

Multiple Regression And Multi-Server Queueing Approach to Public Health Care Systems

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Abstract: In this paper, we investigate the role of queueing models in evaluating and assessing the performance of health care systems. We propose multiple regression technique as machine learning for a multi-server queueing model to analyze the public health care system in order to build a mathematical relationship between predictor-variables and response variable. We apply the multiple regression analysis technique to predict the choice of patients regarding choosing public health care system on the basis of various predictor-variables represented as capacity of health care system, number of health experts, size of waiting line of patients, average cost of treatment etc.

Keywords: Queueing Models, Public Health Care System, Multiple Regression, Null and Alternative Hypothesis.

1. Introduction

Since World War II, the models of operations research have been used as powerful apparatus to fix the problems arising out of real life situations of industry, business, applied science and engineering including public health care systems. Kunwar and Srivastava¹ have attempted to explore the applications of operations research in the area of public health care system. Operations Research has been endowed with several problem solving methods and models. Queueing theory is one of them which has enormous potential to deal with problems wherever waiting lines are formed in service providing system in order to provide and distribute the services as health care to the patients. To this end, we employ a variety of analytical procedures to

ensure a sound decision making across a variety of applicable systems. After the fifties, health care in operations research has become a good option, vide as Fomundam and Herrmann², Green³⁻⁶, Hall *et al.*⁷, Mala and Varma⁸⁻⁹.

Operations research involves a diverse range of techniques and models that have the ability to analyze performance measures of the health care system and accordingly suggests improvement of the system by reorganizing its components in the service organizations of the society. The queued models of operational research focus on the evaluation of various service providing models, often used for improvement in various service delivery systems such as telecommunications, banks, airlines and various health care systems (HCSs), vide as Singh *et al.*¹⁰, Erlang¹¹, Foad and Kamath¹², Khanna¹³, Venkateswarlu and Murty¹⁴.

The selection of a patient's healthcare system depends on its satisfaction. How health care systems are providing services to the patient has an impact on whether the healthcare system's service is delivered in a timely and inefficient manner; and whether it is patient-centered quality delivery of care system. Physicians as medical specialists are the most important element of healthcare system that can analyze and improve healthcare system providing quality health services to patients and patient satisfaction about hospital services etc. Patient satisfaction about hospital services: a study from the outpatient department of tertiary care hospital has been conducted by Sanjana *et al.*¹⁵.

Traditionally, the healthcare system has been a noble business that aims to provide patient health care service, which has now turned into a health care industry. Gradually, the healthcare system has evolved over the years and into the corporate sectors. Its nature is constantly changing after the establishment of nursing homes and hospitals and the entry of health insurance to pay in the form of third parties; and online health care systems available to patients. The patient is considered a customer of the healthcare system, so he can buy this service by paying money. The patient has the right to get the service in exchange for money and thus, the patient can question the quality of health services. There is a need for qualitative interventions to maintain health care selection at high priority to corporate and government health services, vide as Adeleke *et al.*¹⁶, Agarwal and Singh¹⁷, Bailey¹⁸, Kritchanai and Hoeur¹⁹, Sariyer *et al.*²⁰, Mehendiratta²¹.

To maintain the quality of health services in various forms and keeping in mind the preferences and satisfaction of the patients, the health care organizations have to approach good professionals including doctors, paramedics and managers, fair value of health services, and cost of employees. A system oriented towards good behavior and fewer complaints

can lead to the selection of a health care system for patients. Physicians as the medical and health experts of a health care organization are the backbone of the system, if physicians are pleased then patients will also be pleased and hence the work culture environment should be ideal and patient-centered. There is a need to provide good services to doctors, which is efficient and well soothing and health experts should listen to the patient's point and understand their problems. Hospital or nursing home is a stage where the doctor and patients meet in oasis. Apartment and nursing homes are found in many cities in the number of many cities where they work in the very small distances in the local areas. The selection of health care systems for their benefits and prosperity in this situation becomes difficult, vide as Kyong *et al.*²².

