

Non Existence of Scalar Meson Field in Presence of Cosmological Constant in the Bianchi Type-1 Self Creation Cosmology

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Abstract: In the modified theory of general relativity proposed by Barber¹, it is shown that for a spatially homogeneous anisotropic Bianchi type-1 space-time, the massless scalar field ψ does not generate solutions when cosmological constant $\Lambda > 0$. However, the same scalar field generates solutions when $\Lambda < 0$ but the real physical model of the universe does not exist.

Key words: Scalar meson field, modified theory, cosmological constant.

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1. Introduction

Recently there has been a lot of interest in the cosmological term Λg_{ij} in context of quantum field theories, quantum gravity, super gravity theories, Kaluza-klein theories and the inflationary universe scenario. The cosmological term which is measure of the energy of empty space, provides a repulsive force opposing the gravitational pull between the galaxies. If it exists the energy it represents counts as mass because Einstein has shown that mass and energy are equivalent. Moreover, if the cosmological constant is large then the energy involved due to the matter in the universe can sum up to the number that the inflation predicts. In this inflationary era it is quite important to study the cosmological constant. But recent research suggests

that the cosmological term corresponds to a very small value of the order 10^{-58} cm^{-2} (Jhori and Chandra²).

The scalar meson fields represent matter fields with spin less quanta. There are two types of scalar fields viz. zero rest mass scalar fields and massive scalar fields. The zero rest mass scalar fields describe long range interactions, whereas massive scalar fields describe short range interactions. The study of scalar meson fields in general relativity has a focal point for the researchers due to its physical importance in particle physics. The massless scalar field in relativistic mechanics yields some significant results as regards to the singularities. The scalar meson field being a field of a single variable v (say) is the special case of general field and the expression given by

$$T_{ij} = v_i v_j - \frac{1}{2} g_{ij} (v_k v^k - m^2 v^2)$$

is the energy-momentum tensor of Yukawa³ fields (spin zero meson particle) for the metric of (+2) signature in flat space time. The Klein-Gorden equation takes the form

$$g^{ij} v_{;ij} + m^2 v = 0,$$

where v is the real scalar field and m is the rest-mass of scalar meson field. Here (;) semicolon followed by an index denotes covariant differentiation. When $m = 0$, the scalar field v is known as massless scalar field.

It is known that general theory of relativity serves as a basis to construct mathematical models of the universe. Though the theory exists, it has many controversies. Thus several modifications of this theory have been proposed by researchers to unify gravitation and other effects in the universe. Barber¹ is one of the researchers proposed a theory known as the variable G-theory by modifying general relativity. In this modified theory the scalar field does not gravitate directly but simply divides the matter tensor acting as a reciprocal gravitational constant. Pimentel⁴, Soleng^{5,6}, Reddy⁷, Reddy and Venkateswarlu⁸, Venkateswarlu and Reddy⁹, Shanti and Rao¹⁰, Carvalho¹¹, Shri Ram and Singh¹², Mohanty et al.¹³⁻¹⁵, Panigrahi and Sahu¹⁶⁻¹⁸, Sahu and Panigrahi¹⁹, Panigrahi et al.²⁰ and recently Sahu and Mohanty²¹ are some of the authors, who have studied this modified theory in different angles.

In the present paper we have extended the earlier work of Mohanty et al.¹³ to the case of scalar meson field with cosmological constant Λ . The beauty of the work reported here is that cosmological micro models do not exist for both $\Lambda < 0$ and $\Lambda > 0$. Also it is interesting to see that massless scalar field v does not survive in both the cases.

2. Einstein-Barber's Field Equations

Here we consider the space time described by the spatially homogeneous anisotropic Bianchi type-1 metric

$$(1) \quad ds^2 = -dt^2 + A^2 dx^2 + B^2 dy^2 + C^2 dz^2,$$

where A, B and C are functions of cosmic time 't' only.

The Einstein-Barber's relativistic field equations for massless scalar field with cosmological term Λg_{ij} can be written as

$$(2) \quad G_{ij} \equiv R_{ij} - \frac{1}{2} g_{ij} R + \Lambda g_{ij} = -8\pi\phi^{-1} T_{ij}^v$$

and

$$(3) \quad \square\phi = \frac{8\pi\lambda}{3} T^v$$

where ϕ is the Barber's Scalar field, $\square\phi$ is the d' Alembertian operator, T^v is the trace of the energy momentum tensor T_{ij}^v and λ is the coupling constant to be determined from the experiment and $0 < |\lambda| < 10^{-1}$. In the limit $\lambda \rightarrow 0$, this modified theory approaches the standard general relativity theory in every respect and $G = \phi^{-1}$. Because of the homogeneity condition imposed by the space time (1), ϕ is a function of cosmic time t only. In this paper we are interested to study the cosmological effects in microscopic level only. So we consider the region of the space -time consisting massless scalar meson field whose energy momentum tensor T_{ij}^v (Singh and Deo²²) is given by

$$(4) \quad T_{ij}^v = v_i v_j - \frac{1}{2} g_{ij} v_k v^k$$

together with

$$(5) \quad g^{ij} v_{;ij} = \sigma,$$

where the mass parameter $m = 0$ and σ is the source density of the scalar meson field.

