

Performance Evaluation of a Rice Mill System having One Conveyer Belt with no Provision for Shutdown Repair

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Abstract: This paper deals with the performance evaluation of a rice mill system involving three essential components viz. rubber roll, separator and a conveyor belt. Only one repairman is taken for repairing the failed units in this paper. Taking failure rates as exponential and arbitrary repair rates, various system effectiveness measures such as transition probabilities, mean time to system failure, availability, busy period of repairman are calculated. At last profit analysis is done on the basis of above measures.

1. Introduction

Reliability occupies a significant role in the present day technology for achieving healthy economic progress of a country. Reliability deals with the proper functioning of equipment and systems and it makes more effective use of resources at hand and increases the productivity without wastage of money, material and manpower. Reliability technology is acquiring special importance in the innovation of new and improved technological systems in the world as far as the important system measures such as performance, cost etc. are concerned. Reliability theory gives a comprehensive and simple treatment to the study of reliability which is considered essential for the correct utilization and maintenance of engineering equipments and systems. The effectiveness of a system is understood to mean the suitability of the system for the fulfillment of the intended tasks and the efficiency of utilizing the means put into it. System engineers and managers are in the

need of such system(s) which have the best reliability aspect in comparison to others. In a competitive world of manufacturing, there is a great pressure on the manufacturers to modify their products quite frequently. So, in order to meet this requirement, manufacturer acquires a feasible production system. To get this feasible production system, there is an urgent need to know all the parameters which are responsible for the production of equipment of high quality or less cost.

The processing on any typical rice mill in the state of Chhattisgarh is usually carried out using these units. In the past, Dillon et al¹ have done a lot of work in the technical field. Kumar et al² have studied a feeding system of a sugar mill and evaluated mean time to system failure and availability of the system using supplementary variable technique. Shridharan et al³ have discussed its application using two non-identical unit parallel system and calculated several economic related measures such as mean time to system failure, point wise availability, busy period of the repairman etc. Singh et al⁴⁻⁷ have done a lot of work on the effect of shock on the models having protective unit.

This model particularly differs from the other models in the sense that the concept of shut down repair is completely ruled out. The shut down repair not only increases the busy period of the repairman but also puts its effect on the profit adversely. Also the involvement of preventive maintenance in the model increases the reliability of the functioning units.

2. About the Model

The rice mill system consists of three units namely rubber roll (R), Separator (S) and a conveyor belt (C). Rubber roll is employed to remove the husk from the paddy which is the input for the rice mill system. The job of the separator is to separate the unpolished rice and husk and this process is repeated a number of times till the husk is completely removed. All this is done with the help of conveyor belt. Initially, all the three units work without load. As soon as a job arrives, all the units work with load. It is assumed that only one job is taken for processing at a time. There is a single repairman who repairs the failed units on first come first served basis, however the conveyor belt gets preference in repair over other units. Using regenerative point technique several system characteristics such as transition probabilities, mean sojourn times, mean time to system failure, availability and busy period of the repairman are evaluated. In the end the expected profit is also calculated.

3. System Description and assumptions

1. The system consists of three units namely, rubber roll R, separator S and conveyor belt C.
2. Only one job is taken for processing at one time till it passes through all the units.
3. A unit can fail during its working with load only.
4. It is assumed that if either of rubber roll or conveyor belt fails, the system goes down. However, the system does not go down if the separator is in repair mode.
5. After repair a unit works as new.
6. After a random time the whole system goes for preventive maintenance.
7. Arrival and processing time of the job are taken to be negative exponential.
8. Failure time distribution of all units are negative exponential whereas the repair time distribution are taken to be arbitrary.

4. Symbols and Notations

E_0 = State of the system at epoch $t=0$

E = set of regenerative states $S_0 - S_7$

λ = Job arrival Rate

$q_{ij}(t)$ = Probability density function (Pdf) of transition time from S_i to S_j

$Q_{ij}(t)$ = Cumulative distribution function (Cdf) of transition time from S_i to S_j

$\pi_i(t)$ = Cumulative dist. function of time to system failure when starting from state $E_0 = S_i \in E$

$\mu_i(t)$ = Mean Sojourn time in the state $E_0 = S_i \in E$

$B_i(t)$ = Repairman is busy in the repair at time $t / E_0 = S_i \in E$

$r_1 / r_2 / r_3$ = Constant repair rate of conveyor belt / rubber roll / separator

$\alpha / \beta / \gamma$ = Failure rate of conveyer belt / rubber roll / separator

