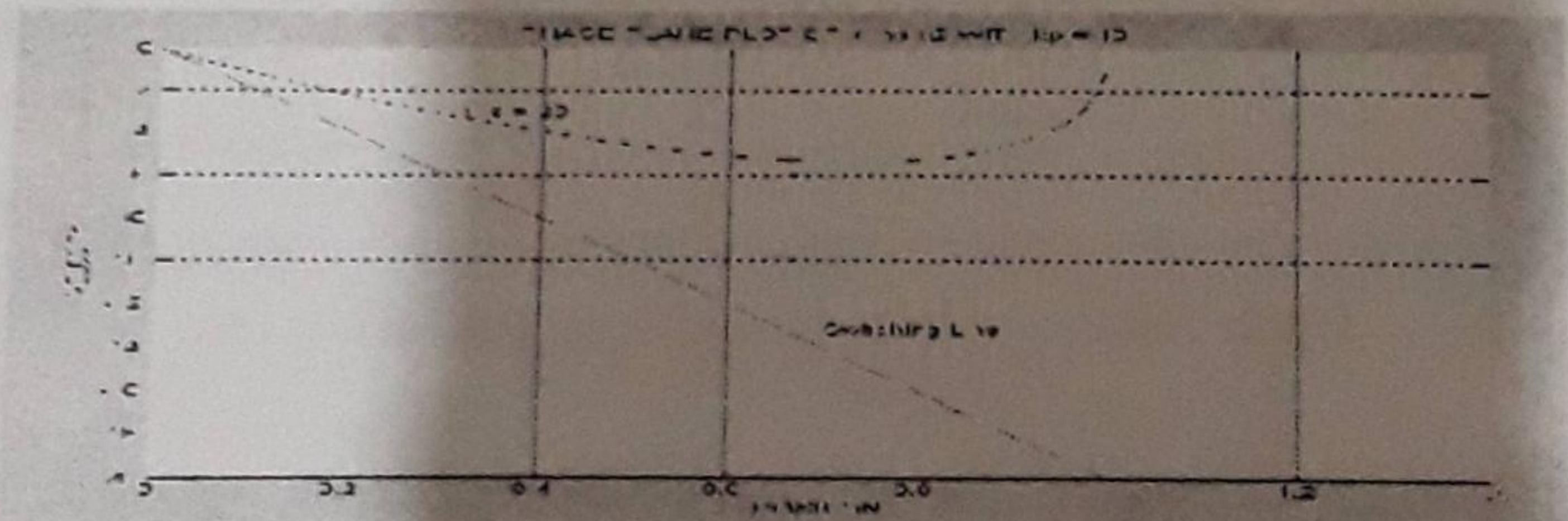


Fig. 5: Phase trajectory of conventional VSC at $C = 20$

4.2. Single Line Switching Simulation

In the simulation of VSC based DC motor using single line switching technique, chattering eliminates completely as shown in following simulation results (Figs. 6-9).

Simulation Program consists of the solution of two equations: One before switching and the other after switching. Various observations made from the response characteristics (Figs. 6-9), can be summarized as:

The single switching makes the system to be highly sensitive to parameter variations and external disturbances. As C decreases hitting time decreases, settling time increases and the system response becomes slow.

Since $K_2 \gg K_1$ in this concept, there is considerable swing in velocity. Reduction in C however, helps in reducing velocity swing because $K_2 = (11C - 25) / 22.42$. So that at lower value of C , response is slow but chattering is completely eliminated as shown in Figs. 6 & 9. Larger the value of C , response is faster. The value of K_2 increases means velocity swing increases as shown in Figs. 7 & 8, but the value of C cannot be increase as desired because of response.

Advantages of this technique are Chattering is eliminated, necessity of fast switching devices is eliminated since only one switching is required, very fast measuring and computing devices are not necessary, position and velocity need not be monitored after switching, response of the system is faster since only one switching is involved.

