

Mathematical Modeling of Two Layered Non-Newtonian Blood Flow Through Artery in Presence of Stenosis

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Abstract: The aim of the present study is to examine a mathematical model of two phased blood flow in human pulmonary artery in the presence of stenosis, keeping in view the nature of pulmonary blood circulation during Lung Cancer. Some of the previous researchers have already considered the blood flow to be of two phase. We have applied power law model (based on non-Newtonian flow) in biofluid mechanical set up. For the purpose, blood has been assumed to be constituted of plasma and blood cells which is realistic so far. The pressure drop along the length of stenosis has been calculated. In present study overall presentation is in tensorial form and the solution technique adopted is analytical as well as numerical. The role of Hematocrit is explicit in determination of blood pressure drop.

Keywords: Mathematical modeling, non-Newtonian blood flow, human pulmonary artery, stenosis, Hematocrit.

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1.Introduction

Lung Cancer is a leading cause of death worldwide. Out of estimated 14.1 million cancer cases around the world in 2012, 7.4 million cases were in men and 6.7 million in women. This number is expected to increase to 24 million by 2035. Lung cancer was the most common cancer worldwide contributing 17% of the total number of new cases diagnosed in 2012. The top three, lung, prostate and colorectal cancers, contributed nearly 42% of all cancers. There were 268 cases of cancer diagnosed per 100,000 in more

developed regions, compared to 148 in less developed regions in 2012 (Cancer Incidence and Mortality Worldwide¹).

A human artery is a blood vessel that conduces blood away from the heart. All arteries have relatively thick walls that can withstand the high pressure of blood ejected from the heart².

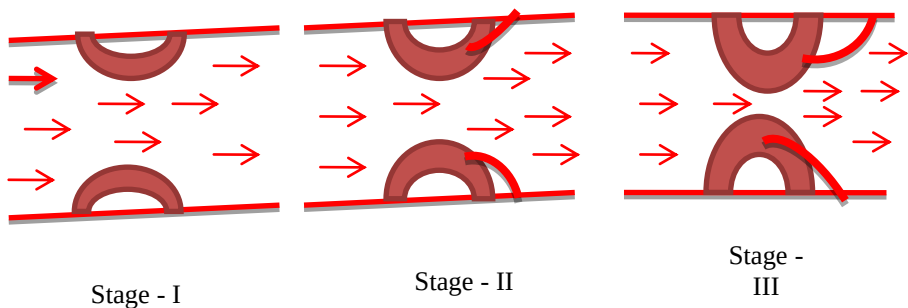
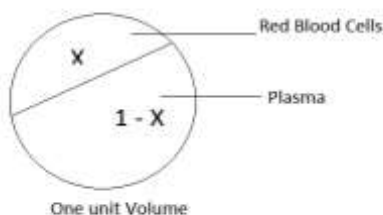


Figure 1. Three stage of Stenosis growth

Stenosis is a narrowing of pulmonary (blood vessels) arteries in the human body. High cholesterol could contribute to the stenosis of an artery when it accumulates on the inner wall of the artery. This accumulation, referred to as atherosclerosis, can build up within the artery to the point where it reduces blood flow to organs of the body. When blood flow is reduced, nutrients and oxygen cannot travel. The presence of stenosis can lead to serious circulatory disorders. There is strong evidence that hydrodynamic factors such as resistance to flow, wall shear stress and apparent viscosity may play a vital role in the development and the progression of arterial stenosis³.

According to Sherman, & Sherman⁴ The first and foremost reason is that the blood is not an ideal fluid but it is a mixture of the two phases one plasma and other one blood cells. These blood cells, semi permeable packages of liquid of a density greater than that of plasma, are capable of changing their shape and size while flowing through different blood vessels. Plasma is a liquid containing semi permeable packages of RBCs. The behavior of blood is almost Newtonian at high shear rate, while at low shear rate the blood exhibits yield stress and non-Newtonian behavior⁵. The flow of blood is affected by the presence of blood cells. This effect is directly proportional to the volume occupied by blood cells.



According to Upadhyay and Chaturvedi⁶, Kumar et al.⁷ the volume portion covered by the plasma of one unit volume containing red blood cells x is $(1-x)$. If mass ratio of cells to plasma is r then clearly: $r = \frac{x\rho_c}{(1-x)\rho_p}$, where

ρ_c and ρ_p are densities of blood cells and plasma respectively. Usually this mass ratio is not a constant. Even then this may be supposed as constant in present context⁸.