Now, a trend has been redirected to discuss the models and their applications across various lines of healthcare organization to improve its quality and efficiency to meet the increasing demand of patients. If we want to increase the efficiency of systems, it becomes imperative to implement it within the mathematical analysis of waiting line theory; vide as, Ishan *et al.*²³, Iyappan²⁴, Preater²⁵, Kembe *et al.*²⁶, Mehendiratta²⁷. In the field of health care system, the application of queue models came into existence in around 1952. Queuing theory and its models have great potential to deal with the increasing costs of healthcare systems, as in the scenario of increasing demand for health care for children and the elderly. The reasons for this also include inefficient delivery and infrastructure in health facilities. Inefficiency in all manner of service delivery not only brings bad name to governance but also has a huge impact on the health status of the society.

Therefore, by attempting the queueing modeling approach, we can deduce the cost of the entire health care system and can identify inefficiencies and mathematically analyze to improve the system. At present, we experience that when we go to the service providing healthcare system we see a huge queue of patients in various hospitals and clinics, vide as Olorunsola *et al.*²⁸, Brahma²⁹, Nosek and Wilson³⁰, Rathore³¹, Sharma *et al.*³².

To deal with this situation fairly well, the data inputs collected together and simulated modeling can be used to separate the data inputs between the waiting times of patients and the service times of physicians and staff for the eventual use of appointment techniques in a variety of settings vide as Belciug and Gorunescu³³. Literature-review on the application of queueing theory and its models has been thoroughly presented and reviewed by Taha³⁴, Mishra and Yadav³⁵ in this regard. Similarly, a brief description of the use of the queueing principle in a health care system, was suggested as a multi-server

queueing model that could be used in a health care situation vide as Mustafa and Unnisa³⁶, Mukarrom *et al.*³⁷.

Recently, a trend has swung to apply queueing models in the field of public health care system to efficiently build the mathematical approach to handle the issues related to public health care system. Review of literature reveals that so far no sufficient work has been done in this area and, therefore, this motivates us to analyze queueing model approach to public health care system by applying multiple regression technique. Very recently, Mishra *et al.*³⁸ have applied artificially intelligent queueing modeling approach to the health care systems and have predicted about the choice of patients in order to respond to various health care systems consisting of number of health experts, the size of waiting line, cost of the treatment and capacity of public health care system as input variables.

In this paper, we propose multiple regression technique as machine learning for a multi-server queueing model to analyze the public health care system in order to build a mathematical structure as relationship between response and input variables. This mathematical relation is also used to predict the choice of patient based on certain input variables connected with public health providing systems. We, in this case, apply the multiple regression analysis technique to predict the choice of patients regarding the selection of public health care system on the basis of various predictor-variables represented as number of health experts, size of waiting line, average cost of treatment and capacity of PHCS. Null hypothesis of no impacts on response variable (choice of patients) is defined and which is later subjected to test with alternative hypothesis that the predictor-variables are statistically significant and impactful. Main advantage of applying the multiple regression technique is that it properly estimates the coefficients of each independent variable which is quantified to really assess the impact of input variables on the output or response variable.

2. Basic Structure of Queue and PHCS

Fundamental aspects are given as under.

2.1 Single Server Queueing Model

In a single server queue model, medical expert as server of PHCS with other related facilities is to provide treatment services for patients. It follows the laws of queued discipline and distribution of its suggested possibilities.

Here, we consider an infinite resource population with discipline where patients emanate to join the queue of PHCS, where both the inter-arrival times and service time follow exponential distributions with arrival rate per unit

time (λ) of patients and treatment rate per unit time (μ) of health expert when it is assumed that there are n patients in the system.

Performance Measures Connected with Waiting Phenomenon of PHCS

The followings are important measures of PHCS.

