

Effect of Chemical Reaction Heat and Mass Transfer on the Flow Past a Semi Infinite Plate in Porous Media

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Abstract: The present paper deals with the study of effect of chemical reaction heat and mass transfer on the flow past a semi-infinite plate in porous media. The fluid is assumed to be viscous and incompressible. The governing equations of the problems subject to the relevant boundary conditions are solved using computer supported numerical method. The velocity, temperature and concentration distributions for different values of permeability and chemical reaction parameters etc. have been obtained, which are presented and discussed with the help of graphs.

Key Words: Viscous and incompressible fluid, porous media, Temperature and concentration distribution, chemical reaction constant.

2000 Mathematics subject classification: 76W05

1. Introduction

The convective problems in porous media have important applications in geothermal reservoirs and geothermal energy extraction processes and a number of natural and industrial happenings around us such as in the field of chemical engineering for filtration and purification processes; in the field of agriculture engineering to study the underground water resources; in petroleum industry to study the movement of natural gasses, oil and water through the oil reservoirs, in metal industry to study the flow pattern of the melted solids at different temperatures etc. The problem becomes more interesting when the viscous and thermal boundary layers are subjected to the action of variable permeability, as naturally occurring porous media are seldom homogeneous. Recent years have also seen an increased interest in these types of problems involving heat and mass transfer. In air or water, some foreign species are present. Gebhart¹ considered the presence of foreign mass, of course of low level, in the fluid flows in details. Later the

effect of suspended particles on convection problems were studied by Scanlon and Segel². These foreign species influence the flow pattern which can not be ignored as the changes in the flow behavior have been observed experimentally also. Some aspects such as heat and mass transfer in the fluid mechanics of reactive flows play a central role in the decision-making pertaining to engineering and technological fields.

Wollkind and Frisch³ considered the chemical instabilities of a heated horizontal layer dissociating flow. The flow problem past a plate has been discussed by many. Soundalkgar⁴ studied free convection effects problem for a vertical plate. Stokes problem for vertical plate for constant heat flux was studied by Soundalgar and Patil⁵. Systematic study of mass transfer effects on free convection flow past a semi-infinite vertical plate was done by Gebhart and Pera⁶. Byron et.al⁷ have considered the situation of heat generation during chemical reaction between two species. Khan and Acharya et.al.^{8,9} have investigated the effect of magnetic field on free convection and mass transfer flow through porous medium with constant suction and constant heat flux. Falkner and Skan¹⁰ developed some approximate solutions of the boundary layer flows. The effect of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction were studied by Das and Deka et.al.¹¹. Effect of chemical reaction, heat and mass transfer on non-linear MHD laminar boundary layer flow over a wedge with suction /injection was studied by Anjali Devi and Kandasamy¹².

Here it has been assumed that the level of species concentration is very low and that the heat generated during chemical reaction can be neglected. The results have been found useful in chemical technology. The application of boundary layer techniques to mass transfer has been of considerable assistance in developing the theory of separation processes and chemical kinetics.

It seems to us that the effects of chemical reaction heat and mass transfer on the flow past a semi infinite plate in porous media has not been given due attention though these effects show their significant role in chemical engineering and aerospace technology. Therefore the aim of the present investigation is to study the effect of chemical reaction heat and mass transfer on the flow past a semi-infinite plate in porous media.

2. Symbols

C : Species concentration in the fluid.

S_c : Schmidt number

C_{∞} : Species concentration with fluid away from the plate.

γ (gamma): Non-dimensional chemical reaction parameter.

C_w : Species concentration near the plate

K (γ): Permeability of the porous medium

D : Chemical molecular diffusivity

δ : Non-dimensional Permeability parameter

G : Acceleration due to gravity

ν : Kinematic viscosity

G_r : Grashof number

β : Coefficient of volume expansion

G_c : Modified Grashof number

β^* : Coefficient of expansion with concentration

k_1 : Chemical reaction constant

T : Temperature of fluid near the plate

R : Reynolds number

T_{∞} : Temperature of the fluid away from the plate

U : Axial velocity

Pr : Prandtl number

α : Thermal diffusivity

3. Mathematical analysis

Let us consider the steady state flow of an incompressible, viscous fluid over a semi-infinite horizontal plate. The x- axis is assumed to be taken along the plate and y-axis normal to the plate. An appropriate mass transfer analog to the problem would be the flow along the flat plate that contains a species M slightly soluble in the fluid N. The concentration at the plate surface is C_w and the concentration of M for away from the plate is C_{∞} . Let the reaction of a species M with N be the first order homogeneous chemical reaction with rate constant k_1 . It is desired to analyze the system by a boundary layer method. It is assumed that the concentration of dissolved species M is small enough and the physical properties ρ , μ and D are

