

Analysis of Two Anti Plane Shear Collinear Cracks in a Porous Piezoelectric Layer Bonded between Two Elastic Half-Spaces

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Abstract: In this paper, an analysis of two collinear anti-plane shear cracks in a porous piezoelectric layer bonded with two similar elastic half-spaces is done. The impermeable crack face conditions are considered. Using Fourier transform, the problem is reduced to three pairs of integral equations. The displacement and electric potentials are expanded as a series of Jacobi polynomials and then the integral equations are solved by making use of Schmidt's method. The analytic expressions, for the intensity factors at the right and left end of the crack, are obtained. A particular case for the piezoelectric layer is also discussed in which the expressions for the intensity factors for the cracks in a piezoelectric layer are derived. Numerical calculations are made to show the effects of geometry and position of cracks on the stress intensity factors and electric displacement intensity factors.

Keywords: Crack; Porous Piezoelectric materials; Shear; Integral equations; Intensity factor.

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1. Introduction

Fracture mechanics of materials has been attracting the attention of researchers in the areas of mechanics of solids, applied mechanics, structural engineering, material science and earth science. In the case of piezoelectric ceramics materials, the initiation and propagation of cracks may be resulted due to mechanical and electrical loads. To check the failure during service and to obtain the reliable service life time of piezoelectric components, the crack analysis of piezoelectric ceramics becomes very important.

The problems of cracks in piezoelectric materials have been studied intensively for the electrically permeable cracks¹⁻² and electrically impermeable cracks³⁻⁴. Narita and Shindo⁵ carried out an analysis of scattering of anti-plane shear waves by a finite crack in piezoelectric laminates. Zhou et al.⁶ made an analysis of the behaviour of two collinear cracks in a piezoelectric layer bonded to two half spaces by Schmidt method for impermeable crack face condition. Using singular integral equation technique, Wang and Mai⁷ discussed the effect of electrical boundary conditions on mechanical and electric field intensity factors for a piezoelectric layer bonded by two elastic layers and Wang et al.⁸ extended this phenomenon to an anti-plane crack problem in a functionally graded piezoelectric materials strip. The dynamic behaviour of two collinear anti-plane shear cracks in a piezoelectric layer was investigated by Zhou and Wang⁹. To et al.¹⁰ discussed propagation of a mode-III interfacial conductive anti-plane crack along a conductive interface between two dissimilar piezoelectric half-spaces. In the above work for the piezoelectric material, out-of-plane mechanical displacement and in-plane electric fields were decoupled. Li and Chen¹¹ performed numerical analysis of interfacial crack tip singularities and crack tip energy release rates for a permeable crack in elastic dielectric/piezoelectric bi-materials. The problem of multiple cracks on the interface between a piezoelectric layer and an orthotropic substrate was solved by Li and Lee¹² with the help of method of dislocation simulation and singular integral equation. Shin and Lee¹³ analyzed the dynamic propagation of a crack in a functionally graded piezoelectric material layer between two dissimilar piezoelectric layers under anti-plane shear by making use of integral transform approaches. Choi and Shin¹⁴ analyzed three collinear impermeable interfacial cracks in bonded dissimilar transversely isotropic piezoelectric materials under electromechanical loadings. Hassan and Song¹⁵ analyzed the influence on the intensity factors in transversely isotropic piezoelectric bi-materials with two symmetrically interfacial cracks near the edges of a circular cavity, subjected to the dynamic incident

anti-plane SH-wave. Govorukha et al.¹⁶ provides a comprehensive survey of works on cracks situated at the interface of two materials, at least one of which has piezoelectric or piezo-electro-magnetic properties. Using an extended finite element method, a static impermeable crack, at interface of piezoelectric materials, was studied by R. R. Bhargava and P.R. Verma¹⁷.

In this paper, a study of behavior of anti-plane shear cracks, in a layer of porous piezoelectric material bonded to similar elastic half spaces, is undertaken. Using integral transform technique, the problem is transformed into sytem of integral equations. The displacement and the electric potential are expanded in the series of Jacobi polynomials and thereafter Schmidt's method is used to find stress intensity factors. Numerical examples are taken to study the effect of cracks geometry, wave frequency, porosity and thickness of the piezoelectric layer on the stress intensity factors.

2. Formulation of the Problem

We consider a model which consists of a porous piezoelectric layer of thickness $2h$ which is bonded between two similar elastic half-spaces. Quantities in the half-spaces are designated with the superscript l . The porous piezoelectric material is assumed to be transversely isotropic having symmetry of type 6mm and poling direction is along the positive z axis. Two symmetric cracks, each of length $(1-b)$, as shown in the Fig. 1, are situated in the mid plane of the porous piezoelectric layer and are parallel to the interfaces. $2b$ is the distance between the cracks. All the field quantities assumed to be time harmonic, i.e. time factor is of the form $e^{(-i\omega t)}$ and that is suppressed for brevity. Here, ω is the circular frequency.

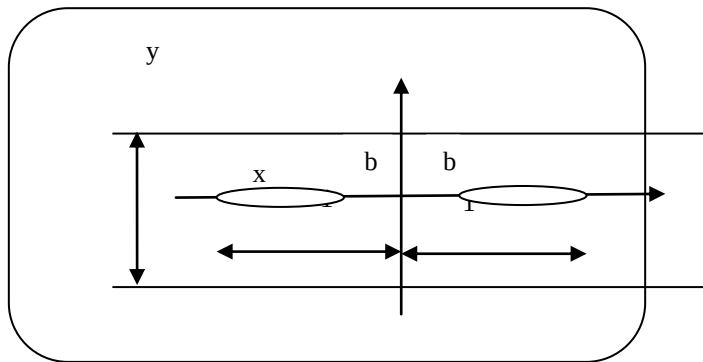


Figure 1. Geometry of the Problem

Following Vashishth and Gupta¹⁸, the appropriate constitutive equations, corresponding to shear hoizontal motion of a porous piezoelectric material, can be written as