Traffic congestion (Patients crowd) (p_1) =	$\frac{\lambda}{\mu}$
Probability of system is idle (PHCS has no work to perform) (p_0) =	$1 - \frac{\lambda}{\mu}$
Probability of n units of patients in the system (p_n) =	$\left(\frac{\lambda}{\mu}\right)^n p_0$
Expected number of customers (patients) in the system (L_s) =	$\frac{\lambda}{\mu - \lambda}$
Expected number of customers (patients) in the queue (L_q) =	$\frac{\lambda^2}{\mu(\mu - \lambda)}$
Average waiting time of a customer (patient) in the system (W_s) =	$\frac{1}{\mu - \lambda}$
Average waiting time of a customer (patients) in the queue (W_q) =	$\frac{\lambda}{\mu(\mu - \lambda)}$
Expected length of non empty queue (L_{nq}) =	$\frac{\mu}{\mu - \lambda}$
Probability that s or more than s customers (patients) in the system ($p \geq s$) =	$\left(\frac{\lambda}{\mu}\right)^s$
Probability that more than s customers (patients) in the system ($p > s$) =	$\left(\frac{\lambda}{\mu}\right)^{s+1}$
Probability that at least one customer (patient) in the system ($p = s$) =	$\left(\frac{\lambda}{\mu}\right)^2$
Waiting cost of customer or patient =	$L_s \cdot C_w$

C_w is a waiting cost of patient per unit per unit time.

2.2 Multi-Server Queueing Model

There are two or more service facilities in parallel to provide the same service in multi - server queue having multi health experts. All customers (patients) of a waiting line can be served by more than one server (public health expert). Assume that there are m channels or servers (public health experts) arranged in parallel and there are n customers (patients) in the system.

Patients emanate to join the queue of PHCS, where both the inter- arrival times and service time follow exponential distributions with arrival rate per

unit time (λ) of patients and treatment rate per unit time (μ) for any single health expert assuming that experts are identical in capability of providing the treatment as service.

Performance Measures Connected with Waiting Phenomenon of PHCS

The measures are given as under.

Expected number of customers (patients) in the system (L_s) =	$\frac{\lambda \mu \left(\frac{\lambda}{\mu}\right)^m}{(m-1)! (m\mu - \lambda)^2} p_0 + \frac{\lambda}{\mu}$
Expected number of customers (patients) waiting in the queue (L_q) =	$\frac{\lambda \mu \left(\frac{\lambda}{\mu}\right)^m}{(m-1)! (m\mu - \lambda)^2} p_0$
Expected number of customers (patient) waiting in the queue (W_s) =	$\frac{\mu \left(\frac{\lambda}{\mu}\right)^m}{(m-1)! (m\mu - \lambda)^2} p_0 + \frac{1}{\mu}$
Average waiting time of a customer (patients) in the queue (W_s) =	$\frac{\mu \left(\frac{\lambda}{\mu}\right)^m}{(m-1)! (m\mu - \lambda)^2} p_0$

where
$$p_0 = \frac{1}{\sum_{n=0}^{m-1} \frac{\left(\frac{\lambda}{\mu}\right)^n}{n!} + \frac{\left(\frac{\lambda}{\mu}\right)^m}{m!} \frac{m\mu}{(m\mu - \lambda)}}$$

Service (treatment) cost of customer (patient) = $m\mu.C_s$, C_s is a treatment cost of patient per unit per unit time, when n is less than equal to m .

Service (treatment) cost of customer (patient) = $n\mu.C_s$, C_s is a treatment cost of patient per unit per unit time, when n is greater than m .

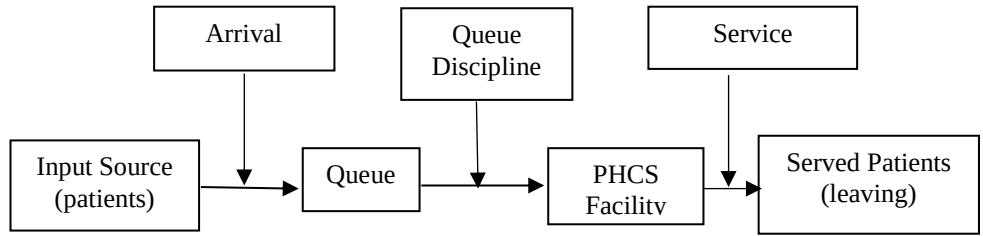


Fig.1: Design of Queuing System

We consider Markovian queueing model, in which both arrival of patients and service of treatment follow the probability law with queue discipline of first come and first served having a single server of health expert and an infinite capacity. There are several operational features or performance measures of waiting phenomenon of the queue model such as traffic congestion of patients, expected number of customers (patients) in queue and systems; waiting time for a patient expected in queue and system; and service utilization factor or busy period etc.