constant throughout the fluid. Under these conditions the relevant governing equations for steady flow through porous medium are: -

$$(3.1) \quad u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2}$$

$$(3.2) \quad u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) - \frac{\nu u}{K(y)}$$

$$(3.3) \quad u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2}$$

$$(3.4) \quad u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - k_1 C$$

where u and v are corresponding velocity component along and perpendicular to the plate surface.

The boundary conditions are

$$(3.5) \quad u = v = 0, C = C_w, T = T_w \text{ at } y = 0$$

$$(3.6) \quad u = U_\infty, C \rightarrow C_\infty, T \rightarrow T_\infty \text{ at } y \rightarrow \infty$$

We introduce the following change of variables

$$\eta(x, y) = \frac{y}{2} \sqrt{U_\infty / \nu} x$$

$\psi(x, y) = 2\sqrt{U_\infty \nu x} f(\eta)$, where $f(\eta)$ is dimensionless form of stream function.

The velocity components are $u = \frac{\partial \psi}{\partial y}$ and $v = \frac{\partial \psi}{\partial x}$

Introducing the following non-dimensional quantities

$$(3.7) \quad \left\{ \begin{array}{l} \theta(\eta) = T - T_\infty / T_w - T_\infty, \quad \phi(\eta) = C - C_\infty / C_w - C_\infty \\ R = U_\infty x / \nu, \quad S_c = \frac{\nu}{D}, \quad \gamma(\text{gamma}) = \frac{\nu k_1}{U_\infty^2} \\ Gr = \nu g \beta (T_w - T_\infty) / U_\infty^3, \quad Gc = \nu g \beta^* (C_w - C_\infty) / U_\infty^3 \\ Pr = \nu / \alpha, \quad \delta(\text{delta}) = \frac{x \nu}{K(\eta) U_\infty} \end{array} \right.$$

then equations (3.1) to (3.4) become

$$(3.8) \quad f''' + 4GrR\theta + 4GcR\phi - 4\delta f' + 2ff' = 0$$

$$(3.9) \quad \theta'' - 4f' \text{Pr} \theta + 2f \text{Pr} \theta' = 0$$

$$(3.10) \quad \phi'' + 2Scf' \phi' - 4Scf' \phi - 4\gamma ScR\phi = 0$$

where prime denote differentiation w.r.t η .

The corresponding boundary conditions reduce to

$$(3.11) \quad \begin{cases} f(0) = 0, f'(0) = 0, \phi(0) = 1, \theta(0) = 1 \\ f'(\infty) = 1, \phi(\infty) = 0, \theta(\infty) = 0 \end{cases}$$

4. Method of Solution

The equations (3.8) to (3.10) are coupled differential equations, we therefore solve them numerically with boundary condition (3.11) using BVP4C method. During the course of numerical calculations of velocity temperature and species concentration functions, the value of Prandtl number Pr is chosen to be 0.71, which represent air at 20°C . When the values of physical parameters are fixed at Reynolds number $R=3$, Grash of number $\text{Gr}=1.0$, Modified Garsh of number $\text{Gc}=3.0$, for different values of chemical reaction parameter γ and permeability parameter δ , while $\text{Sc}=1.0$. Numerical results are displayed with the help of graphical illustrations.

5. Results and Discussion

Effects of chemical reaction over the velocity, temperature and concentration are shown in Figures 1-3. Figure 1 depicts the dimensionless velocity profile $f'(\eta)$ for different values of chemical reaction parameter. It is observed that the velocity decrease with increase in chemical reaction parameter. Figure 2 represents the dimensionless temperature profile; the temperature increases with increasing chemical reaction parameter. Figure 3 shows the dimensionless concentration of the fluid. The concentration of the fluid decreases with increase in chemical reaction parameter. An increase in permeability parameter leads to increase in the velocity and temperature distribution, these are shown through Figures 4-5. Figure 6 represents the dimensionless concentration, the concentration of the fluid decreases with increasing permeability parameter.