The scalar field v is a function of cosmic time t due to the homogeneous nature of the space time and so also the source density σ . Here the equation (2) connects the distribution of matter and energy with the geometry of the space -time by relating the energy momentum tensor T_{ij}^v to the fundamental

metric tensor g_{ij} and its derivatives. It is the business of relativistic mechanics to investigate with the help of this equation, the principle which govern the energy momentum tensor and hence determine the behavior of energy and matter.

Now the field equations (2) and (3) for the metric (1) are obtained as

$$(6) \quad \frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} + \Lambda = -4\pi \phi^{-1} v_4^2,$$

$$(7) \quad \frac{C_{44}}{C} + \frac{C_{44}}{A} + \frac{C_4 A_4}{CA} + \Lambda = -4\pi \phi^{-1} v_4^2,$$

$$(8) \quad \frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} + \Lambda = -4\pi \phi^{-1} v_4^2,$$

$$(9) \quad \frac{A_4 B_4}{AB} + \frac{B_4 C_4}{BC} + \frac{C_4 A_4}{CA} + \Lambda = 4\pi \phi^{-1} v_4^2,$$

$$(10) \quad \phi_{44} + \phi_4 \left[\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} \right] = -\frac{8\pi}{3} \lambda v_4^2.$$

Here and afterwards the subscript '4' after a field variable represents ordinary differentiation with respect to time t .

Adding equations (6), (7) and (8) to three times of equation (9), we find

$$(11) \quad (ABC)_{44} + 3\Lambda ABC = 0.$$

3. Solutions

In order to solve the field equations (6) - (10) for exact and explicit solutions, we consider the following three cases i.e.:

(i) $\Lambda = 0$, (ii) $\Lambda > 0$ and (iii) $\Lambda < 0$.

Case I: When $\Lambda = 0$.

In this case the results reduce to that of Mohanty et al.¹³.

Case II: When $\Lambda > 0$.

In this case integrating equation (11), we get

$$(12) \quad ABC = \alpha_1 \cos \sqrt{3\Lambda} t + \alpha_2 \sin \sqrt{3\Lambda} t$$

where α_1 and α_2 are arbitrary constants of integration.

From equation (12), we can write the explicit form for A, B and C as

$$(13) \quad \begin{cases} A = (\alpha_1 \cos \sqrt{3\Lambda}t + \alpha_2 \sin \sqrt{3\Lambda}t)^{m_1}, \\ B = (\alpha_1 \cos \sqrt{3\Lambda}t + \alpha_2 \sin \sqrt{3\Lambda}t)^{m_2} \\ \text{and} \\ C = (\alpha_1 \cos \sqrt{3\Lambda}t + \alpha_2 \sin \sqrt{3\Lambda}t)^{m_3} \end{cases}$$

where m_i ($i = 1, 2, 3$) are real constants and satisfying the relation

$$(14) \quad (m_1 + m_2 + m_3) = \sum_{i=1}^3 m_i = 1.$$

By substitution of the values of A, B and C from equation(13) in equation(9), we obtain

$$(15) \quad \sum_{\substack{i,j=1 \\ i \neq j}}^3 m_i m_j = \left[\frac{8\pi\phi^{-1}v_4^2}{3\Lambda} \right] \cdot \left[\frac{\alpha_1 \cos \sqrt{3\Lambda}t + \alpha_2 \sin \sqrt{3\Lambda}t}{\alpha_2 \cos \sqrt{3\Lambda}t - \alpha_1 \sin \sqrt{3\Lambda}t} \right]^2.$$

As m_i , $i = 1, 2, 3$ are constants, so $\sum_{\substack{i,j=1 \\ i \neq j}}^3 m_i m_j$ is a constant which is the

L.H.S. of equation (15). Again as ϕ and v are both time-dependent, so first part of the product of the R.H.S. of equation (15) can be set equal to a constant. Also the second part of the product of the R.H.S. of equation (15) cannot be set equal to a constant as the coefficients of $\cos \sqrt{3\Lambda}t$ or $\sin \sqrt{3\Lambda}t$ in numerator and denominator are not same. Hence R.H.S. of equation (15) cannot be equal to a constant. Therefore this relation cannot hold in equation (15) due to the reality condition.

Thus for $\Lambda > 0$, we cannot obtain the solutions of the field equations. Hence the real physical model of the universe in modified theory of general relativity does not exist when $\Lambda > 0$.