3. Model and Methods

Regression analysis involves many techniques for modeling and analyzing multiple variables when a dependent variable focuses on the relationship between one or more independent variables. More specifically, regression analysis helps us to understand how the specific value of dependent variable changes when one of independent variables varies; while others independent variables are constant. Regression analysis is widely used for predicting and forecast, where it is overlapped enough with the area of its machine learning. The use of regression analysis is also done to understand as to which independent variables are related to dependent variables, and to explore the forms of these relations. In restricted conditions, regression analysis can be used to estimate the relations between the independent and dependent variables; vide as Chatterjee and Hadi³⁹, Draper and Smith⁴⁰, Kutner *et al.*⁴¹.

3.1 Multiple Regression Analysis

Multiple regression analysis is a statistical technique that uses many explanatory or predictor variables to predict the result of a reaction variable. The goal of multiple linear regression (MLR) is to model the linear relation between the explanatory or predictor variable and the response (dependent) variable give here mathematical part of multiple regression vide as Uyanik and Guler⁴².

The multiple regression model equation is

$$y_i = b_0 + b_1x_{i1} + b_2x_{i2} + b_3x_{i3} + \dots + b_jx_{ij} + \varepsilon$$

where, $E(\varepsilon) = 0$ and $Var(\varepsilon) = \sigma^2$.

And also, it is assumed that ε is normally distributed.

y_i = dependent variable

x_{i1} = explanatory or predictor variables

b_0 = constant term

b_j = denotes slope coefficients for each explanatory or predictor variable

ε = denotes the model's error term (Residuals)

Regression analysis is used to discuss the relationship between response variable and predictor variables. Predictor variables are also known as independent variables or as input variables. Similarly, response variable is also known as outcome variable. With the help of regression analysis, we estimate impact of predictor-variables on the response variable. On the response variable, regression analysis measures the impact of relation by various coefficients of independent variables.

Multiple regression is a kind of a machine learning algorithm which is very popularly used for estimating the response or output variable based on

input variables. This technique is mainly used for three pronged advantages. The first one is establishing relationship between a dependent and more than one independent variables. Secondly, it is used to predict the dependent or response variable based on input variables and thirdly, it is very frequently used in the analysis of time series.

Regression analysis is always intended to develop mathematical expression between response variable and predictors. In this mathematical expression, we have regression coefficients or weights associated with each predictor which are determined in such a way so that error in the estimation of response variable is always minimized.

Different coefficients of predictors have been computed by using R software which tend to indicate the contribution towards the relationship which may be positive or negative of each and every predictor –variable with response variable as choice of patients.

Multiple regression as machine learning is a case of supervised learning which if we compare with artificial neural network it is both supervised and unsupervised learning which we have already used in a previous problem of public health care system as only supervised learning, vide Mishra *et al.*⁴³.

We use sampled data of choice collected by primary data collection method-interview (direct & indirect) through localized inquiry obtained from Mishra *et al.*⁴⁴. There are six predictor variables and one response variable. Predictor-variables include capacity of PHCS, service or treatment cost, size of waiting line of patients, servers or public health care experts. Response variable is considered as choice or satisfaction of patients.

3.2 Definition and Design of Hypotheses

The hypothesis is a perception that is created on the basis of some evidence. This is the initial point of any inquiry that changes the research questions in prediction. It includes components such as relation between variables. A research hypothesis is a hypothesis that is used to test the relationship between two or more variables.