$g_i(t) (i = 1 - 3)$ = Pdf of repair time of conveyer belt / rubber roll / separator

$\bar{G}_i(t) (i = 1-3)$ = Cdf of repair time of conveyer belt / rubber roll / separator

$a(t)$ = Probability density function of preventive maintenance

$b(t)$ = Probability density function of preventive maintenance completion time

$\bar{A}(t)$ = Cumulative dist. function of preventive maintenance

$\bar{B}(t)$ = Cumulative dist. function of preventive maintenance completion time

5. Symbols used for state of the system

$C_o / C_g / C_r$ --Conveyer belt under operation /good and non-operative mode/ repair

$R_o / R_g / R_r$ --Rubber Roll under operation / good and non-operative mode/ repair

$S_o / S_g / S_r / S_{wr}$ --Separator under operation / good and non-operative mode/ repair / waiting for repair

P.M.--System under preventive maintenance.

Up States- $S_0 = (C_g, R_g, S_g), S_1 = (C_o, R_o, S_o), S_4 = (C_o, R_o, S_r)$

Down States-

$S_2 = (C_r, R_g, S_g), S_3 = (C_g, R_r, S_g), S_5 = (C_r, R_g, S_{wr}), S_6 = (C_g, R_r, S_{wr})$

6. Transition Probabilities

Simple probabilistic consideration yields the following expression for non-zero transition probabilities

$$1. \quad p_{12} = \frac{\alpha}{\alpha + \beta + \gamma} [1 - a^*(\alpha + \beta + \gamma)]$$

$$2. \quad p_{13} = \frac{\beta}{\alpha + \beta + \gamma} [1 - a^*(\alpha + \beta + \gamma)]$$

$$3. \quad p_{14} = \frac{\gamma}{\alpha + \beta + \gamma} [1 - a^*(\alpha + \beta + \gamma)]$$

$$4. \quad p_{17} = \int_0^{\infty} e^{-(\alpha + \beta + \gamma)t} a(t) dt = a^*(\alpha + \beta + \gamma)$$

$$5. \quad p_{41} = g_3^* (\alpha + \beta)$$

$$6. \quad p_{45} = \frac{\alpha}{\alpha + \beta} [1 - g_3^* (\alpha + \beta)]$$

$$7. \quad p_{46} = \frac{\beta}{\alpha + \beta} [1 - g_3^* (\alpha + \beta)]$$

$$8. \quad p_{01} = p_{21} = p_{31} = p_{54} = p_{64} = p_{70} = 1,$$

and the mean sojourn times are given by

$$9. \quad \mu_0(t) = \int_0^\infty e^{-\lambda t} dt = \frac{1}{\lambda},$$

$$10. \quad \mu_1(t) = \int_0^\infty e^{-x_1 t} \bar{A}(t) dt,$$

$$11. \quad \mu_2(t) = \int_0^\infty \bar{G}_1(t) dt = \mu_5(t)$$

$$12. \quad \mu_3(t) = \int_0^\infty \bar{G}_2(t) dt = \mu_6(t),$$

$$13. \quad \mu_4(t) = \int_0^\infty e^{-x_2 t} \bar{G}_3(t) dt,$$

$$14. \quad \mu_7(t) = \int_0^\infty \bar{B}(t) dt$$

7. Mean time to System failure

Time to system failure can be regarded as the first passage time to the failed state. To obtain it we regard the down state as absorbing. Using the argument as for the regenerative process, we obtain the following recursive relations.

$$(7.1) \quad \pi_0(t) = Q_{01}(t) \boxed{s} \pi_1(t),$$

$$(7.2) \quad \pi_1(t) = Q_{12}(t) + Q_{13}(t) + Q_{14}(t) \boxed{s} \pi_4(t) + Q_{17}(t),$$

$$(7.3) \quad \pi_4(t) = Q_{41}(t) \boxed{s} \pi_1(t) + Q_{45}(t) + Q_{46}(t).$$

Taking Laplace-stiltjes transform of above equation and writing in matrix form; the mean time to system failure when the system starts from the state S_0 is given by

$$(7.4) \quad \text{MTSF} = E(T) = -\left[\frac{d}{ds} \tilde{\pi}_0(s)\right]_{s=0} = \frac{D_1'(0) - N_1'(0)}{D_1(0)} \\ = \frac{\mu_0(1 - p_{14}p_{41}) + \mu_1(1) + \mu_4p_{14}}{1 - p_{14}p_{41}}$$