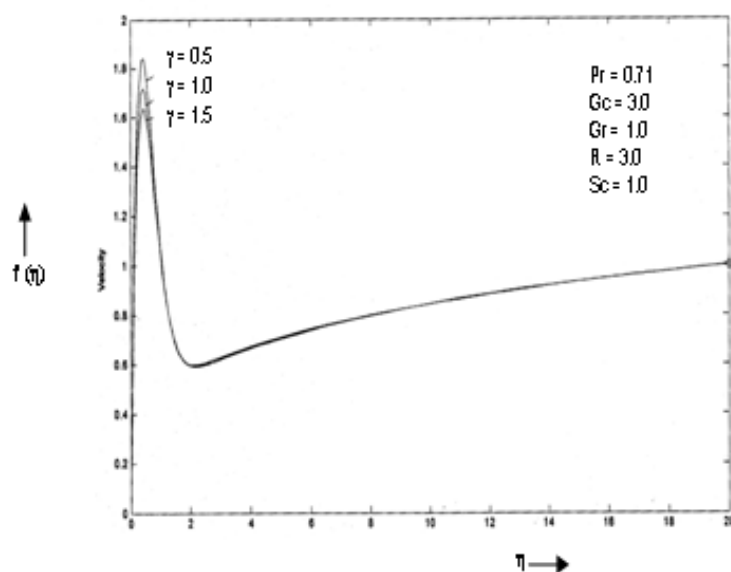


Fig. 1

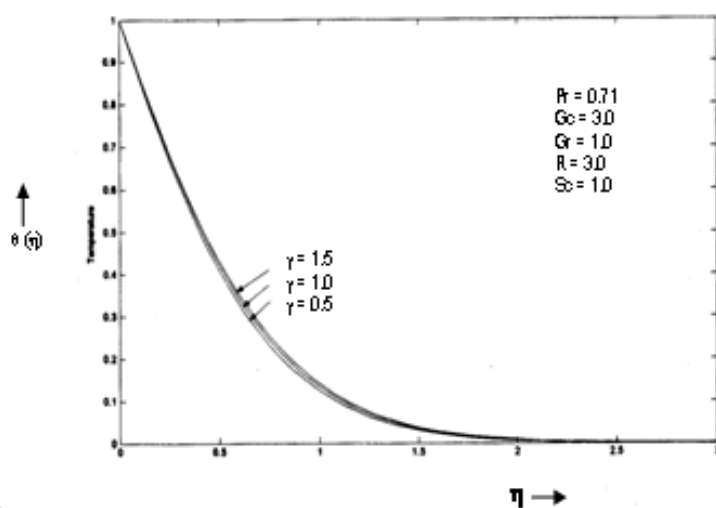


Fig. 2

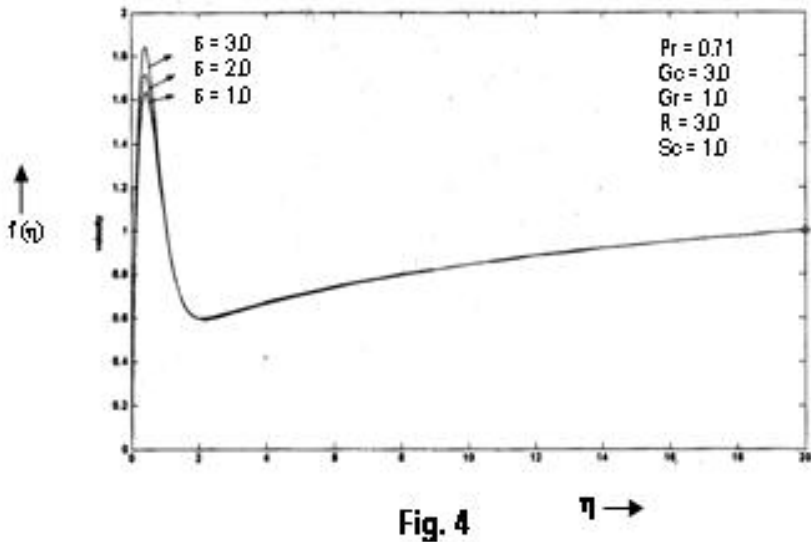
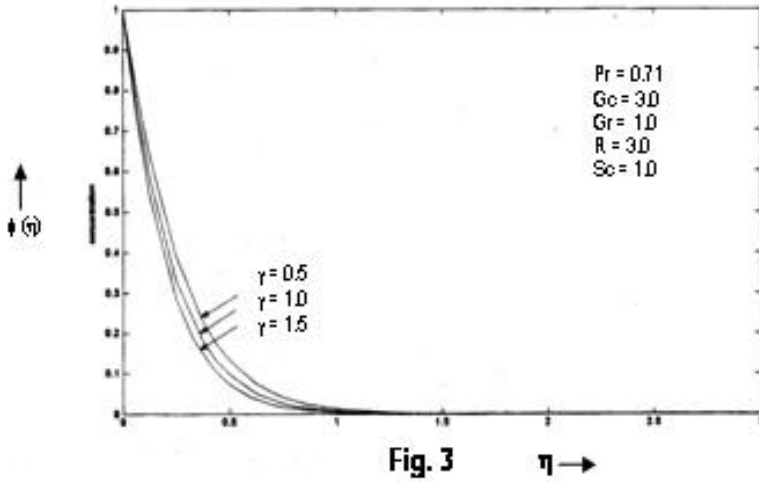


Figure 1 and Figure 4 show that the velocity increases rapidly in the close vicinity of the plate and then settles steadily away from the plate. Also, temperature and concentration profiles show that concentration gradient is higher than the temperature gradient, which means that the concentration of the species diminishes faster much before the effect of the temperature of the plate vanishes.