Case III: When $\Lambda < 0$ say $\Lambda = \frac{-\omega^2}{3}$. In this case equation (11) reduces to

$$(16) \quad (ABC)_{44} - \omega^2 ABC = 0..$$

On integration, equation (16) yields

$$(17) \quad ABC = \alpha_3 e^{\omega t} + \alpha_4 e^{-\omega t},$$

where α_3 and α_4 are arbitrary constants of integration.

Now from equation(17), we can write the explicit form for A,B and C as

$$(18) \quad \begin{cases} A = (\alpha_3 e^{\omega t} + \alpha_4 e^{-\omega t})^{r_1}, \\ B = (\alpha_3 e^{\omega t} + \alpha_4 e^{-\omega t})^{r_2}, \\ \text{and} \\ C = (\alpha_3 e^{\omega t} + \alpha_4 e^{-\omega t})^{r_3}, \end{cases}$$

where r_i ($i = 1, 2, 3$) are real constants satisfying the condition

$$(19) \quad (r_1 + r_2 + r_3) = \sum_{i=1}^3 r_i = 1.$$

Putting the values of A, B and C from equation (18) in equation (9), we get

$$(20) \quad \sum_{\substack{i,j=1 \\ i \neq j}}^3 r_i r_j = \left[\frac{\alpha_3 e^{\sqrt{-3\Lambda}t} + \alpha_4 e^{-\sqrt{-3\Lambda}t}}{\alpha_3 e^{\sqrt{-3\Lambda}t} - \alpha_4 e^{-\sqrt{-3\Lambda}t}} \right]^2 \cdot \left[\frac{4\pi\phi^{-1}v_4^2 - \Lambda}{-3\Lambda} \right].$$

Since r_i ($i = 1, 2, 3$) are real constants, so

$$(21) \quad \sum_{\substack{i,j=1 \\ i \neq j}}^3 r_i r_j = \text{a real constant.}$$

Now three sub cases do arise in view of equation (20).

Sub Case-1:

When

$$(21) \quad \text{a) } \alpha_3 \neq 0, \quad \text{b) } \alpha_4 = 0 \quad \text{and} \quad \text{c) } \frac{4\pi\phi^{-1}v_4^2 - \Lambda}{-\Lambda} = 1,$$

then, equation (20) reduces to

$$(22a) \quad \sum r_i r_j = \frac{1}{3}.$$

Thus from equation (21c), we obtain

$$(22) \quad v = \alpha_5$$

where α_5 is a constant of integration. Now equations (19) and (22a) lead to

$$(23) \quad r_1 = r_2 = r_3 = \frac{1}{3}.$$

Subsequently in view of equation (22a), the explicit expressions of A, B and C are

$$(24) \quad A = B = C = (\alpha_3 e^{\sqrt{-3\Lambda}t})^{1/3}.$$

Using equations (24) and (22) in equation (10), we obtain

$$(25) \quad \phi_{44} + \omega \phi_4 = 0.$$

On integration, equation (25) yields

$$(26) \quad \phi = \frac{\alpha_6}{-\sqrt{-3\Lambda}} \cdot \frac{1}{e^{\sqrt{-3\Lambda}t}} + \alpha_7,$$

where $\alpha_6 (\neq 0)$ and α_7 are constants of integration.

The energy density associated with ν (Anderson²³) is given as

$$(27) \quad \rho = \frac{1}{2} \nu_4^2.$$

The source density σ of the scalar field ν given by equation (5) is

$$(28) \quad \sigma = -\nu_{44} - \nu_4 \frac{(ABC)_4}{ABC}.$$

Using equation (22) in (27) and (28), we obtain

$$(29) \quad \rho = 0$$

and

$$(30) \quad \sigma = 0.$$

Thus the corresponding metric of our solution can be expressed as

$$(31) \quad ds^2 = -dt^2 + (\alpha_3 e^{\sqrt{-3\Lambda}t})^{2/3} (dx^2 + dy^2 + dz^2).$$

Sub case 2:

When

$$(32) \quad \text{a) } \alpha_3 = 0, \text{ b) } \alpha_4 \neq 0 \text{ and c) } \frac{4\pi\phi^{-1}\nu_4^2 - \Lambda}{-\Lambda} = 1,$$

then equation (20) reduces to $\sum_{i,j=1, i \neq j}^3 r_i r_j = \frac{1}{3}$, which is same as equation (22a).