8. Availability Analysis

Let $M_i(t)$ denote the probability that system is up initially in regenerative state $S_i \in E$, has no transition till time t . Then by probabilistic arguments, we have the following recursive relations giving point wise availability, as follows:

$$(8.1) \quad A_0(t) = M_0(t) + q_{01}(t) \boxed{c} A_1(t),$$

$$(8.2) \quad A_1(t) = M_1(t) + \sum_{i=2,3,4,7} q_{1i}(t) \boxed{c} A_i(t),$$

$$(8.3) \quad A_2(t) = q_{21}(t) \boxed{c} A_1(t)$$

$$(8.4) \quad A_3(t) = q_{31}(t) \boxed{c} A_1(t)$$

$$(8.5) \quad A_4(t) = M_4(t) + \sum_{i=1,5,6} q_{4i}(t) \boxed{c} A_i(t),$$

$$(8.6) \quad A_5(t) = q_{54}(t) \boxed{c} A_4(t),$$

$$(8.7) \quad A_6(t) = q_{64}(t) \boxed{c} A_4(t),$$

$$(8.8) \quad A_7(t) = q_{70}(t) \boxed{c} A_0(t)$$

Thus the steady-state availability of the system when the system starts from $S_i \in E$ is obtained as follows:

$$(8.9) \quad A_0^*(\infty) = \lim_{t \rightarrow \infty} A_0^*(t) = \lim_{s \rightarrow 0} sA_0^*(s) = \frac{N_2(s)}{D_2'(s)}$$

where

$$(8.10) \quad N_2(0) = \mu_0p_{41}p_{17} + \mu_1p_{41} + \mu_4p_{14}$$

$$\begin{aligned}
 (8.11) \quad D_2'(0) = & \mu_0 p_{41} p_{17} + \mu_1 p_{41} + \mu_2 p_{41} p_{12} + \mu_3 p_{13} p_{41} \\
 & + \mu_4 p_{14} + \mu_5 p_{14} p_{45} + \mu_6 p_{14} p_{46} + \mu_7 p_{41} p_{17}.
 \end{aligned}$$

9. Busy Period Analysis

(a) Let $W_i(t)$ denote the probability that the repairman is busy initially with repair in regenerative state S_i and remains busy at epoch t without transiting to any other state or returning to itself through one or more regenerative states.

By probabilistic arguments, we have

$$(9.1) \quad W_2(t) = W_5(t) = \bar{G}_1(t),$$

$$(9.2) \quad W_3(t) = W_6(t) = \bar{G}_2(t),$$

$$(9.3) \quad W_4(t) = \bar{G}_3(t)$$

Developing similar relationship in availability, we have

$$(9.4) \quad B_0(t) = q_{01}(t) \boxed{c} B_1(t),$$

$$(9.5) \quad B_1(t) = \sum_{i=2,3,4,7} q_{1i}(t) \boxed{c} B_i(t),$$

$$(9.6) \quad B_2(t) = W_2(t) + q_{21}(t) \boxed{c} B_1(t),$$

$$(9.7) \quad B_3(t) = W_3(t) + q_{31}(t) \boxed{c} B_1(t),$$

$$(9.8) \quad B_4(t) = W_4(t) + \sum_{i=1,5,6} q_{4i}(t) \boxed{c} B_i(t),$$

$$(9.9) \quad B_5(t) = W_5(t) + q_{54}(t) \boxed{c} B_4(t),$$

$$(9.10) \quad B_6(t) = W_6(t) + q_{64}(t) \boxed{c} B_4(t),$$

$$(9.11) \quad B_7(t) = q_{70}(t) \boxed{c} B_0(t).$$

Taking Laplace- Stieltjes transform of above equations, in the long run the fraction of time for which the repairman is busy in simple repair, we get

$$(9.12) \quad B_0^{1*}(\infty) = \lim_{t \rightarrow \infty} B_0^{1*}(t) = \lim_{s \rightarrow 0} s B_0^{1*}(s) = \frac{N_3(0)}{D_2'(0)}$$

where

$$(9.13) \quad N_3(0) = \mu_2 p_{41} p_{12} + \mu_3 p_{13} p_{41} + \mu_4 p_{14} + \mu_5 p_{14} p_{45} + \mu_6 p_{14} p_{46}$$

(b) Busy period of the repairman in the performing preventive maintenance in $(0, t]$

Let $W_i(t)$ denote the probability that the repairman is busy initially with repair in regenerative state S_i and remain busy at epoch t without transiting to any other state or returning to itself through one or more regenerative states.