Null hypothesis is designed as a hypothesis of no difference which is always tested for possible rejection under the assumption that it is true. It is represented by H_0 . It is always tested against an alternative hypothesis represented by H_1 which is always against the null hypothesis.

Here, H_0 : there are no difference i.e. no impact of predictor-variables on response one.

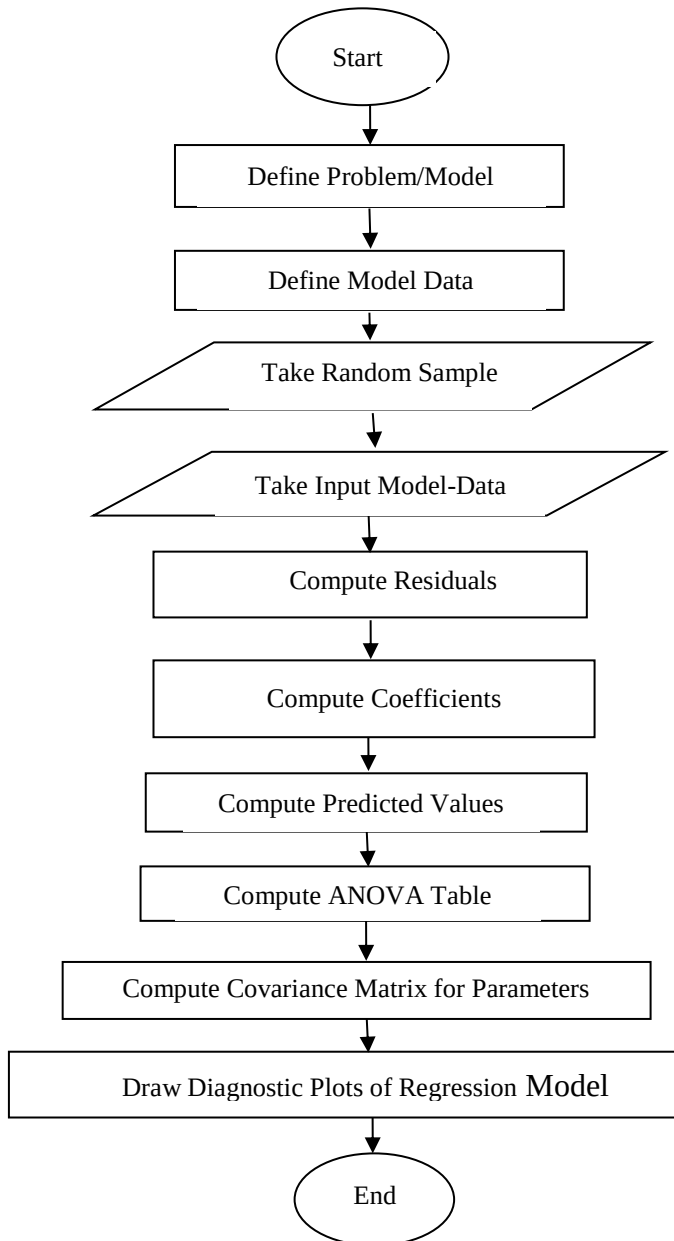
Later, it will be tested for statistical significance. Statistically significant result shows there is an impact of predictor variable on response one otherwise it leaves no impact of predictor-variables on response one.

For analysis of variance for various predictor-variables, we define null hypothesis and alternative hypothesis as

H_0 : All predictor variables have equal means.

H_1 : All predictor variables have no equal means.

3.3 Flowchart of Computing Algorithm



4. Computing of Results and their Tabulation

The following results are given after using R software on the data-information using Mishra *et al.*³⁰ as given below.