2. Mathematical Formulation

According to Upadhyay and Pandey⁹⁻¹⁰, we take the blood flow in pulmonary arteries remote from heart to be non-Newtonian power law model. It is also supposed to be steady. The stenosis is taken to be axially-symmetric whose surface is given by the following equation

$$\frac{R(z)}{R_0} = 1 - \frac{\delta}{2R_0} \left(1 + \frac{\cos \pi z}{z_0} \right), \quad -z_0 \leq z \leq z_0.$$

We shall assume further that $\frac{\delta}{R_0} \ll 1$ and $R_e \frac{\delta}{R_0} \ll 1$, where R_e the

Reynolds number of fluid flow. Now we write the basic equations of blood flow in tensorial form as follows:

Equation of continuity and Equation of motion are

$$\frac{1}{\sqrt{g}} \left(\sqrt{g} v^i \right)_{,i} = 0$$

$$\rho_m \frac{\partial v^i}{\partial t} + (\rho_m v^i) v_{,j}^i = T_{,j}^{ij}$$

The constitutive equations for the power law non-Newtonian blood flow is as follows

$$T^{ij} = -pg^{ij} + \eta_m (e^{ij}) + T^{ij}.$$

By caring out the order of magnitude analysis on these basic equations of flow transferred into cylindrical form, it can be observed that the radial velocity can be neglected in relation to axial velocity v which is determined by

$$(2.1) \quad -\frac{\partial \rho}{\partial r} = 0,$$

$$(2.2) \quad -\frac{\partial p}{\partial z} + \frac{\eta_m}{r} \frac{\partial}{\partial r} \left\{ r \left(\frac{\partial v}{\partial r} \right)^n \right\}.$$

We have used here the no slip condition on the stenosis surface given by

$$(2.3) \quad \begin{aligned} v &= 0 \text{ at } r = R(z), \quad -z_0 \leq z \leq z_0 \\ v &= 0 \text{ at } r = R_0 |z| \geq z_0 \end{aligned}$$

3. Solution

Pressure gradient $P = -\frac{\partial p}{\partial z}$ and axial velocity are functions of z also.

In view of these, we obtained equations

$$(3.1) \quad -P(z) = \frac{\eta_m}{r} \frac{\partial}{\partial r} \left\{ r \left(\frac{\partial v}{\partial r} \right)^n \right\}.$$

Integrating we get

$$(3.2) \quad r \left(\frac{\partial v}{\partial r} \right)^n = -\frac{P(z) r^2}{2\eta_m} + A(z).$$

Applying the boundary conditions, the equation is transformed into following form

$$(3.3) \quad r \left(\frac{\partial v}{\partial r} \right)^n = - \frac{P(z)r^2}{2\eta_m} - \frac{\partial v}{\partial r} = \left(\frac{P(z)r}{2\eta_m} \right)^{\frac{1}{n}}.$$

Integrating this equation we get,

$$(3.4) \quad v = - \left(\frac{P(z)r}{2\eta_m} \right)^{\frac{1}{n}} \frac{n}{n+1} r^{\frac{1}{n}+1} + B(z).$$

Applying the boundary condition we get the value arbitrary constant

$$(3.5) \quad B(z) = \frac{P(z)}{2\eta_m} \frac{n}{n+1} R(z)^{\frac{1}{n}+1}$$

Hence the equation (3.5) reduces in the form of

$$(3.6) \quad v = \frac{n}{n+1} \left(\frac{P(z)r}{2\eta_m} \right)^{\frac{1}{n}} \left(R(z)^{\frac{1}{n}+1} - r^{\frac{1}{n}+1} \right)$$

The formula (3.5) gives the velocity of blood flow passing through stenosis.

$$Q = \int_0^{R(z)} \frac{n}{n+1} \left(\frac{P(z)r}{2\eta_m} \right)^{\frac{1}{n}} \left(R(z)^{\frac{1}{n}+1} - r^{\frac{1}{n}+1} \right) 2\pi dr$$

$$Q = \frac{\pi n}{3n+1} \times \left(\frac{P(z)}{2\eta_m} \right)^{\frac{1}{n}} (R(z))^{\frac{1}{n}+3}$$

The total flow-flux of blood through the transverse section of the arteries is given as

(3.7)

$$Q=\int_0^R v.2\pi r dr,$$
$$Q=\int_0^{R(z)} \frac{n}{n+1}\left(\frac{P(z)r}{2\eta_m}\right)^{\frac{1}{n}}\left(R(z)^{\frac{1}{n}+1}-r^{\frac{1}{n}+1}\right)2\pi dr,$$
$$Q=\frac{\pi n}{3n+1}\times\left(\frac{P(z)}{2\eta_m}\right)^{\frac{1}{n}}(R(z))^{\frac{1}{n}+3}.$$

4. Result and Discussion

Now according to Kumar et.al.⁷, we have $Q=425\frac{ml}{min}$, $Q=0.00708333m^3/sec$ and $R=1, r_p=\frac{1}{3}$. According to Gustafson and Daniel ¹¹ $\eta_p=0.0013$ pascal second. According to Glenn ^{Eler12} $\eta_m=0.0271$ pascal second. Pulmonary artery average length $(z_f-z_i)=0.15cm$ or $0.15\times10^{-9}m^3$ (According to J.T. Ottesen, M. S. Olufsen, and J. K. Larsen¹³). If 20 % stenosed