By probabilistic arguments, we have

$$(9.14) \quad W_7(t) = \bar{B}(t).$$

Developing similar relationship as in 9(a) and taking Laplace transformation of the equations and writing in matrix form, in the long run the fraction of time for which the repairman is busy in preventive maintenance, we get

$$N_4(s) = W_7^* q_{01}^* q_{17}^* (1 - q_{45}^* q_{54}^* - q_{46}^* q_{64}^*).$$

Let $s \rightarrow 0$, then we get

$$(9.15) \quad N_4(0) = \mu_7 p_{41} p_{17}$$

$$(9.16) \quad B_0^{2*}(\infty) = \lim_{t \rightarrow \infty} B_0^{2*}(t) = \lim_{s \rightarrow 0} s B_0^{2*}(s) = \frac{N_4(0)}{D_2'(0)}.$$

10. Particular cases

When all repair time distribution are n-phase Erlangian distribution i.e.

$$\text{Density function } g_i(t) = \sum_{i=1}^3 \frac{nr_i (nr_i t)^{n-1} e^{-nr_i t}}{n-1!},$$

$$\text{and Survival function } \bar{G}_i(t) = \sum_{j=0}^{n-1} \frac{(nr_i t)^j e^{-nr_i t}}{j!}$$

and other distribution are negative exponential

$$a(t) = \theta e^{-\theta t}, b(t) = \eta e^{-\eta t}, \bar{A}(t) = e^{-\theta t}, \bar{B}(t) = e^{-\eta t}$$

$$\text{For } n=1, \quad g_i(t) = r_i e^{-r_i t}, \quad \bar{G}_i(t) = e^{-r_i t}$$

$$\text{If } i=1, 2, 3 \quad \begin{array}{lll} g_1(t) = r_1 e^{-r_1 t} & g_2(t) = r_2 e^{-r_2 t} & g_3(t) = r_3 e^{-r_3 t} \\ \bar{G}_1(t) = e^{-r_1 t} & \bar{G}_2(t) = e^{-r_2 t} & \bar{G}_3(t) = e^{-r_3 t} \end{array}$$

$$\text{Also } p_{01} = 1 = p_{21} = p_{31} = p_{54} = p_{64} = p_{70},$$

$$p_{12} = \int_0^{\infty} \alpha e^{-x_1 t} e^{-\theta t} dt = \frac{\alpha}{x_1 + \theta}, \quad p_{13} = \frac{\beta}{x_1 + \theta}, \quad p_{14} = \frac{\gamma}{x_1 + \theta}, \quad p_{17} = \frac{\theta}{x_1 + \theta},$$

$$p_{41} = \int_0^{\infty} e^{-(\alpha + \beta)t} r_3 e^{-r_3 t} dt = \frac{r_3}{x_2 + r_3}, \quad p_{45} = \frac{\alpha}{x_2 + r_3}, \quad p_{46} = \frac{\beta}{x_2 + r_3}$$

$$\mu_0 = \frac{1}{\lambda}, \quad \mu_1 = \frac{1}{x_1 + \theta}, \quad \mu_2 = \frac{1}{r_1}, \quad \mu_3 = \frac{1}{r_2},$$

$$\mu_4 = \frac{1}{x_2 + r_3}, \quad \mu_5 = \frac{1}{r_1}, \quad \mu_6 = \frac{1}{r_2}, \quad \mu_7 = \frac{1}{\eta},$$

$$\text{MTSF} = \frac{(\theta + x_1)(r_3 + x_2) - \gamma r_3 + (r_3 + x_2 + \gamma)\lambda}{\lambda[(\theta + x_1)(r_3 + x_2) - (\gamma + r_3)]},$$