Thus proceeding as in sub case - 1, we find

$$\nu = \alpha_5, \quad r_1 = r_2 = r_3 = 1/3 \text{ and}$$

$$(33) \quad A = B = C = (\alpha_4 e^{-\sqrt{-3\Lambda}t})^{1/3}.$$

Using $\nu = \alpha_5$ and equation (33) in equation (10), we obtain

$$(34) \quad \phi_{44} - \omega \phi_4 = 0.$$

On integration, equation (34) yields

$$(35) \quad \phi = \frac{\beta e^{\sqrt{-3\Lambda}t}}{\sqrt{-3\Lambda}} + \beta_1,$$

where $\beta \neq 0$ and β_1 are constants of integration. As in sub case -1, here also energy density ρ and source density σ become zero.

Hence the geometry of the space time (1) takes the form

$$(36) \quad ds^2 = -dt^2 + (\alpha_4 e^{-\sqrt{-3\Lambda}t})^{2/3} (dx^2 + dy^2 + dz^2).$$

Sub Case 3:

$$\text{If } \alpha_3 \neq 0, \alpha_4 = 0 \text{ and } \frac{4\pi\phi^{-1}v_4^2 - \Lambda}{\Lambda} = 1$$

or

$$\alpha_3 = 0, \alpha_4 \neq 0 \text{ and } \frac{4\pi\phi^{-1}v_4^2 - \Lambda}{\Lambda} = 1,$$

then equation (20) reduces to $\sum r_i r_j = -1/3 < 0$.

But for the reality of r , it is necessary that $r_1 r_2 + r_1 r_3 + r_2 r_3 \geq 0$.

So in this case we cannot obtain the real physical model of the universe.

It is interesting to see that the energy density ρ and the source density σ associated with massless scalar field v in the model (31) and (36) are each zero. Also it is interesting to see that the scalar field v in the model (31) and (36) becomes a constant. Hence it is inferred that the real physical model of the universe does not exist for $\Lambda < 0$.

4. Conclusion

In this paper, we have taken an attempt to construct micro models for anisotropic spatially homogeneous Bianchi type-I space time for two different cases corresponding to cosmological constant Λ . In both the cases it is found that the real physical model of the universe does not exist. Also it is found that massless scalar field v does not exist. Hence Bianchi type-I cosmological micro models in modified theory of general relativity do not exist when cosmological constant Λ is not equal to zero.

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References

1. G. A. Barber, *Gen Real Gav.* 14 (1982) 117.
2. V. B Jhori and R. Chandra, *J. Math. Phys. Sci. India* 17 (1983) 73.
3. H. Yukawa, *Modern Physics* (edited by J.C. Slater), *Asian Student Edition, McGraw Hill Book Inc.*, 301 (1935).
4. L. O. Pimentel, *Astrophys. Space Sci.*, 116 (1985) 395.
5. H.H. Soleng, *Astrophys. Space Sci.*, 137 (1987a) 373.
6. H. H. Soleng, *Astrophys. Space Sci.*, 139 (1987b) 13.
7. D. R. K. Reddy, *Astrophys. Space Sci.*, 133 (1987) 189.
8. D. R. K. Reddy and R. Venkateswarlu, *Astrophys. Space Sci.*, 155 (1989) 135.
9. R. Venkateswarlu and D. R. K.Reddy, *Astrophys. Space Sci.*, 168 (1990) 193.
10. K. Shanti and V. U. M. Rao, *Astrophys. Space Sci.*, 179 (1991) 147.
11. J. C. Carvalho, *Int. J. Theo. Phys.*, 35 (1996) 2019.
12. Shri Ram and C. P. Singh, *Astrophys. Space Sci.*, 257 (1998) 123.
13. G. Mohanty, U. K. Panigrahi and R. C. Sahu, *Astrophys. Space Sci.*, 281 (2002) 633.
14. G. Mohanty, R. C. Sahu and U. K. Panigrahi, *Astrophys. Space Sci.*, 284 (2003) 1055.
15. G. Mohanty, R. C. Sahu and U. K. Panigrahi, *Journal of Orissa Mathematical Society*, 23-24 (2004-05) 293.
16. U. K. Panigrahi and R. C. Sahu, *National Academy Science Letters*, 25 (11-12) (2002).
17. U. K. Panigrahi and R. C. Sahu, *Theoret. Appl. Mech.*, 30 (2003) 163.
18. U. K. Panigrahi and R. C. Sahu, *Czech Journal of Physics.*, 54(5) (2004).
19. R. C. Sahu and U. K. Panigrahi, *Astrophysics. Space Sci.*, 288 (2003) 601.
20. U. K. Panigrahi, R. N. Patra and R. C. Sahu, *Science Journal of GVISH.*, 11 (2005) 1-5.
21. R. C. Sahu and G. Mohanty, *Astrophys. Space Sci.*, 306 (2006) 179.
22. R. K .T. Singh and Shridhar Deo, *Acta Physica Hungarica.*, 59 (1986) 321.
23. J. L. Anderson, *Principles of Relativity Physics*, (1967) 289.