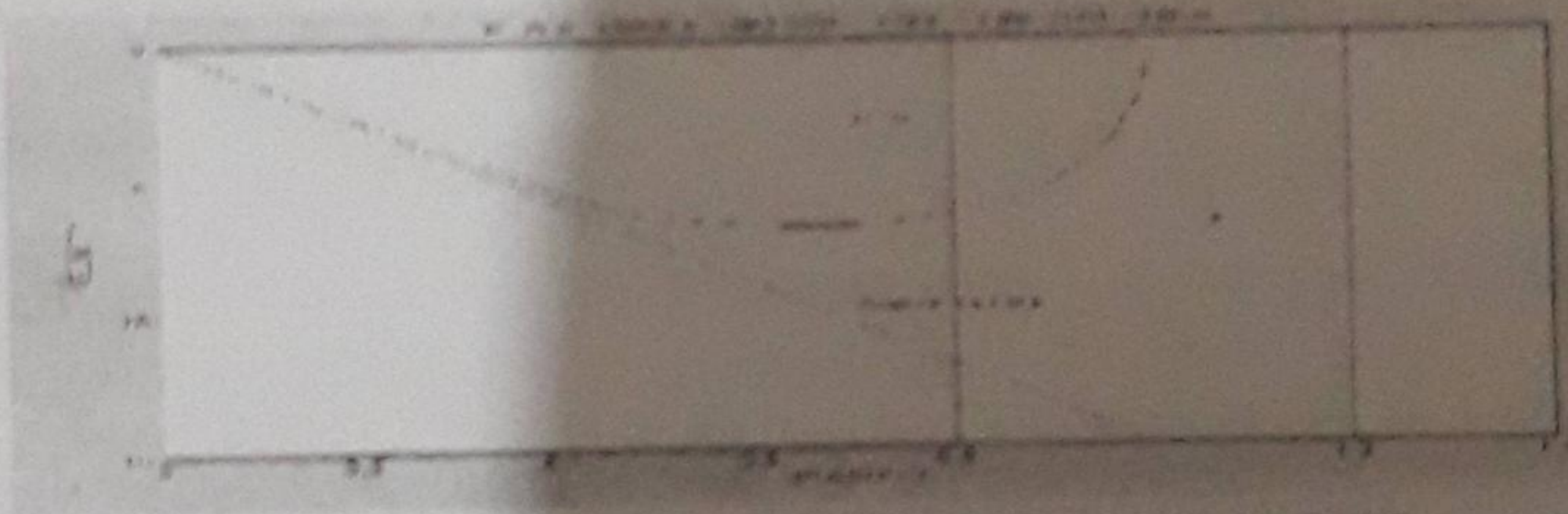


Fig. 6: Phase trajectory of single line switching at $C=15$

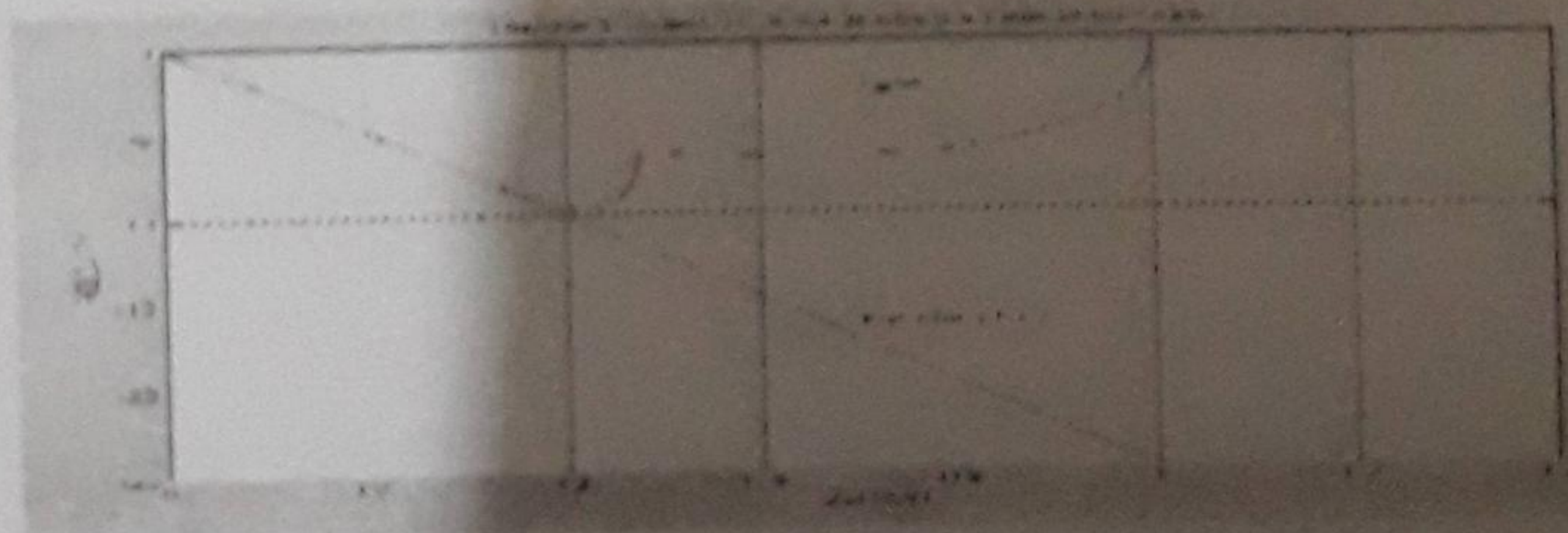


Fig. 7: Phase trajectory of single line switching at $C=25$

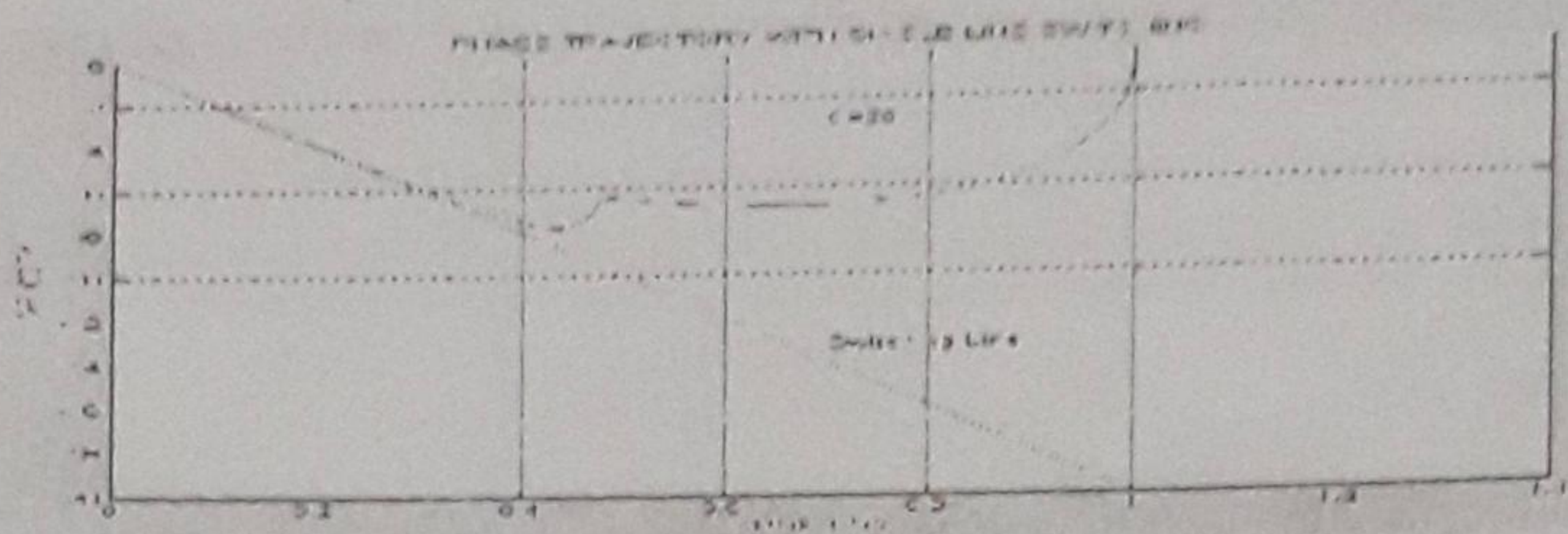


Fig. 8: Phase trajectory of single line switching at $C=20$

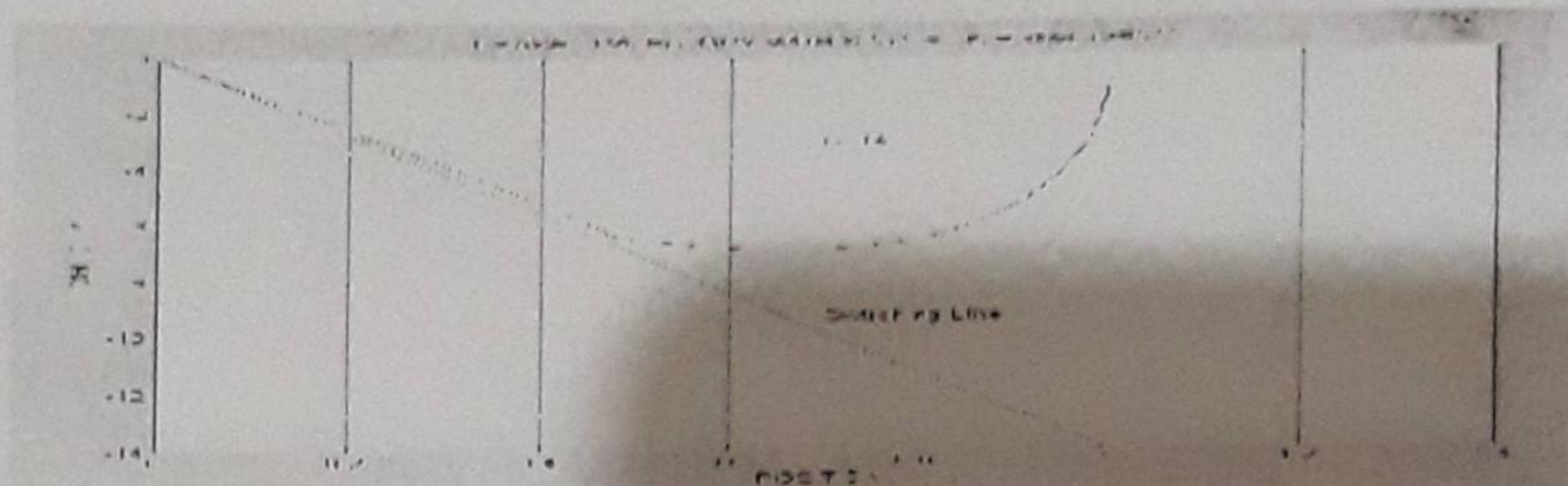


Fig. 9: Phase trajectory of single line switching at $C=14$

4.3. Comparisons of Conventional VSC and SLST

At lower value of C the chattering easily seen in the graphs of simulation of conventional VSC Figs. 3 & 4 at $C = 05$ & $C = 10$ respectively. As C increases the chattering is eliminated but the response is so faster that it is not moving on switching line as shown in Figs. 2 & 5 at $C = 15$ & $C = 20$ respectively. In the simulation SLST, at lower value of C the chattering is completely eliminated as shown in Figs. 6 & 9 at $C = 15$ & $C = 14$ respectively and response moving on the switching line. If the value of C increases the velocity swing increases as shown in Figs. 7 & 8 at $C = 25$ & $C = 20$ respectively due to response capability of physical system.

5. Conclusion

The chattering can be reduced using Single line switching techniques in the present paper. The concept of single line switching improves the responses of the system by combining the features of under damped and

over damped responses. The advantages of this technique are that small rise time and good stability characteristics; which reduces dead time of the system. In this technique the chattering is completely eliminated. But it has some limitations that it is applicable only in low moment of inertia system, which has negligible mechanical time constant and smooth operation in the entire range; the system to be highly sensitive to parameter variations and external disturbances.

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