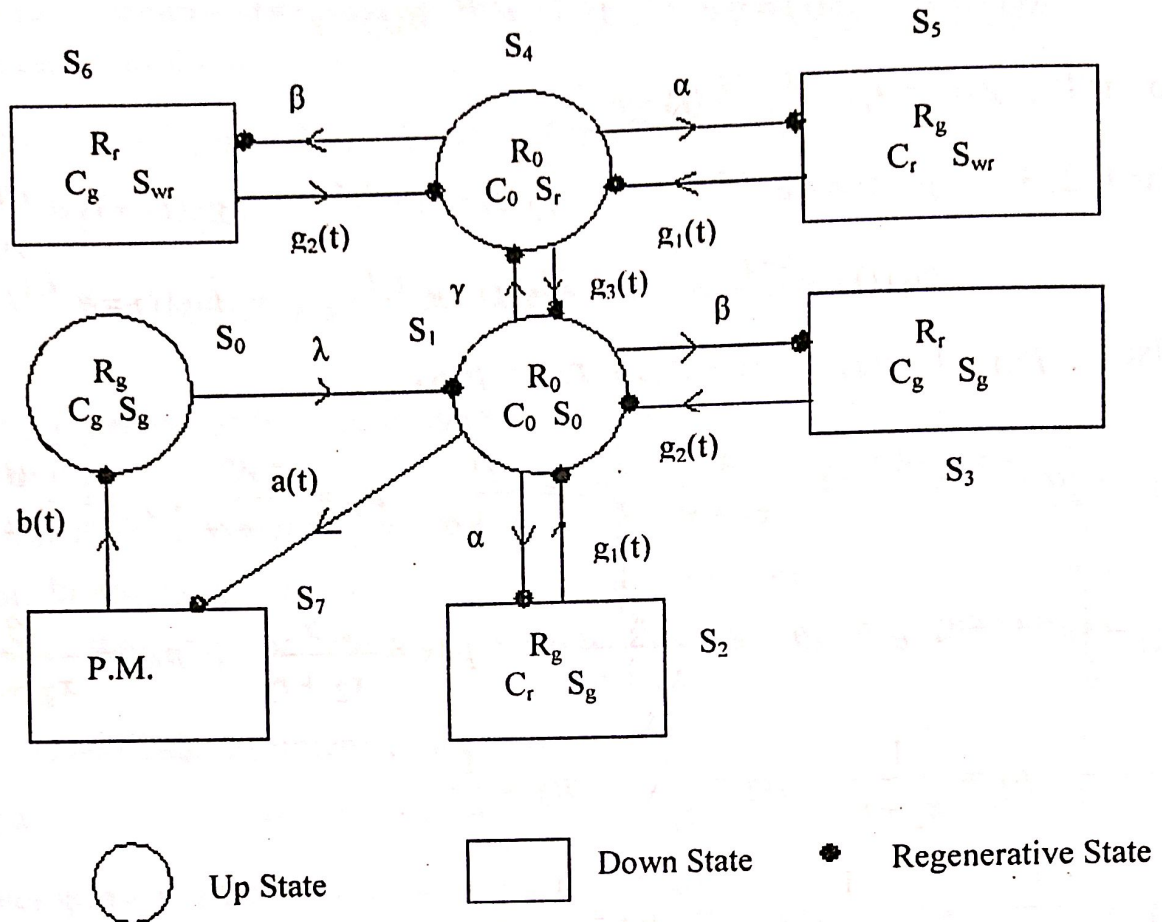
$$\text{Availability } A^*_0(\infty) = \frac{\theta r_3 + (r_3 + \gamma)\lambda}{\theta r_3 + L_3 \lambda},$$

$$\text{Busy Period } B^{1*}_0(\infty) = \frac{\frac{\alpha r_3}{r_1(x_1 + \theta)(x_2 + r_3)} + \frac{\alpha \gamma}{r_2(x_1 + \theta)(x_2 + r_3)}}{\theta r_3 + L_3 \lambda},$$

$$B^{2*}_0(\infty) = \frac{\frac{\theta r_3}{\eta(x_1 + \theta)(x_2 + r_3)}}{\theta r_3 + L_3 \lambda},$$

$$\text{where } L_3 = r_3 + \frac{\alpha r_3}{r_1} + \gamma + \frac{\alpha r_3}{r_1(x_2 + r_3)} + \frac{\beta \gamma}{r_3} + \frac{\theta r_3}{\eta} + \frac{\beta(x_2 + r_3)}{r_2},$$

$$x_1 = \alpha + \beta + \gamma, \quad x_2 = \alpha + \beta.$$



State Transition Diagram

11. Profit Analysis

The profit analysis of the system can be carried out by considering the expected busy period of the repairman in repair of the unit in $(0, t]$. Therefore

$G(t)$ = Expected total revenue earned by the system in $(0, t]$ - Expected repair cost of the failed units - Expected repair cost of the repairman in preventive maintenance.

$$= C_1 \mu_{up}(t) - C_2 \mu_{b1} - C_3 \mu_{b2}$$

$$\text{where } \mu_{up}(t) = \int_0^t A_0(t) dt; \mu_{b1}(t) = \int_0^t B_0^1(t) dt; \mu_{b2}(t) = \int_0^t B_0^2(t) dt,$$

C_1 is the revenue per unit time and C_2, C_3 are the cost per unit time for which the Rice mill is under simple repair, preventive maintenance respectively.

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