$$(2.1) \quad \tau_{z k} = c_{44} w_{,k} + e_{15} \phi_{,k} + \zeta_{15} \phi^*_{,k},$$

$$(2.2) \quad \tau^* = 0,$$

$$(2.3) \quad D_k = e_{15} w_{,k} - \xi_{11} \phi_{,k} - A_{11} \phi^*_{,k},$$

$$(2.4) \quad D_k^* = \zeta_{15} w_{,k} - A_{11} \phi_{,k} - \xi_{11}^* \phi^*_{,k}, \quad (k = x, y)$$

$$(2.5) \quad c_{44} \nabla^2 w + e_{15} \nabla^2 \phi + \zeta_{15} \nabla^2 \phi^* = \rho \ddot{w},$$

$$(2.6) \quad e_{15} \nabla^2 w - \xi_{11} \nabla^2 \phi - A_{11} \nabla^2 \phi^* = 0,$$

$$(2.7) \quad \zeta_{15} \nabla^2 w - A_{11} \nabla^2 \phi - \xi_{11}^* \nabla^2 \phi^* = 0,$$

where τ_{zk} , D_k and τ^* , D_k^* ($k = x, y$) are the shear stress and in plane electric displacement for solid and liquid phases of the porous piezoelectric materials. c_{44} is the elastic constant; e_{15} , ζ_{15} are the piezoelectric constants and ξ_{11} , A_{11} , ξ_{11}^* are the dielectric constants. w , W are the mechanical displacements and ϕ , ϕ^* are the electric potentials for solid and liquid

phases of the porous piezoelectric materials. $\rho = \rho_{11} - \frac{\rho_{12}^2}{\rho_{22}}$,

$\nabla^2 \equiv \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)$ and ρ_{11} , ρ_{12} , ρ_{22} are the mass densities of porous piezoelectric material.

The constitutive equations for elastic half space are

$$(2.8) \quad \tau_{xz}^I = c_{44}^I w_{,x}^I,$$

$$(2.9) \quad \tau_{yz}^I = c_{44}^I w^I_{,y},$$

$$(2.10) \quad c_{44}^I \nabla^2 w^I = \rho^I \ddot{w}^I,$$

τ_{xz}^I are the shear stresses, w^I is the shear displacement, ρ^I is the mass density and c_{44}^I is the rigidity in the elastic half-spaces. Because of assumed symmetry in geometry and loading, it is sufficient to consider the problem in the domain $0 \leq x \leq \infty$, $0 \leq y \leq \infty$.

Using Fourier transform, the solutions of Eqs. (2.5)-(2.7) and (2.10) can be expressed as

$$(2.11) \quad w(x, y, t) = \frac{2}{\pi} \int_0^\infty \left\{ A_1(s) e^{-\gamma_1 y} + A_2(s) e^{\gamma_1 y} \right\} \cos(sx) ds,$$

$$(2.12) \quad w^I(x, y, t) = \frac{2}{\pi} \int_0^\infty A_3(s) e^{-\gamma_2 y} \cos(sx) ds,$$

$$(2.13) \quad \phi(x, y, t) - \frac{\eta_3}{\eta_4} w(x, y, t) = \frac{2}{\pi} \int_0^\infty \left\{ B_1(s) e^{-sy} + B_2(s) e^{sy} \right\} \cos(sx) ds,$$

$$(2.14) \quad \phi^*(x, y, t) - \frac{\eta_1}{\eta_2} w(x, y, t) = \frac{2}{\pi} \int_0^\infty \left\{ C_1(s) e^{-sy} + C_2(s) e^{sy} \right\} \cos(sx) ds,$$

where $\gamma_1, \gamma_2, \eta_1, \eta_2, \eta_3, \eta_4$ are given in the Appendix A and $A_1(s), A_2(s), B_1(s), B_2(s), C_1(s), C_2(s)$, are unknown functions which are to be determined from field equations and boundary conditions.

The boundary conditions for the considered model are

$$(2.15) \quad \tau_{yz}(x, 0, t) = -\tau_0, \quad (b \leq |x| \leq 1)$$

$$(2.16) \quad D_y(x, 0, t) = -D_0, \quad (b \leq |x| \leq 1)$$

$$(2.17) \quad D_y^*(x, 0, t) = -D_0^*, \quad (b \leq |x| \leq 1)$$

$$(2.18) \quad w(x, 0, t) = \phi(x, 0, t) = \phi^*(x, 0, t) = W(x, 0, t) = 0, \quad (|x| < b, |x| > 1)$$

$$(2.19) \quad \tau_{yz}(x, \pm h, t) = \tau_{yz}^l(x, \pm h, t),$$

$$(2.20) \quad (1 - \beta)w(x, \pm h, t) + \beta W(x, \pm h, t) = w^l(x, \pm h, t),$$

$$(2.21) \quad D_y(x, \pm h, t) = 0,$$

$$(2.22) \quad D_y^*(x, \pm h, t) = 0,$$

$$(2.23) \quad w(x, y, t) = w^l(x, y, t)$$

$$= \phi(x, y, t) = \phi^*(x, y, t) = 0, \text{ for } \sqrt{x^2 + y^2} \rightarrow \infty.$$