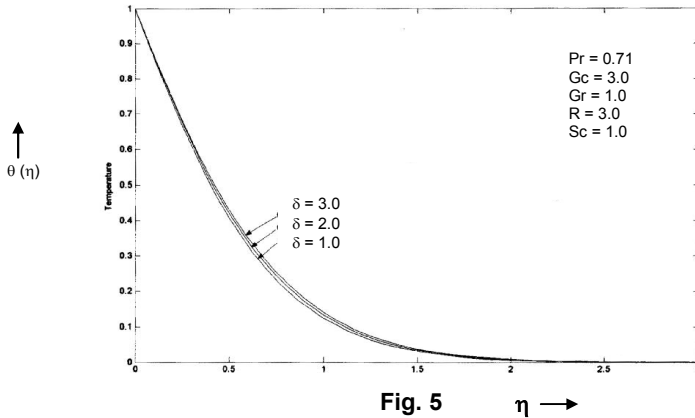


Fig. 5

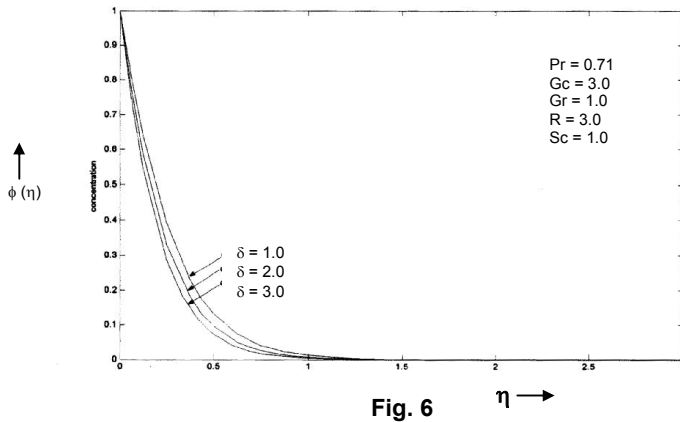


Fig. 6

6. Conclusions

It is concluded that the velocity of fluid decreases with increase of chemical reaction parameter. The temperature of fluid increases but concentration decreases with increase of chemical reaction parameter. Porosity of the medium has considerable effect on velocity and temperature distribution, both the profiles increase with increase in permeability parameter but it is reversed in the case of concentration profile.

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