Table 1: Basic Data-Information

S.No.	Choice	Capacity	Service cost	Waiting line	Servers
1	0	37	19	29	8
2	1	67	14	29	7
3	1	79	21	9	2
4	1	63	15	39	10
5	0	51	13	39	11
6	1	75	17	19	5
7	1	55	15	9	2
8	0	39	19	19	5
9	1	53	24	11	8
10	0	69	25	19	5
11	0	38	20	30	9
12	1	68	15	30	8
13	1	80	22	10	3
14	1	64	16	40	11
15	0	52	14	40	12
16	1	76	18	20	4
17	1	56	16	10	3
18	0	40	20	20	6
19	1	54	25	12	9
20	0	70	26	20	6
21	0	39	21	31	10
22	1	69	16	31	9
23	1	81	23	11	4
24	1	65	17	41	12
25	0	53	15	41	13
26	1	77	19	21	5
27	1	57	17	11	4
28	0	41	21	21	7
29	1	55	26	13	10
30	0	71	27	21	7

Table 2: Residuals

Min	1Q	Median	3Q	Max
-0.58250	-0.07015	0.01475	0.13871	0.47654

Table 3: Coefficients

	Estimate	Std. Error	t value	Pr(> t)
Intercept	1.207306	0.477074	2.531	0.018054 *
capacity	0.025757	0.004267	6.036	2.63e-06 ***
sercost	-0.092780	0.017955	-5.167	2.42e-05 ***
waitline	-0.060934	0.011667	-5.223	2.10e-05 ***
servers	0.146160	0.036873	3.964	0.000544 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.2854 on 25 degrees of freedom. Smaller residual standard error shows prediction is better. Multiple R-squared: 0.7172, Adjusted R-squared: 0.672 .F-statistic: 15.85 on 4 and 25 DF, p-value: 1.387e-06

Table 4: Coefficients of (fit)

Intercept	Capacity	Service cost	Waiting line	Servers Max
1.20730609	0.02575694	-0.09278007	-0.06093433	0.14616035

Table 5: Confidence Interval (fit level=0.95)

	2.5 %	97.5 %
(Intercept)	0.22475372	2.18985847
Capacity	0.01696877	0.03454511
Service cost	-0.12975992	-0.05580022
Waiting line	-0.08496205	-0.03690661
Servers	0.07021980	0.22210090

Table 6: Predicted Values

S. No.	Predicted Values	S. No.	Predicted Values	S. No.	Predicted Values
1	-0.20032107	11	-0.18211817	21	-0.16391528
2	0.89012718	12	0.90833008	22	0.92653297
3	1.03763481	13	1.05583770	23	1.07404060
4	0.52345712	14	0.54166001	24	0.55986291
5	0.54609430	15	0.56429720	25	0.58250009
6	1.13486509	16	0.86074728	26	0.87895018
7	0.97614861	17	0.99435151	27	1.01255440
8	0.02205504	18	0.04025794	28	0.05846083
9	0.84470755	19	0.86291044	29	0.88111334
10	0.23808289	20	0.25628578	30	0.27448867

CV computed = 0.03415

Table 7: Residuals

S. No.	Residuals Values	S. No.	Residuals Values
1	0.200321069	16	0.139252715
2	0.109872817	17	0.005648491
3	-0.037634807	18	-0.040257937
4	0.476542881	19	0.137089558
5	-0.546094303	20	-0.256285780
6	-0.134865087	21	0.163915280
7	0.023851385	22	0.073467029
8	-0.022055042	23	-0.074040596
9	0.155292452	24	0.440137092
10	-0.238082886	25	-0.582500091
11	0.182118174	26	0.121049821
12	0.091669923	27	-0.012554403
13	-0.055837701	28	-0.058460831
14	0.458339986	29	0.118886663
15	-0.564297197	30	-0.274488675

Table 8: Analysis of Variance (Response: Choice)

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
capacity	1	2.50231	2.50231	30.7233	9.213e-06 ***
Service cost	1	0.34271	0.34271	4.2078	
Waiting line	1	1.03907	1.03907	12.7576	0.0014740 **
Servers	1	1.27975	1.27975	15.7127	0.0005435 ***

Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Table 9: Covariance Matrix for Model Parameters