Where β is the porosity of porous piezoelectric material. Using Eqs. (2.1)-(2.10) and (2.11)-(2.14) in these boundary conditions, we get

$$(2.24) \quad \frac{2}{\pi} \int_0^\infty A(s) \cos(sx) ds = 0, \quad (0 \leq x < b, 1 < x)$$

$$(2.25) \quad \frac{2}{\pi} \int_0^\infty \gamma_1 F_1(s) A(s) \cos(sx) ds = \frac{1}{\mu^{**}} \left[\tau_0 + \frac{e_{15} D_0}{\xi_{11}} - \frac{D^{**} \mu^*}{\alpha} \right], \quad (b \leq x \leq 1)$$

$$(2.26) \quad \frac{2}{\pi} \int_0^\infty B(s) \cos(sx) ds = 0, \quad (0 \leq x < b, 1 < x)$$

$$(2.27) \quad \frac{2}{\pi} \int_0^\infty s F_2(s) B(s) \cos(sx) ds = -\frac{1}{\xi_{11}} \left[D_0 + \frac{A_{11} D^{**}}{\alpha} \right], \quad (b \leq x \leq 1)$$

$$(2.28) \quad \frac{2}{\pi} \int_0^\infty C(s) \cos(sx) ds = 0, \quad (0 \leq x < b, 1 < x)$$

$$(2.29) \quad \frac{2}{\pi} \int_0^\infty s F_3(s) C(s) \cos(sx) ds = \frac{D^{**}}{\alpha}, \quad (b \leq x \leq 1)$$

where $\mu^{**}, \mu_1, \mu_2, \mu_3, \beta^*, \alpha, D^{**}, F_1(s), F_2(s), F_3(s), A(s), A_2(s), B(s), B_2(s), C(s), C_2(s)$, are given in the Appendix A

3. Solution of Integral Equations

The system of integral equations (2.24)-(2.29) is solved by using Schmidt's method (Morse and Feshbach¹⁹). The displacement and the electric potentials for solid and liquid phase of the porous piezoelectric layer can be written as

$$(3.1) \quad \left\{ w, \phi, \phi^* \right\} (x, 0, t) = \sum_{n=0}^{\infty} \left\{ a_n, b_n, c_n \right\} P_n \left(\frac{1}{2}, \frac{1}{2} \right) \\ \times \left(\frac{x - \frac{1+b}{2}}{\frac{1-b}{2}} \right) \left[1 - \frac{(x - \frac{1+b}{2})^2}{(\frac{1-b}{2})^2} \right]^{\frac{1}{2}}, \text{ for } b \leq x \leq 1, y = 0$$

$$(3.2) \quad \left\{ w, \phi, \phi^* \right\} (x, 0, t) = \{0, 0, 0\}, \text{ for } x < b, x > 1, y = 0$$

where a_n, b_n and c_n are unknown coefficients to be determined and

$P_n \left(\frac{1}{2}, \frac{1}{2} \right) (x)$ is a Jacobian polynomial.

Taking Fourier cosine transforms of Eq. (3.1), we get

$$(3.3) \quad \{A(s), B(s), C(s)\} = \sum_{n=0}^{\infty} \left\{ a_n, b_n - \frac{\eta_3}{\eta_4} a_n, c_n - \frac{\eta_1}{\eta_2} a_n \right\} \\ \times Q_n G_n(s) \frac{1}{s} J_{n+1} \left(s \frac{1-b}{2} \right),$$

where

$$\begin{aligned}
 (3.4) \quad Q_n &= 2\sqrt{\pi} \frac{\Gamma(n+1+\frac{1}{2})}{n!}, \\
 G_n(s) &= \begin{cases} (-1)^{\frac{n}{2}} \cos(s\frac{1+b}{2}), & \text{for } (n=0, 2, 4, 6, 8, \dots), \\ (-1)^{\frac{n+1}{2}} \sin(s\frac{1+b}{2}), & \text{for } (n=1, 3, 5, 7, 9, \dots) \end{cases}
 \end{aligned}$$

and $\Gamma(x)$, $J_n(x)$ are the Gamma and Bessel functions, respectively. Substituting the value of $A(s)$, $B(s)$ and $C(s)$ in Eqs.(2.24)-(2.29) and integrating with respect to x in $[b, x]$, we get

$$\begin{aligned}
 (3.5) \quad & \sum_{n=0}^{\infty} a_n Q_n \int_0^{\infty} \frac{1}{s} G_n(s) J_{n+1}\left(s\frac{1-b}{2}\right) \left[1 + \frac{g_1(s)}{sg_2(s)}\right] \\
 & \times [\sin(sx) - \sin(sb)] ds = \frac{\pi\tau_0}{2\mu^{**}} [1 + \lambda_1 + \lambda_2](x-b),
 \end{aligned}$$

$$\begin{aligned}
 (3.6) \quad & \sum_{n=0}^{\infty} (b_n - \frac{\eta_3}{\eta_4} a_n) Q_n \int_0^{\infty} \frac{1}{s} G_n(s) J_{n+1}\left(s\frac{1-b}{2}\right) \\
 & \times \{1 + [F_2(s) - 1]\} [\sin(sx) - \sin(sb)] ds \\
 & = -\frac{\pi}{2\xi_{11}} \left[D_0 \left(1 + \frac{A_{11}^2}{\alpha\xi_{11}}\right) - \frac{A_{11}}{\alpha} D_0^* \right] (x-b),
 \end{aligned}$$

$$\begin{aligned}
 (3.7) \quad & \sum_{n=0}^{\infty} (c_n - \frac{\eta_1}{\eta_2} a_n) Q_n \int_0^{\infty} \frac{1}{s} G_n(s) J_{n+1}\left(s\frac{1-b}{2}\right) \\
 & \times \{1 + [F_3(s) - 1]\} [\sin(sx) - \sin(sb)] ds \\
 & = \frac{\pi}{2\alpha} \left[\frac{A_{11}}{\xi_{11}} D_0 - D_0^* \right] (x-b),
 \end{aligned}$$

where $\lambda_1, \lambda_2, g_1(s)$ and $g_2(s)$ are given in the Appendix A. Eqs. (3.5)-(3.7) can now be solved for the coefficients a_n , b_n and c_n by Schmidt method.