Table 1. Hemoglobin & blood pressures in clinical data

Date	HB (Hemoglobin) in (gram/dl)	Hematocrit In (3×HB) (kg/I)	Blood Pressure(BP) in (mmhg)	Arteries Pressure Drop In Pascal-second
1/2/2017	13.4	0.037924528	110/70	-2662.64
5/4/2017	13.1	0.037075472	100/70	-1996.98 s
17/5/2017	12.9	0.036509434	130/80	-3328.3
3/6/2017	12.7	0.035943396	140/85	-3661.13
27/6/2017	12.6	0.03566037	110/65	-2995.47
18/7/2017	12.3	0.03481132	110/70	-2662.64

If 40 % stenosed
 $\delta=40\%$ of radius of pulmonary artery $=0.015\times0.4=0.006m$
 $R(z)=R_0-\delta=0.015-0.006=0.009m$. In according to used clinical data (Table I.) (Hematocrit) $H=0.036509434$ and Pressure drop $(P_f-P_i)=3328.3$ pascal

second. Using relation $\eta_m=\eta_c X+\eta_p(1-X)$ and get η_c , $\eta_c=70.66796659$ Pascal second. Again from relation $\eta_m=\eta_c X+\eta_p(1-X)$ again and change in to the hematocrit form

$$\eta_m=0.7066796659H+0.001299525 \; .$$

Now putting all values in equation (3.7) and find n , solving by Numerical method (using Trial and Error Method)

$$0.00708333=\frac{\pi n}{3n+1}\times\left(\frac{33283}{2\times0.0271\times0.05}\right)^{\frac{1}{n}}(0.009)^{\frac{1}{n}+3}.$$

$n=1.0118392$, now apply $n=1.0118392$, again using equation

$$Q=\frac{\pi n}{3n+1}\times\left(\frac{P(z)}{2\eta_m}\right)^{\frac{1}{n}}(R(z))^{\frac{1}{n}+3} \text{ and get } \Delta p$$

$$0.00708333=\frac{1.0118392\times3.14}{3\times1.0118392+1}\times\left(\frac{P(z)}{2\eta_m}\right)^{\frac{1}{n}}(0.009)^{\frac{1}{1.0118392}+3}\times(0.009)^3,$$

$$\Delta p=2\eta_m\times(z_f-z_i)\times1715106.411,$$

$$\Delta p=\eta_m\times1715106.411,$$

$$\Delta p=(1715106.411)(0.7066796659H+0.001299525),$$

$$\Delta p=121203.0826H+222.8824306 \; .$$

Table 2 Blood pressures drop v/s Hematocrit in Clinical data

Date	Hematocrit in ($3\times HB$ (kg/m ³))	BPD (Blood Pressure drop) In Pascal-second
1/2/2017	0.037924528	4819.452129
5/4/2017	0.037075472	4716.543925
17/5/2017	0.036509434	4647.938374
3/6/2017	0.035943396	4579.332824
27/6/2017	0.035660377	4545.030048
18/7/2017	0.034811321	4442.121844

Graph: a. (table I.) and b. (table II.)



Figure 2. Real Clinically BPD

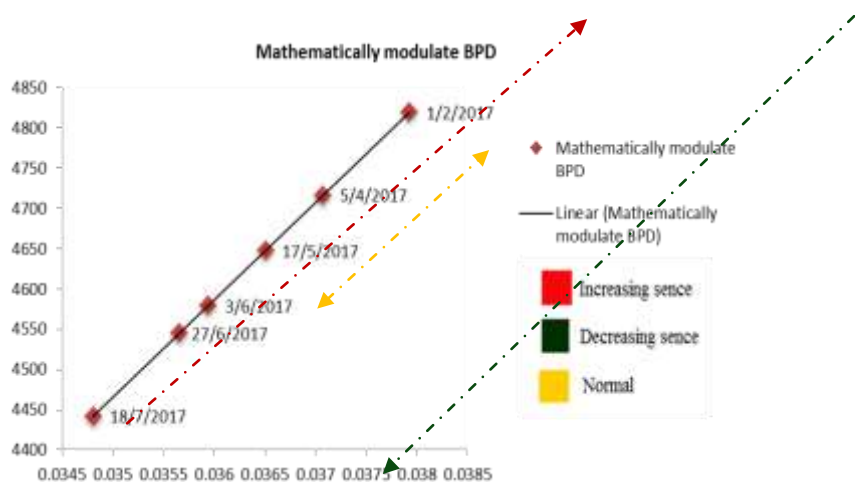


Figure 3. Mathematically Modulate BPD

5. Observation

Graph (a) shows that these 6 different dates were observed minimum about 1996.98 on dated 5/4/2017 and maximum value obtain 3661.13 on dated 3/6/2017 (BDP). The value from 0.034811321 to 0.035660377 of hematocrit value, the blood pressure drop shows slightly increasing sence and the value from 0.035660377 to 0.036509434 via 0.035943396 of hematocrit value, the blood pressure drop shows upper convex in decreasing sence but the value from 0.036509434 to 0.037924528 via 0.037075472