Intercept	Intercept	Capacity	Service Cost	Waiting line	Servers
	0.2275996379	-9.918801e-04	-6.259172e-03	-2.488067e-03	1.720316e-03
Capacity	-0.000991880	1.820779 e-05	-1.323055e-05	-1.203269e-05	6.087058e-05
Sercost	-0.006259172	-1.323055e-05	3.223966e-04	1.413461e-04	-3.375189e-04
Waitline	-0.002488067	-1.203269e-05	1.413461e-04	1.361085e-04	-3.717127e-04
Servers	0.0017203160	6.087058e-05	-3.375189e-04	-3.717127e-04	1.359589e-03

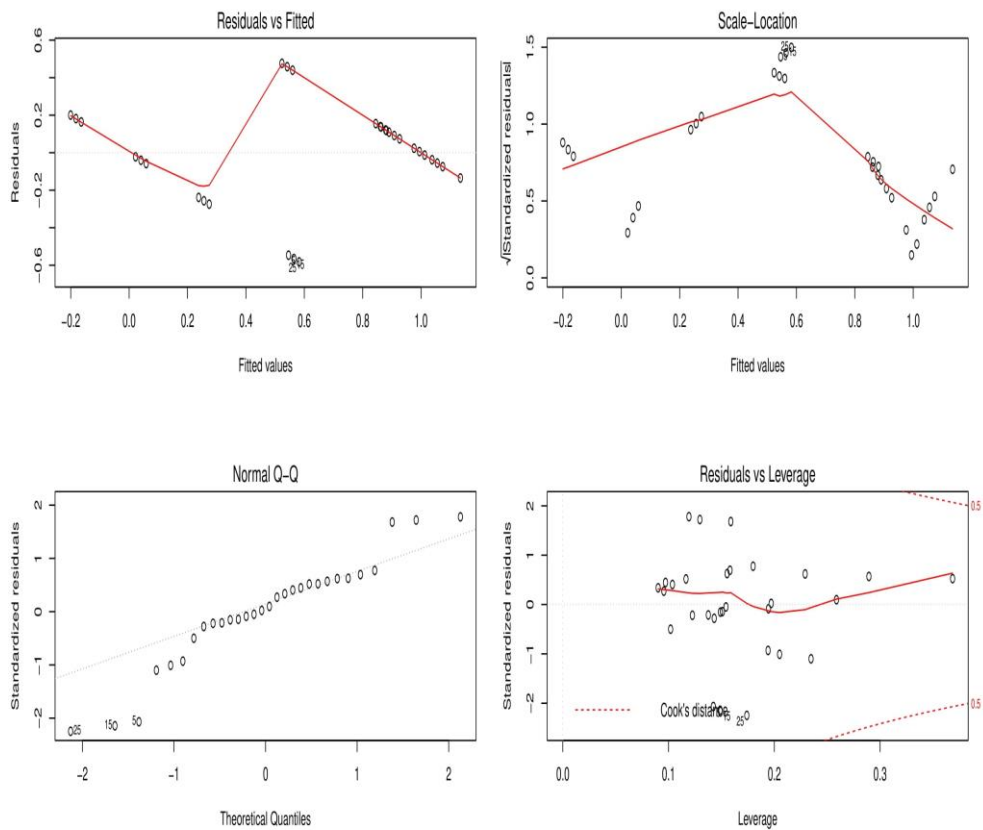


Fig. 2: Diagnostic Plots for Present Model

5. Discussion and Conclusion

Table 1 provides us input values of possible variables for analyzing the PHCS. Table 2 depicts minimum, first and second quartiles and maximum values of residuals. In view of observations given in table 3, the predictor-variables including capacity of PHCS, treatment cost, size of waiting line and number of health care experts are highly significant with respect to response variable i.e. choice of the patient. Out of four variables, two variables are highly and positively significant and two variables are highly and negatively significant with respect to response variable. Capacity of PHCS and number of PHCS experts are highly and positively significant; and where as treatment cost and size of waiting line are highly and negatively significant. Thus, null hypothesis is summarily rejected. In view of the section of multi-server queue and its performance measures, two predictor-variables: capacity of PHCS and