4. Intensity Factors

Coefficients a_n , b_n and c_n are determined, so that entire perturbation stress field, electric displacement for solid and liquid respectively can be obtained. Intensity factors play a dominating role because they characterize the intensity of the singular piezoelectric crack fields (stress and strain). The knowledge of intensity factors is based on the near field solutions and is useful because intensity factors give information for the strength and life time prediction of structures. Thus, by making use of Eqs. (2.11)-(2.14) and Eq. (3.3) in the Eqs. (2.1)-(2.4), the expressions of τ_{23} , D_2 and D_2^* near the crack tips, are obtained as

$$(4.1) \quad \tau_{yz}(x, 0, t) = -\frac{2\mu^{**}}{\pi} \sum_{n=0}^{\infty} a_n Q_n \int_0^{\infty} G_n(s) \left\{ 1 + \left[\frac{\gamma_1}{s} F_1(s) - 1 \right] \right\}$$

$$\times J_{n+1} \left(s \frac{1-b}{2} \right) \cos(sx) ds - \frac{2e_{15}}{\pi} \sum_{n=0}^{\infty} \left(b_n - \frac{\eta_3}{\eta_4} a_n \right) Q_n$$

$$\times \int_0^{\infty} G_n(s) \{ 1 + [F_2(s) - 1] \} J_{n+1} \left(s \frac{1-b}{2} \right) \cos(sx) ds - \frac{2\zeta_{15}}{\pi}$$

$$\times \sum_{n=0}^{\infty} \left(c_n - \frac{\eta_1}{\eta_2} a_n \right) Q_n \int_0^{\infty} G_n(s) \{ 1 + [F_3(s) - 1] \}$$

$$\times J_{n+1} \left(s \frac{1-b}{2} \right) \cos(sx) ds$$

$$(4.2) \quad D_y(x, 0, t) = \frac{2\zeta_{11}}{\pi} \sum_{n=0}^{\infty} \left(b_n - \frac{\eta_3}{\eta_4} a_n \right) Q_n \int_0^{\infty} G_n(s)$$

$$\times \{ 1 + [F_2(s) - 1] \} J_{n+1} \left(s \frac{1-b}{2} \right) \cos(sx) ds$$

$$\begin{aligned}
& + \frac{2A_{11}}{\pi} \sum_{n=0}^{\infty} \left(c_n - \frac{\eta_1}{\eta_2} a_n \right) Q_n \int_0^{\infty} G_n(s) \\
& \times \left\{ 1 + [F_3(s) - 1] \right\} J_{n+1} \left(s \frac{1-b}{2} \right) \cos(sx) ds, \\
(4.3) \quad & D_y^*(x, 0, t) = \frac{2A_{11}}{\pi} \sum_{n=0}^{\infty} \left(b_n - \frac{\eta_3}{\eta_4} a_n \right) Q_n \int_0^{\infty} G_n(s) \\
& \times \left\{ 1 + [F_2(s) - 1] \right\} J_{n+1} \left(s \frac{1-b}{2} \right) \cos(sx) ds \\
& + \frac{2\xi_{11}^*}{\pi} \sum_{n=0}^{\infty} \left(c_n - \frac{\eta_1}{\eta_2} a_n \right) Q_n \int_0^{\infty} G_n(s) \\
& \times \left\{ 1 + [F_3(s) - 1] \right\} J_{n+1} \left(s \frac{1-b}{2} \right) \cos(sx) ds,
\end{aligned}$$

From the above equations, the singular part of the stress field and the singular part of the electric displacements can be expressed as

$$(4.4) \quad \tau = -\frac{1}{\pi} \sum_{n=0}^{\infty} (c_{44}a_n + e_{15}b_n + \zeta_{15}c_n) Q_n H_n(b, x),$$

$$(4.5) \quad D = -\frac{1}{\pi} \sum_{n=0}^{\infty} (\xi_{11}b_n - e_{15}a_n + A_{11}c_n) Q_n H_n(b, x),$$

$$(4.6) \quad D^* = -\frac{1}{\pi} \sum_{n=0}^{\infty} (A_{11}b_n - \zeta_{15}a_n + \xi_{11}^*c_n) Q_n H_n(b, x),$$

where

$$H_n(b, x) = (-1)^{n+1} f_1(b, x, n), \quad (n = 0, 1, 2, 3, 4, 5, \dots) \text{ for } 0 < x < b,$$

$$H_n(b, x) = (-1) f_2(b, x, n), \quad n = 0, 1, 2, 3, 4, 5, \dots, \text{ for } 1 < x,$$

$$f_1(b, x, n) = \frac{2(1-b)^{n+1}}{\sqrt{(1+b-2x)^2 - (1-b)^2} \left[1+b-2x + \sqrt{(1+b-2x)^2 - (1-b)^2} \right]^{n+1}},$$

$$f_2(b, x, n) = \frac{2(1-b)^{n+1}}{\sqrt{(2x-1-b)^2 - (1-b)^2} \left[2x-1-b + \sqrt{(2x-1-b)^2 - (1-b)^2} \right]^{n+1}}.$$

Using Eqs. (4.4)-(4.6), the stress intensity factor K_L , electric displacement intensity factor D_L solid phase and electric displacement intensity factor D_L^* for liquid phase at the left end of the right crack are obtained as

$$(4.7) \quad \left\{ K_L, D_L, D_L^* \right\} = \lim_{x \rightarrow b^-} \sqrt{2\pi(b-x)} \left\{ \tau, D, D^* \right\},$$

$$(4.8) \quad \left\{ K_L, D_L, D_L^* \right\} = \sqrt{\frac{2}{\pi(1-b)}} \sum_{n=0}^{\infty} (-1)^n Q_n (c_{44}a_n + e_{15}b_n + \zeta_{15}c_n, \\ e_{15}a_n - \xi_{11}b_n - A_{11}c_n, \zeta_{15}a_n - A_{11}b_n - \xi_{11}^*c_n).$$