hematocrit value, the blood pressure drop shows down convex in increasing sence. Graph (b) shows that these 6 different dates were observed minimum about 4442.121844 on dated 18/7/1/2017 and maximum value obtains 4819.452129 on dated 1/2/2017 (BPD). At the value from 0.037924528 to 0.034811321 via 0.037075472, 0.036509434, 0.035943396 & 0.035660377 of hematocrit value, the blood pressure drop straightly decreases on dated 1/2/2017 to 18/7/2017 via 5/4/2017, 17/5/2017, 3/6/2017 & 27/6/2017.

6. Conclusion

If 40 % stenosed pulmonary artery presented figure; graph b (table. II.) shows from 1/2/2017 to 18/7/2017 via 5/4/2017, 17/5/2017, 3/6/2017 & 27/6/2017 decreasing sence and again. When graph shows increasing sence (in different vessels on different cases) then we cannot suggest for serious dose and when graph shows decreasing sence then we suggest for serious dose but according to steepness of slops (triad line) at different condition (critical, middle, normal). We have suggested for successful operation but subject to the condition that the clinical data is collected in the duration of declared operation.

7. Achieves of Research Work

With the help of a medical expert and a computer software expert, through our research, we can create such software. With the help of which we can find out the trend of blood fluctuation of patients in a few seconds which will be very helpful in the operation and the likelihood of dying after the impression of the patients or after the operation can be reduced - (A suggested project work).

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References

1. . Cancer Incidence and Mortality Worldwide: IARC Cancer Base No. 11. Lyon, France: International Agency for Research on Cancer; 2014. Available from: <http://globocan.iarc.fr>, accessed on 16/01/2015.
2. Browne Donna, *Cardivascular System Module 5: Structure and Function of Blood Vessels*, The Connexions Project and licensed under the Ceative Commons Attribution License 4., 2014.

3. Pankaj Mathur and Surekha Jain, Mathematical Modeling of Non-Newtonian Blood Flow Through Artery in the Presence of Stenosis, *Int. Research Pub. House*, **4 (1)** (2013) 1-12.
4. I. J. Sherman and V. G. Sherma, Biology- a human approach, *oxford univ. press New York, oxford*, (1989), 276-277.
5. Sapn Shah and Ratan, Mathematical analysis of blood flow throw atherosclerotic arterial segment having non-symmetric and mild stenosis, *International journal of research in pure & applied physics*, (2011).
6. V. Upadhyay, S. K. Chaturvedi and A. Upadhyay, A Mathematical Model on Effect of Stenosis in Two Phase Blood Flow in Arteries Remote from the Heart, *Journal of International Academy of Physical Sciences*, **16(3)** (2012), 247-257.
7. Dheerendra Kumar, Surya Kant Chaturvedi, V. Upadhyay and P. N. Pandey, Analysis of Two Phase Blood Flow in Human Pulmonary Artery During Lung Cancer by Mathematical Modeling, *American International Journal of Research in Science, technology Engineering & Mathematics*, **20(1)** (2017), 44-49.
8. V. Upadhyay, *Some Phenomena in Two Phase Blood Flow*, Ph. D. Thesis, Central University, Allahabad, (2000), 123.
9. V. Upadhyay and P N Pandey, A Power Law Model of Two Phase Blood Flow in Arteries Remote from the Heart, *Proceedings of the Third Conference of International Academy of Physical Sciences*, (1999/ 2000), 325- 330.
10. V. Upadhyay and P. N. Pandey, Newtonian Model of Two – Phase Pulsatile Blood Flow in Aorta and Arteries Proximate to the Heart, *Proceedings of the Third Conference of International Academy of Physical Sciences*, (1999/ 2000), 235 – 242.
11. Gustafson and R. Daniel, *Physics, Health and the Human Body*, Wadsworth, 1980.
12. Glenn Elert, *Viscosity, the physics*, 2010.
13. J. T. Ottesen, M. S. Olufsen and J. K. Larsen, *Applied Mathematical Models in Human Physiology* (Table 2.3: Physiological data for the various parameters in the circulatory system Caro *et al.*, 1978).
14. Berkow and Robert, *Merck Manual of Medical Information*, Whitehouse Station, N J: Merck Research Laboratorie, 1997.