number of PHCS experts are highly and positively significant i.e. they are positively correlated. This is fairly commensurate with real life situations of public health care services and patients' choice and satisfaction. But about other two predictor-variables: treatment cost and size of waiting line are highly and negatively correlated and because of it, this becomes conspicuous for researchers to explore more. If we look at the section of multi-server queue and its performance measures, whenever waiting line increases, the waiting cost of a patient increases and this point falls against the choice and satisfaction of patients. Similarly, whenever treatment cost increases it costs dear to the pocket of a patient which against the choice and satisfaction of a patient. Multiple R squared given below the table 3 reveals the proportion of variance attached with response variable of choice of patients of multiple regression models is possible to be explained by predictor-variables. It lies in between zero and one. Its value is computed here 71 percent which is a justifiable value for this model. Adjusted R square is a kind of modified multiple R squared which represents its decreasing and increasing values as compared to multiple R squared. If new variables are added to the model and no any effect is observed, the adjusted R squared is decreased. In this model, adjusted R squared is decreased and that comes out to be 67 percent from 71 percent. Table 4 provides the coefficients of fit for predictor-variables. Confidence intervals for ninety five percent are presented in the table 5.

We also draw the observations from the table 6; the fitted values against the choice of patients are good and quite valid because its coefficient of variation is much less around 0.03415. Table 7 presents individual residuals one by one for the response variable of choice of patients. Analysis of variance given in the table 8 shows that predictor-variables have different significant impacts on response variable of choice of patients because their respective means are different. In other words, we say that our null hypothesis of no equal means of predictor-variables is summarily rejected. Table 9 predicts the covariance matrix for model parameters. This individual covariance ideally must be much less between other variables as well as itself. It is evident from the table 9 that the covariance computed between parameter is much –much less leading to validity of the model.

In figure 2, the diagnostic plots are drawn by R software. In the plot of residual vs fitted, it is shown that residuals (difference between observed and estimated values of response variable) characterize a scatter-diagramme to identify non-linearity and outliers etc. which is fairly evident from the plot itself.

In the scale-location plot, observations are made whether residuals are equally distributed along the values of predictor-variables. We also intend to

test the violation of homoscedasticity (assumption of equal variance). Present model is good because points are equally and randomly distributed along the horizontal line. Scale here refers to a series of marks at fixed intervals. Therefore, this plot is also called as spread (marks) location.

Normal Q-Q plot intends to compare the quantiles (it may be quartile, quintile, percentile in particular) of a data distribution with the quantiles of a theoretical standardized distribution. A straight line with 45 degree on x-axis shows a normal distribution in nature. Lower part down the line indicates a negatively skewed normal distribution. Similarly, upper part above the line indicates a positively skewed normal distribution. If lower and upper parts of a line are down and above respectively, it indicates too peaked in middle of a normal distribution. In other words, we can say that normal Q-Q plot is a measurement of normality nature of the observations.

Last plot of residuals vs leverage is intended to tell us to compare residuals with leverage. In this section of diagnostic plot, leverage means to measure a distance (away) of values of independent variables (predictor variables) from other observations. This diagnostic plot of residuals vs leverages endeavours to identify influential points on the regression model. Outliers with large residuals are easily identified. In the plot, on y-axis, any point above 0.5 has a high Cook's distance value. Similarly, on x-axis, any point beyond 0.167 has a high value of leverage. A point with high Cook's distance is considered to have a closer analysis and can be deleted. A general rule of thumb is that observations (data points) with Cooks distance having more than three times the mean can be detected as a potential outlier. In this plot of the paper, highest Cooks distance is 0.5 and within the plot. For observations in the plot, mean is 0.01475 and it's three times comes out to be 0.04425 which is less than 0.5000 but very near to it. Such points situated at top in the plot, are potential outliers. This finally draws the concluding remark that the higher the leverage value of an observation, the more potential it has to impact the fitted model.

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