Similarly, at the right end of the right crack, the intensity factors are obtained as

$$(4.9) \quad \left\{ K_R, D_R, D_R^* \right\} = \lim_{x \rightarrow 1^+} \sqrt{2\pi(x-1)} \left\{ \tau, D, D^* \right\},$$

$$(4.10) \quad \left\{ K_R, D_R, D_R^* \right\} = \sqrt{\frac{2}{\pi(1-b)}} \sum_{n=0}^{\infty} Q_n (c_{44}a_n + e_{15}b_n + \zeta_{15}c_n, \\ e_{15}a_n - \xi_{11}b_n - A_{11}c_n, \zeta_{15}a_n - A_{11}b_n - \xi_{11}^*c_n)$$

5. Reduced Case

In order to validate the results obtained in the present model of study, intensity factors are derived for piezoelectric layered structure as special case of porous piezoelectric layered structure. When we take

$\zeta_{15} = 0, A_{11} = 0, \xi_{11}^* = 0$, then porous piezoelectric material model reduce to piezoelectric material model. The Eqs. (4.8) and (4.10) then reduce to the form

(5.1) $\{K_L, D_L\} = \sqrt{\frac{2}{\pi(1-b)}} \sum_{n=0}^{\infty} (-1)^n (c_{44}a_n + e_{15}b_n, e_{15}a_n - \xi_{11}b_n) Q_n,$

(5.2) $\{K_R, D_R\} = \sqrt{\frac{2}{\pi(1-b)}} \sum_{n=0}^{\infty} (c_{44}a_n + e_{15}b_n, e_{15}a_n - \xi_{11}b_n) Q_n.$

Eqs. (5.1) and (5.2) gives the stress intensity factors and electric displacement intensity factors at the left and right end of the right crack respectively and these expressions coincide with those as obtained by Zhou and Wang⁶.

6. Numerical Discussions

In this section, the intensity factors are calculated numerically for porous piezoelectric material. The results have been found with a good level of performance by using Schmidt’s method. A sufficient and satisfactory level of approximation is achieved when first ten terms of the infinite series are retained. The magnitudes of the unknown coefficients a_n decrease rapidly with the increase in the number of terms n . The material constants for the porous piezoelectric material (PZT-5H) and elastic half-space (Aluminium) are given in Table 1.

Table 1. Material Constants

Materials	Elastic constant ($10^{10} N / m^2$)	Piezoelectric constant c / m^2	Dielectric constant $10^{-9} C / Vm^2$	Density Kg / m^3
PZT-5H	$c_{44} = 2.3$	$e_{15} = 17$	$\xi_{11} = 27.7$	$\rho_{11} = 4950$
		$\zeta_{15} = 7.8$	$\xi_{11}^* = 29.9$	$\rho_{12} = -1125$
			$A_{11} = 11.2$	$\rho_{22} = 4800$
Aluminium	$c_{44}^I = 2.65$	--	--	$\rho^I = 2706$

The stress intensity factors (K_L, K_R) and the electric displacement intensity factors for solid phase (D_L, D_R) and the electric displacement intensity

factors for fluid phase (D_L^*, D_R^*) at the crack tips are calculated numerically and plots are shown in Figures 2- 8. It is observed that the dynamic intensity factors not only depend on the crack length (b), the width of the porous piezoelectric layer (h), the electric loading (λ_1, λ_2) the frequency of the wave (ω/c_{SH}) porosity (β) but also on materials' characteristics. Figures 2(i) and 3(i) show that the stress intensity factors increase with the increase of the electric loading (λ_1) for $h=1.0$ but decrease for $h < 1.0$. Thus the probability of mechanical failure with electric loading in piezoelectric plate increases with depth of the layer up to a certain limit. The electric displacement intensity factors for solid phase increases (Figure 2(ii), 3(ii)) while the electric displacement intensity factors for fluid phase remains constant (Figures 2(iii), 3(iii)) with increasing electric load (λ_1). When h is taken very large, then results were found not varying with λ_1 .

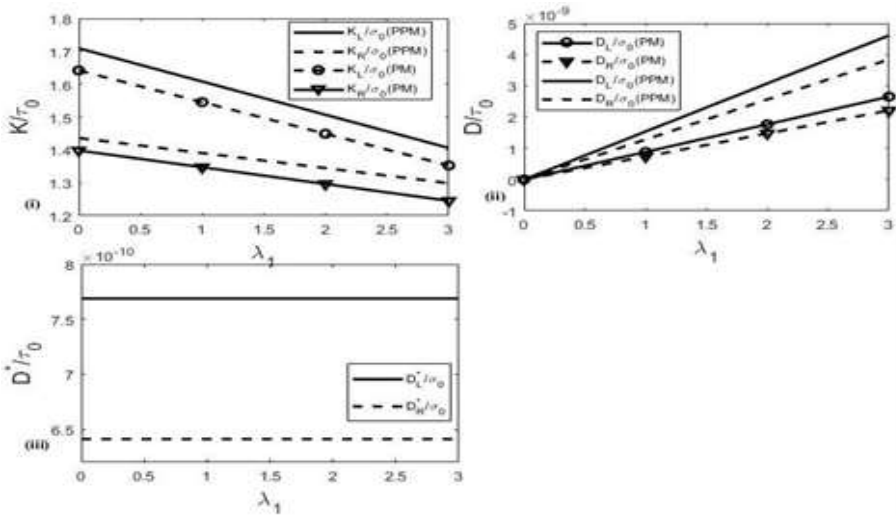


Figure 2. Variation of intensity factors with λ_1 for $b = 0.1, \omega/c_{SH} = 0.5, \lambda_2 = 0.5, \beta = 0.2, h = 0.5$. The graphs with the legend ($\circ, \nabla$) represent intensity factors for piezoelectric

This observation is consistent with the fact that the stress intensity factor is independent of the electric loading for infinite/semi-infinite porous piezoelectric materials. The comparison of intensity factors for piezoelectric and porous piezoelectric materials in reference of electric loading (λ_1) is

shown in Figures 2(i) and 2(ii) and it is observed that the intensity factors in porous piezoelectric materials are greater than that in piezoelectric material. In the Figure 4, the effect of porosity on the intensity factors with increasing λ_1 is shown. The dynamic stress intensity factors increase with increase in porosity (β) but electric displacements for solid and fluid phase remain constant.

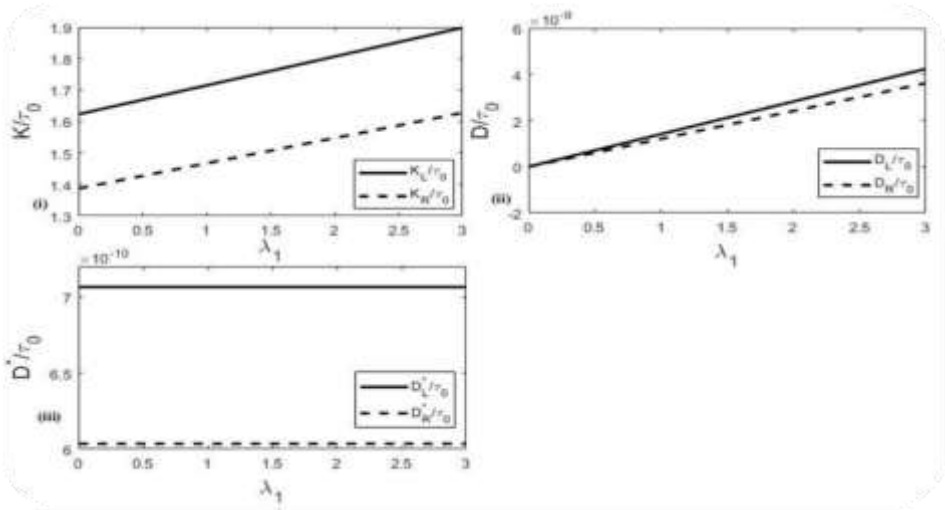


Figure 3. Variation of intensity factors with λ_1 for $b = 0.1$, $\omega/c_{SH} = 0.5$, $\lambda_2 = 0.5$, $\beta = 0.2$, $h = 1$.

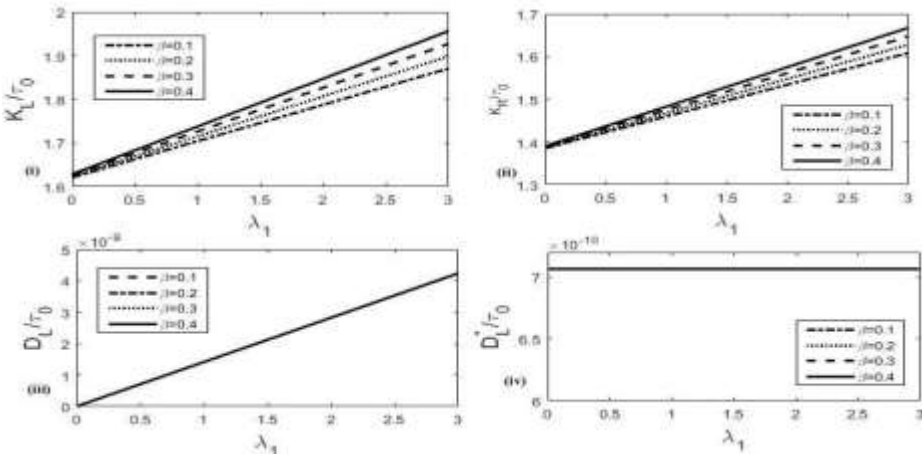


Figure 4. Variation of intensity factors with λ_1 for $b = 0.1$, $\omega/c_{SH} = 0.5$, $\lambda_2 = 0.5$, $h = 1$ with different values of porosity (β)

Figure 5 describes the variation of intensity factors with λ_2 . The stress intensity factors decrease while electric displacements intensity factors for fluid phase increase as λ_2 increases. The electric displacements intensity factors for solid phase remain unchanged with λ_2 .

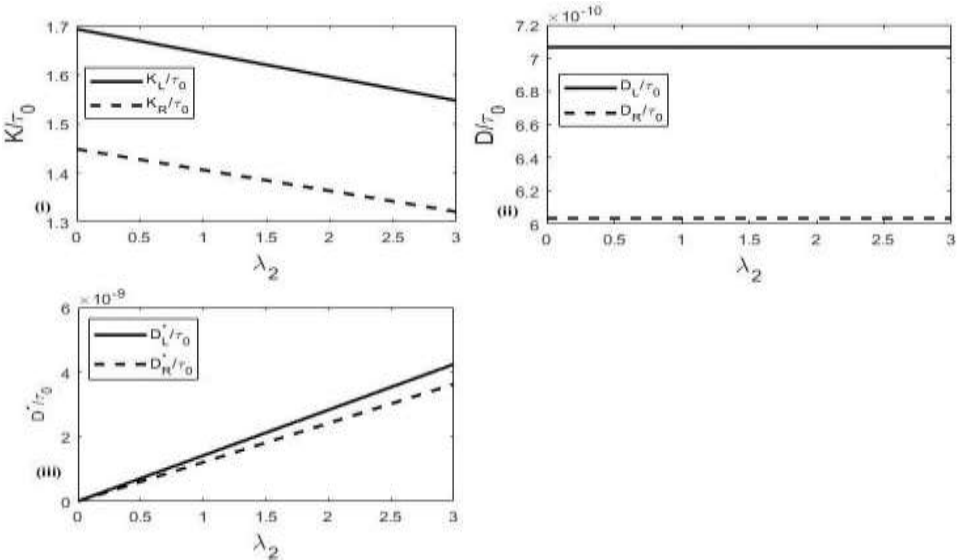


Figure 5. Variation of intensity factors with λ_2 for $b = 0.1$, $\omega/c_{SH} = 0.5$, $\lambda_1 = 0.5$, $\beta = 0.2$, $h = 1$.

Figure 6 shows that the porosity affects only the stress intensity factors while D_L and D_L^* remains unaffected. The same type of observation was made in case of λ_1 . Figure 7 shows that the dynamic stress intensity factors tend to increase with the frequency (ω/c_{SH}) reaching a peak and then decrease. However, when the frequency $\omega/c_{SH} > 2$, the dynamic stress intensity factors tend to increase with the frequency again. This phenomenon is brought up by the free waves. However, the electric displacement intensity factors do not experience variation with frequency. In Figure 7(i), the stress intensity factors for porous piezoelectric materials and piezoelectric material in reference ω/c_{SH} are compared and it is found that the intensity factors are more for porous piezoelectric materials than that for piezoelectric materials.

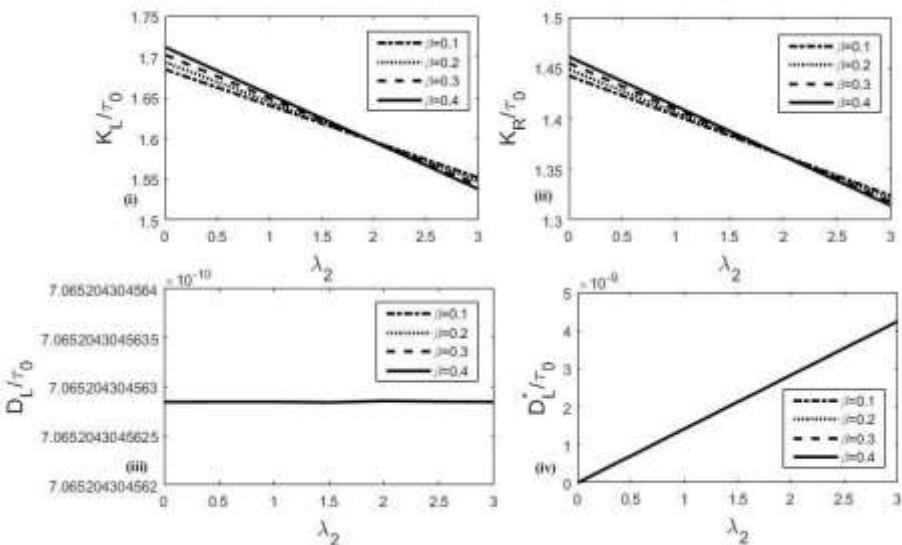


Figure 6. Variation of intensity factors with λ_2 for $b = 0.1$, $\omega/c_{SH} = 0.5$, $\lambda_1 = 0.5$, $h = 1$ with different values of porosity (β)

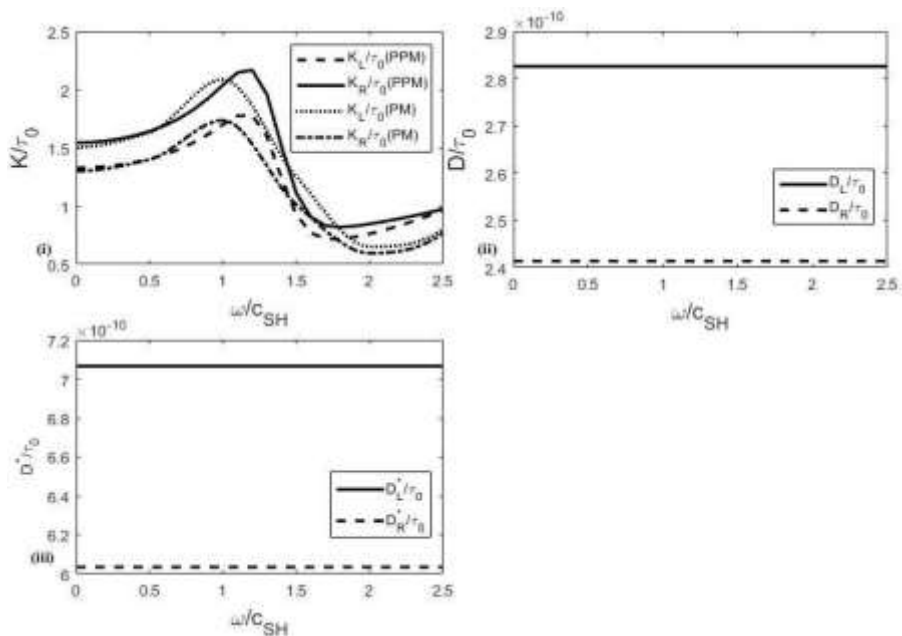


Figure 7. Variation of intensity factors with ω/c_{SH} for $\lambda_1 = 0.2$, $b = 0.1$, $\lambda_2 = 0.5$, $h = 1$, $\beta = 0.2$

Figure 8 depicts the variation of intensity factors with b . The stress and electric displacement intensity factors decrease with the decrease of the crack's length. The stress intensity factors and the electric displacement intensity factors at the inner crack tips are larger than ones at the outer crack tips. It is observed that intensity factors K_L and K_R , D_L and D_R , D_L^* and D_R^* are almost equal when $b > 0.6$. It can, therefore, be concluded that intensity factors at left and right ends of the tips of a crack of small length are almost same.

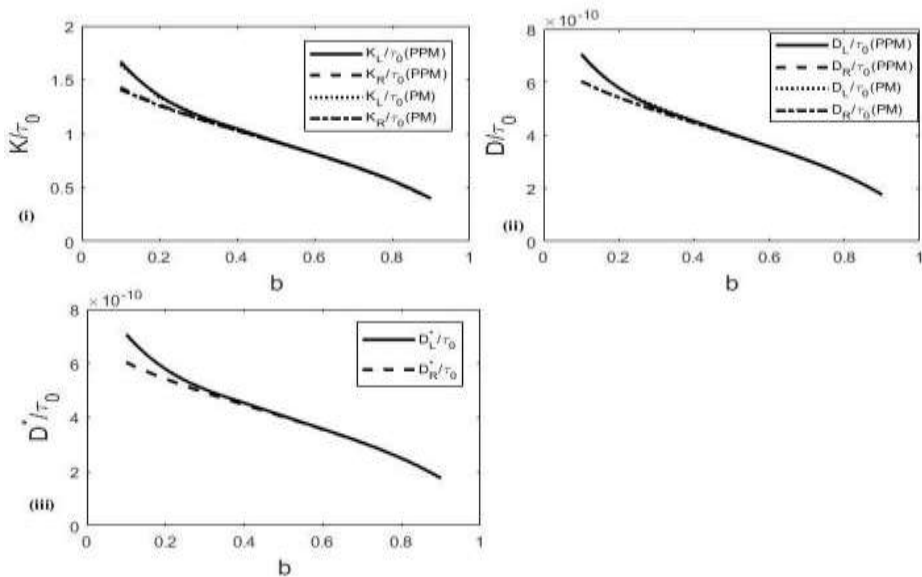


Figure 2. Variation of intensity factors with b for $\omega/c_{SH} = 0.5$, $\lambda_1 = 0.5$, $\lambda_2 = 0.5$, $\beta = 0.2$, $h = 1$ for porous piezoelectric and piezoelectric materials

7. Conclusions

The dynamic response of two symmetric cracks embedded in a porous piezoelectric layer bonded between two elastic half-spaces, under anti-plane shear impact is studied in this paper. The cracks are considered parallel to the interface and lie in the mid plane of the porous piezoelectric layer. Fourier transforms are used to reduce the problem to the solution of pair of three integral equations which are further solved by using Schmidt's method. The analytic solutions of the dynamic stress intensity factor and dynamic electric displacement intensity factor, for solid and fluid phase, have been obtained and numerical computation has been done. It is observed that the dynamic intensity factors depend on the crack length, width of the

porous piezoelectric layer, electric loading, frequency of the wave and porosity along with materials characteristics. The influence of the electric loading on the results is considerable for a thin porous piezoelectric layer in comparison to a thick layer. Thus the probability of mechanical failure with electric loading in piezoelectric plate increases with depth of the layer up to a certain limit. The stress intensity factors and the electric displacement intensity factors at the inner crack tips are larger than the ones at the outer crack tips but for $b > 0.6$, the intensity factors at left and right ends of the tips of a crack of small length are almost same.

Appendix A:

$$\mu = c_{44} + \frac{e_{15}^2}{\xi_{11}}, \mu' = \mu + \mu^* \frac{\eta_1}{\eta_2}, \mu^* = \zeta_{15} - \frac{e_{15} A_{11}}{\xi_{11}},$$

$$\gamma_1 = \sqrt{s^2 - \left(\frac{\omega}{c_{SH}}\right)^2}, \gamma_2 = \sqrt{s^2 - \left(\frac{\omega}{c_{SH}^I}\right)^2}, c_{SH} = \sqrt{\frac{\mu'}{\rho}},$$

$$c_{SH}^I = \sqrt{\frac{c_{44}^I}{\rho^I}}, \rho = \rho_{11} - \frac{\rho_{12}^2}{\rho_{22}}, \eta_1 = \frac{e_{15}}{\xi_{11}} - \frac{\zeta_{15}}{A_{11}},$$

$$\eta_2 = \frac{A_{11}}{\xi_{11}} - \frac{\xi_{11}^*}{A_{11}}, \eta_3 = \frac{e_{15}}{A_{11}} - \frac{\zeta_{15}}{\xi_{11}^*}, \eta_4 = \frac{\xi_{11}}{A_{11}} - \frac{A_{11}}{\xi_{11}^*},$$

$$\mu^{**} = c_{44} + e_{15} \frac{\eta_3}{\eta_4} + \zeta_{15} \frac{\eta_1}{\eta_2}, \beta^* = 1 - \beta \left(1 + \frac{\rho_{12}}{\rho_{22}} \right),$$

$$\alpha = \xi_{11}^* - \frac{A_{11}^2}{\xi_{11}}, D^{**} = -D_0^* + \frac{A_{11} D_0}{\xi_{11}}, F_1(s) = \frac{1 - \mu_3 e^{-2\gamma_1 h}}{1 + \mu_3 e^{-2\gamma_1 h}},$$

$$F_2(s) = \frac{1 - e^{-2sh}}{1 + e^{-2sh}}, F_3(s) = \frac{1 - e^{-2sh}}{1 + e^{-2sh}}, A(s) = (1 + \mu_3 e^{-2\gamma_1 h}) A_1(s),$$

$$A_2(s) = \mu_3 e^{-2\gamma_1 h} A_1(s), \quad B(s) = (1 + e^{-2sh}) B_1(s),$$

$$B_2(s) = e^{-2sh} B_1(s), \quad C(s) = (1 + e^{-2sh}) C_1(s), \quad \mu_3 = \frac{\mu_1}{\mu_2},$$

$$C_2(s) = e^{-2sh} C_1(s), \quad \mu_1 = \gamma_1 - \frac{c_{44}^l \gamma_2}{\mu} \beta^*, \quad \mu_2 = \gamma_1 + \frac{c_{44}^l \gamma_2}{\mu} \beta^*,$$

$$\lambda_1 = \left(\frac{e_{15}}{\xi_{11}} - \frac{A_{11} \mu^*}{\xi_{11} \alpha} \right) \frac{D_0}{\tau_0}, \quad \lambda_2 = \frac{\mu^* D_0}{\alpha \tau_0},$$

$$g_1(s) = (\gamma_1 - s) - (\gamma_1 + s) \mu_3 e^{-2\gamma_1 h}, \quad g_2(s) = 1 + \mu_3 e^{-2\gamma_1 h}.$$

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